EVALUATION OF SHEAR AND FLEXURAL DEFORMATIONS OF FLEXURAL TYPE SHEAR WALLS

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SUMMARY

This paper presents an evaluation method of distributing the total deformation of a shear wall into the flexural and shear deformations. The paper also proposes an analytical method of evaluating flexural and shear deformations of a flexural type shear wall. In the analysis, such a shear wall is represented as a truss system having a non-prismatic elasto-plastic truss member. The analytical results agree well with results of the shear wall tested in the US-Japan Cooperative Research Program.

INTRODUCTION

In order to predict the inelastic response of R/C structures under dynamic earthquake loading, hysteretic behavior of their structural components, i.e. beams, columns, and shear walls, must be evaluated appropriately. Various hysteretic load (moment) versus deformation (curvature) models, such as the Takeda model, have already been proposed for beams and columns. Hysteretic behavior of shear walls, on the other hand, remains unclear in many respects. Shear deformation of flexural type shear walls is one of those yet to be determined.

Many experimental studies of shear walls have been carried out, but most of their load-deformation data are presented in terms of the load versus total deformation. Very few data refer to the shear and flexural deformations primarily because of the difficulty in separating the total deformation into these two deformations. Also, practical theory to analyze shear and flexural deformations for flexural type shear walls has not been presented yet.

FLEXURAL DEFORMATION AND SHEAR DEFORMATION

In shear wall tests, shear deformation is sometimes conventionally estimated from changes in the length of the two diagonals. However, the shear deformation given by this method contains flexural deformation because of the existence of a moment gradient along the height of shear walls.

This chapter describes the relationships between horizontal and vertical displacements at the four corners of a shear wall, and flexural deformation, shear deformation, and expansion. It also proposes a simple method to evaluate each deformation.

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COMPONENTS OF DEFORMATION

Displacements of a shear wall subjected to a lateral load are illustrated in Fig. 1. With an aim towards simplification of the development of equations, horizontal and vertical displacements at the base are modified to be zero. It is also assumed that these displacements can be represented by three components, i.e. shear deformation (which includes slip), flexural deformation, and expansion shown in Fig. 2. The following relations are also assumed.

$$u_{LS} = u_{RS} = u_S \qquad \text{-----}(1), \qquad u_{RE} = -u_{LE} = u_E \qquad \text{-----}(2)$$

$$u_{LB} = u_{RB} = u_B \qquad \text{-----}(3), \qquad v_{RE} = v_{LE} = v_E \qquad \text{-----}(4)$$

The shear deformation can be given by Eq. (5) or Eq. (6) from the relationships between these displacements and those shown in Fig. 1.

$$u_S = \frac{d}{2\ell}(\delta_1 - \delta_2) - \{u_B + \frac{h}{2\ell}(v_R - v_L)\}$$

$$= \frac{d}{2}(\delta_1 - \delta_2) - \{u_B - \frac{h}{2}\theta) \qquad \text{-----}(5)$$

or, $$u_S = \frac{1}{2}(u_R + u_L) - u_B \qquad \text{-----}(6)$$

where,

$$\theta = \frac{1}{\ell}(v_L - v_R) = \frac{1}{\ell}(v_{LB} - v_{RB}) \qquad \text{-----}(7)$$

EXPRESSION OF FLEXURAL DEFORMATION AND SHEAR DEFORMATION IN TERMS OF ROTATION

Eqs. (5) and (6) imply that in order to estimate the shear deformation, flexural deformation must be inevitablly estimated with sufficient accuracy. Flexural deformation can be assumed to be given by Eq. (8).

$$u_B = h\int_0^1 \theta_\eta d\eta = \int_0^1 \frac{h}{\ell}(v_{L\eta} - v_{R\eta})\, d\eta \qquad \text{-----}(8)$$

where,

$\eta = y/h$

θ_η = rotation of a shear wall at η

$v_{R\eta}$, $v_{L\eta}$ = vertical displacements of right-hand side and left-hand side boundary columns at η, respectively.

