

ON ULTIMATE SHEAR STRENGTH OF REINFORCED CONCRETE SHEAR UNIT WALLS

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SUMMARY

This paper presents the ultimate shear strength of reinforced concrete unit walls relating to the mechanism of shear resistance and failure. The effects of the ratio of reinforcing bar in wall, the ratio of shear wall and additional axial loading of surrounding frame to the ultimate shear strength were discussed, comparing the theoretical values with the experimental values of 28 specimens of past experiments. The formulas of ultimate shear strength modified by multiple regression analysis coincide fairly well with the values of experiments, of which the multiple correlation coefficient is 0.98.

INTRODUCTION

As an instruction of the disaster caused Tokachioki earthquake, security of durability as well as strength had been advocated for the aseismatic design of buildings in Japan and the fifth revision of "standard for structural calculation of reinforced concrete structures" was completed in 1971. In the revision, the article for the quake-resisting wall has also been revised. Although the revised formula is advanced one, substantial deviation from the experiment has been noticed not to be completely satisfactory. From the above point of view, this paper deals with a quake-resisting wall of reinforced concrete of a story and a bay and a formula for the ultimate shearing strength is obtained. Then, comparing it with experimental result, the characteristics of the resisting factor is made clear and a formula for the ultimate strength is proposed for the use of practical designing.

MODELING OF RESISTANCE AND FAILURE MECHANISM

As shown in Fig. 1, a quake-resisting wall of reinforced concrete of a story and a bay is considered to be subjected to shearing force due to external forces of tension and compression imposed simultaneously at joints in two diagonal directions.

The notation is as follows.

- t : thickness of wall plate
- l : distance between center lines of columns of surrounding frame of wall plate
- h : distance between center lines of beams of surrounding frame of wall plate
- l' : inside length of wall plate
- h' : inside height of wall plate
- D_c : total depth of column
- D_b : total depth of beam
- B_c : width of column
- B_b : width of beam

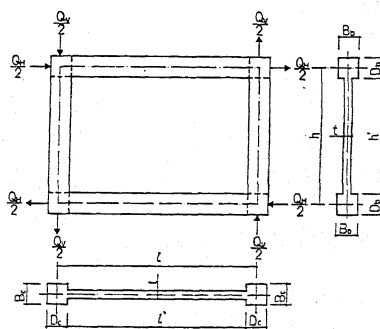


Fig. 1 Shape of Quake-Resisting Wall and External Force

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Q_H : horizontal component of external force at a joint
 Q_V : vertical component of external force at a joint

Constructing a model for resistance and failure mechanism after occurrence of cracks in a wall plate of a quake-resisting wall shown in Fig. 1, the following items of assumption are provided. (1) The angle between direction of cracks in a wall plate and horizontal or vertical direction is 45 degrees. (2) Cracks in a wall plate occurs uniformly and densely over its all area. (3) Resistance of a wall plate after occurrence of cracks owes to compression braces of concrete in the direction of cracks and yielded tension braces of reinforcement of a wall plate in the direction perpendicular to that of cracks. (4) A surrounding frame is made of straight members placed along the center lines of members. Based on the above assumption, the equilibrium of a triangular element with width of $\Delta x/\sqrt{2}$ shown in Fig. 2 gives:

$$\begin{aligned}\tau_1 &= p_s \sigma_y + \sigma_R \dots \dots \dots (1) \\ \tau_2 &= \sigma_c - \sigma_R \dots \dots \dots (2)\end{aligned}$$

where

- τ_1 : shearing stress of a wall plate expressed with tensile stress of a tension brace of reinforcement and constraining stress of a surrounding frame
- τ_2 : shearing stress of a wall plate expressed with compressive stress of a compression brace of concrete and constraining stress of a surrounding frame
- σ_y : yielding stress of shear reinforcing bar of a wall plate
- σ_c : compressive stress of a compression brace of concrete
- σ_R : constraining stress of a surrounding frame
- p_s : ratio of shear reinforcing bar of wall plate

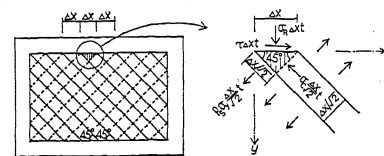


Fig. 2 Equilibrium of Compression Brace of Concrete

— Compression Brace of Concrete
 ---- Tension Brace of Reinforcement

Therefore, for $\tau_1 > \tau_2$, namely $2\sigma_R > \sigma_c - p_s \sigma_y$, compressive-shearing failure of a plate, so-called slip failure occurs and for $\tau_1 < \tau_2$, namely $2\sigma_R < \sigma_c - p_s \sigma_y$, constraining failure of a surrounding frame occurs. As the constraining failure of a surrounding frame, shearing failure and tensile-bending failure are considered. However, constraining failure of a surrounding frame is mainly shearing in general and tensile-bending failure is seldom. This paper deals solely the shear failure of a surrounding frame.

FORMULA OF THE ULTIMATE SHEAR STRENGTH

A quake-resisting wall shown in Fig. 3(a) is decomposed into those in Fig. 3(b) and 3(c) and dealt as a problem of a surrounding frame being subjected to shearing force and constraining reaction from a wall plate shown in Fig. 3(b). As shearing failure of a surrounding frame, two cases are conceivable. One is the shearing failure of beams and the other, that of columns.

Equilibrium Condition

In the beginning, shearing failure of the beam is dealt. In order to establish an equilibrium equation of a surrounding frame of Fig. 3(b), horizontal section m-m in the vicinity of joint A, B is taken and the forces acting

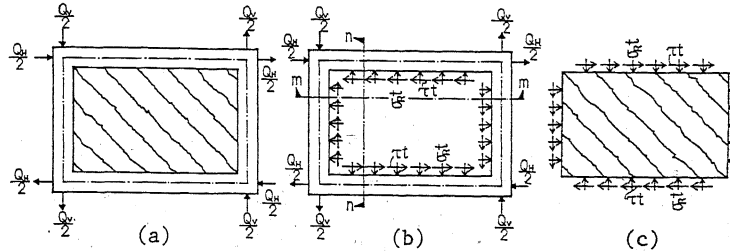


Fig. 3 Resistance-Failure Mechanism of Reinforced Concrete Quake-Resisting Wall after Occurrence of Cracks
 (a) Reinforced Concrete Quake-Resisting Wall after Occurrence of Cracks
 (b) Surrounding Frame of Reinforced Concrete Quake-Resisting Wall after Occurrence of Cracks
 (c) Wall Plate of Reinforced Concrete Quake-Resisting Wall after Occurrence of Cracks

on the section are expressed as shown in Fig. 4(a). The equilibrium equations of the force with respect to x , y -direction and the moment with respect to point B are expressed as follows:

$$Q_H = \int_0^l \tau dx + A Q_c + B Q_c \dots \dots \dots (3)$$

$$A N_{co} + B N_{co} = \int_0^l \sigma_R dx \dots \dots \dots (4)$$

$$-A N_{co} \cdot l - A M_c - B M_c + \int_0^l \sigma_R dx \cdot \alpha l - \frac{Q_v}{2} \cdot l = 0 \dots \dots \dots (5)$$

where

- $A Q_c (B Q_c)$: shearing stress of column at a joint A(B)
- $A N_{co} (B N_{co})$: axial force of column at a joint A(B)
- $A M_c (B M_c)$: end moment of column at a joint A(B)
- α : ratio of distance between a point B and position of the center of all constraining reaction acting on the beam to the beam span