An example of the distribution of θ_η along the height of a cantilever shear wall is illustrated in Fig. 3. Curvature is dramatically increased around the base where the occurrence of cracks and yielding of steels is usually observed.

Eq. (8) can be rewritten by Eq. (10) using a new factor α defined by Eq. (9).

$$\alpha = \frac{\int_0^1 \theta_\eta d\eta}{\theta} \qquad \text{-----}(9), \qquad u_B = \alpha\theta h \qquad \text{----}(10)$$

α is the ratio of the shaded area to that surrounded by solid lines ABCD in Fig. 3. Therefore, it is reduced to the prediction of this ratio to evaluate flexural deformation. With regard to α, there is generally a relationship as shown in Eq. (11), when the point of contraflexure is located above the story being discussed of a shear wall.

$$1/2 < \alpha < 1 \qquad \text{----(11)}$$

By substituting Eq. (10) into Eq. (5) or Eq. (6), shear deformation is given as follows:

$$u_S = \frac{d}{2\ell}(\delta_1 - \delta_2) - (\alpha - \frac{1}{2})\,\theta h \qquad \text{----(12)}$$

or, $$u_S = \frac{1}{2}(u_R + u_L) - \alpha\theta h \qquad \text{----(13)}$$

Eq. (12) proves that shear deformation given approximately by only changes in the length of diagonals, corresponds to shear deformation given by 1/2 of α. However, if 1/2 of α is assumed, the shaded area in Fig. 3 would be approximately represented by the area of the triangle ABC, which occurs only in the case of pure bending, and this therefore results in an overestimation of shear deformation.

Fig. 3 is nothing but a conceptual illustration, but it does not seem to be difficult to evaluate α with certain accuracy because of the general behavior of rotation. For example, if either rotation θ_M, or right-hand side and left-hand side vertical displacements at mid-height of the story being discussed of a shear wall were measured, considerable improvement of accuracy in evaluation of α could be expected. Fig. 4 shows such examples. In these examples, there is generally relationships given by Eqs. (14) through (17) between approximate values α_1 and α_2 and the exact value α.

$$\alpha_1 = \frac{1}{2}(0.5 + \frac{\theta_M}{\theta}) \text{----(14)}, \qquad \alpha_2 = \frac{1}{4}(0.5 + 3\frac{\theta_M}{\theta}) \text{----(15)}$$

$$0.75 < \alpha_1 / \alpha < 1 \quad \text{----(16)}, \qquad 0.875 < \alpha_2 / \alpha < 1.167 \quad \text{----(17)}$$

The maximum possible error amounts to 25% for α_1 and 16.7% for α_2. However, α_2 seems to estimate α with good accuracy as is expected from Eq. (17) and Fig. 4.

FLEXURAL AND SHEAR DEFORMATIONS OF FLEXURAL TYPE SHEAR WALLS

In this chapter, stress of flexural reinforcing bars of boundary column under tension is expressed by H. Bachmann's theory[1]. Shear wall is represented as a truss system having a non-prismatic truss member whose cross sectional area is determined by the stress along the height of the boundary column. Then flexural and shear deformations are evaluated.

TRUSS MODEL OF SHEAR WALLS

Fig. 5 shows crack pattern of a three story shear wall tested in US-Japan Cooperative Research Program[2]. When reinforcing bars of a boundary column under tension yield, cracks developed in the tension side column extend obliquely to the bottom part of the compression side column through in-filled panel wall.

In this paper, shear walls after yielding of flexural reinforcing bars of the boundary column under tension is expressed as a truss model shown in Fig. 6. The main object of the analysis is placed to deformations at the first story, so truss members above the first story are assumed to be rigid. It is also assumed that shortening of the boundary column under compression is neglegible, and tensile chord member (boundary column under tension) is expressed as a non-prismatic elasto-plastic member whose cross sectional area is given by Eq. (18) so as for exsisting stress to satisfy Eq. (19) which is determined from equilibrium of the forces at an inclined cracked surface and at a base (see. Fig. 7)[1].

$$A_\eta = \frac{\sigma_0}{\sigma_\eta} A_0 = \frac{\sigma_0}{\sigma_0 - r\sigma_y \eta^2} A_0 \qquad \text{----(18)}$$

$$\sigma_\eta = \frac{T_\eta}{a_t} = \sigma_0 - r\sigma_y \eta^2 \quad \text{----(19)}, \qquad r = \frac{a_w h}{2a_t \ell} \qquad \text{----(20)}$$

where,

A_η, A_0 = cross sectional areas of the tensile chord member, at y-height and at base, respectively, of the assumed truss system

a_t = sum of the cross sectional area of reinforcing bars of the boundary column under tension

a_w = sum of horizontal reinforcement area of the in-filled panel wall

σ_η, σ_0 = stress of reinforcing bars of the boundary column under tension at y-height and at the base, respectively

σ_y = tensile yielding stress.

The deformation of this truss system consists of deformation due to stretching of the tension side column, that due to shortening of the diagonal compression member, and that due to stretching of the beam (horizontal tensile member). Figs. 8 and 9 show their components. There are following relationships between these components.

$$u = u_S + u_B = u_1 + u_2 + u_3 / 2 \qquad \text{----(21)}$$

$$u_B = \alpha\theta h = \alpha u_1 \quad \text{----(22)}, \qquad u_S = u_{S1} + u_{S2} + u_{S3} \qquad \text{----(23)}$$

$$u_{S1} = (1-\alpha)\,\theta h = (1-\alpha)\,u_1 = \left(\frac{1}{\alpha} - 1\right) u_B \qquad \text{----(24)}$$

$$u_{S2} = \frac{d}{\ell}\,\delta \quad \text{----(25)}, \qquad u_{S3} = u_3 / 2 \qquad \text{----(26)}$$

where,

u = average horizontal displacement of the right-hand side and left-hand side horizontal displacements of the first story (sum of flexural and shear deformations)

u_{S1}, u_{S2}, u_{S3} = shear deformation due to stretching of the tension side column, that due to shortening of the diagonal compresion member, and that due to stretching of the beam, respectively.

VERTICAL DISPLACEMENT OF THE BOUNDARY COLUMN UNDER TENSION AND FLEXURAL DEFORMATION

Vertical displacement v of the boundary column under tension is given by Eq. (27), and α and flexural deformation u_B are given by Eqs. (28) and (29).

$$v = h\int_0^1 \varepsilon_\eta \, d\eta \quad \text{----(27)}, \qquad \alpha = \frac{u_B \ell}{vh} \qquad \text{----(28)}$$

$$u_B = h \int_0^1 \theta_\eta d\eta = \frac{h}{\ell} \int_0^1 v_\eta d\eta = \frac{h^2}{\ell} \int_0^1 \int_0^\eta \varepsilon_\eta d\eta d\eta \qquad \text{----(29)}$$

where,

ε_η = strain of tensile chord member at y height.

The ε_η is determined based on the stress σ_η given by Eq. (19).

COMPARISONS OF ANARITICAL RESULTS WITH EXPERIMENTAL ONES

Figures 11 and 12 show the comparisons between experimental and analytical results of the shear wall at first story of the full scale seven story structure tested in the US-Japan Cooperative Research Program[2]. The stress versus strain relationship for reinforcing bars of the tension side column is represented as shown in Fig. 10 according to tensile test result. The other constants such as height (up to bottom of 2nd floor slab), width of first story shear wall, amount of reinforcing bars are:

$E_S / E_{SH} = 34.8$, $\varepsilon_{SH} / \varepsilon_y = 7.03$, $h = 363\text{cm}$, $\ell = 500\text{cm}$

$\sigma_y = 3780\text{kg/cm}^2$, $a_t = 30.96\text{cm}^2$, $a_w = 25.03\text{cm}^2$

In the analytical results, u_{S2} and u_{S3} are not considered in shear deformation because their values are of little amount campared to u_{S1}. Shear and flexural deformations increase according to tip drift angle. Furthermore, the ratio of shear deformation to total deformation dramatically increases according to the increase of tip drift angle, and in the large deformation region, shear deformation is nearly half of total deformation. The analytical results agree well with experimental results.

PRIMARY CURVE OF THE LOAD VERSUS DEFORMATION RELATIONSHIP

The primary curve for the load versus flexural deformation relationship has three breaking points as shown in Fig. 13. These breaking points are flexural cracking, yielding and maximum strengths.

The primary curve for the load versus shear deformation relationship before yielding is represented by bi-linear, and that after yielding in flexure is done by a curved line which is given by Eq. (30) by considering the u_{S1} versus u_B relationship shown in Eq. (24), and u_{S2} and u_{S3}(see. Fig. 14).

$$u_S = u_{S1} + u_{S2} + u_{S3} = \left(\frac{1}{\alpha} - 1\right) u_B + u_{S2} + u_{S3}$$

$$= \left(\frac{1}{\alpha} - 1\right) \left\{ u_{By} + \frac{P - P_y}{P_u - P_y} \left(u_{Bu} - u_{By} \right) \right\} + u_{S2} + u_{S3} \qquad \text{----(30)}$$

CONCLUSIONS

Following conclusions have been reached.

1. Shear deformation is overestimated, and consequently flexural deformation is underestimated if the shear deformation is determined simply as a difference in length of two diagonals.
2. Flexural and shear deformations are estimated with excellent accuracy by using the rotation at the story mid-height of a shear wall.

3. Shear deformation increases by the rotational mechanism having a rotation center at the base of the column under compression.
4. Shear deformation significantly increases after reinforcing bars of the boundary column under tension yield. The ratio of the shear deformation to the flexural deformation is analytically determined by a truss model which has a non-prismatic elasto-plastic member.

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1) H. Bachmann, "Influence of Shear and Bond on Rotational Capacity of Reinforced Concrete Beams," Publications, International Association for Bridge and Structural Engineering, vol. 30, Part II, Zurich, 1970, pp. 11-28.

2) S. Okamoto, S. Nakata, Y. Kitagawa, M. Yoshimura, and T. Kaminosono, "A Progress Report on the Full-Scale Seismic Experiment of a Seven Story Reinforced Concrete Building-Part of the US-Japan Cooperative Program," ISSN 0453-4972, Building Research Institute, Research Paper No. 94, March 1982.

3) H. Hiraishi, M. Yoshimura, H. Isoishi and S. Nakata, "Planar Tests on Reinforced Concrete Shear Wall Assemblies - U.S.- Japan Cooperative Research Program," ISSN 0453-4972, Building Research Institute, Research Paper No. 98, Jan. 1983.

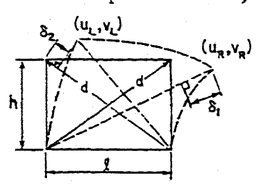


Fig. 1 Deformation of Shear Wall Subjected to Lateral Load

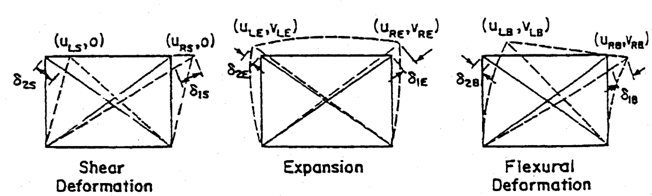


Fig. 2 Components of Deformation

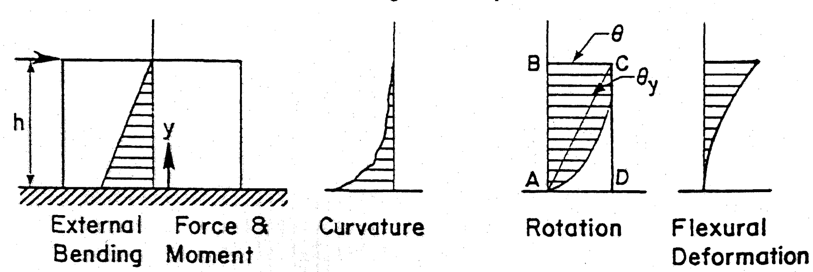


Fig. 3 Distribution of Rotation of a Cantilever Type Shear Wall (Pull out of Steel at Base is Neglected)

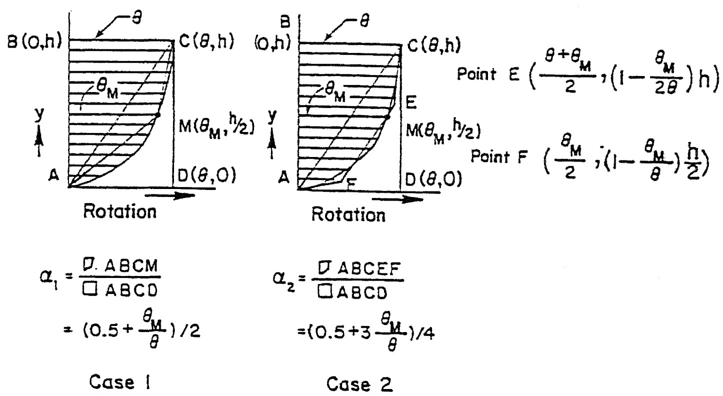


Fig. 4 Methods of Estimation for α

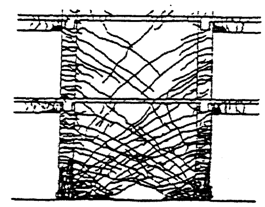


Fig. 5 Final Crack Pattern of Half-Scale Three Story Shear Wall Tests[3]

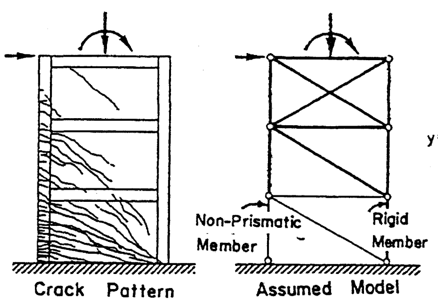


Fig. 6 Assumed Truss Model

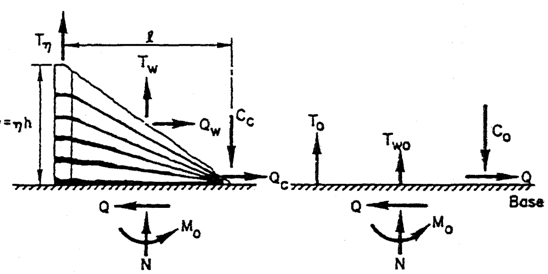
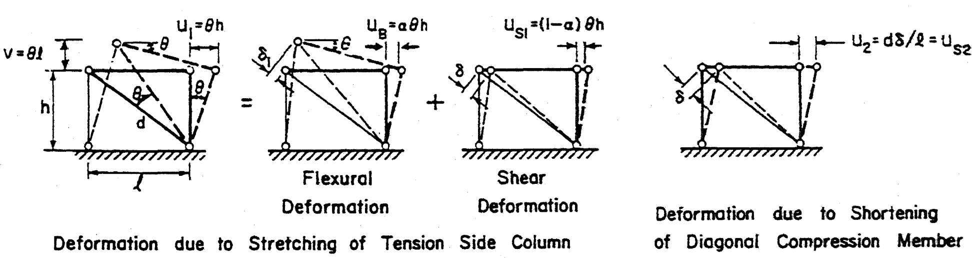


Fig. 7 Forces and Moment at a Cracked Surface and at the Base



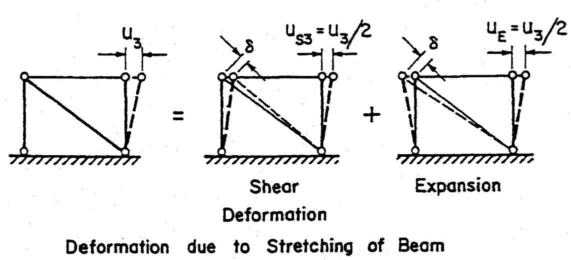


Fig. 8 Deformations due to Stretching and Shortening of Each Truss Member

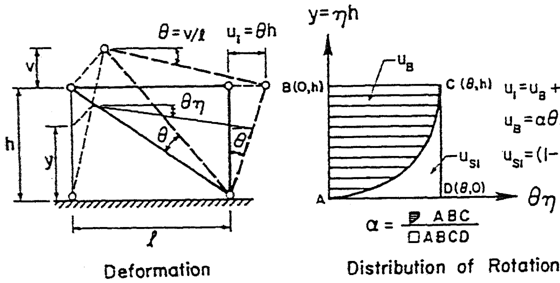


Fig. 9 Relationship of Rotation versus Flexural and Shear Deformation of u_1

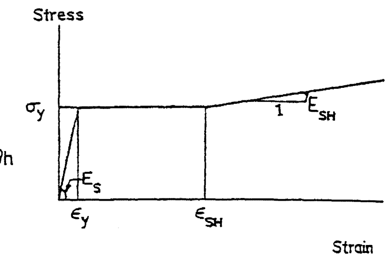


Fig. 10 Assumed Stress versus Strain Relationship for Steel

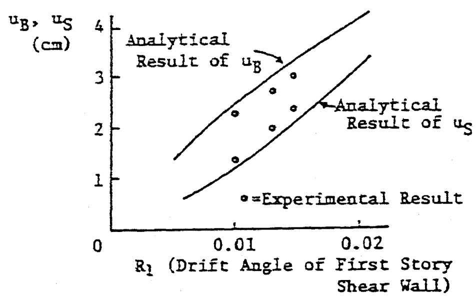


Fig. 11 Comparison of Flexural and Shear Deformation of Analytical Results with Experimental Results of Full-Scall Seven Story Building

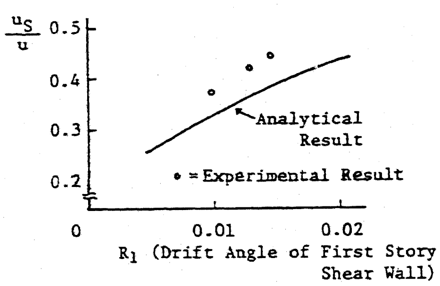


Fig. 12 Comparison of Ratio of Shear Deformation to Story Drift of Analytical Result with Experimental Result of Full-Scale Seven Story Building

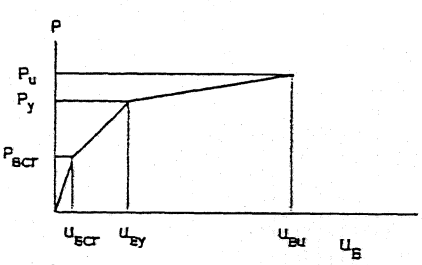


Fig. 13 Idealized Load versus Flexural Deformation Relationship

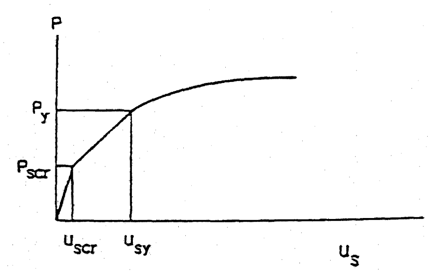


Fig. 14 Idealized Load versus Shear Deformation Relationship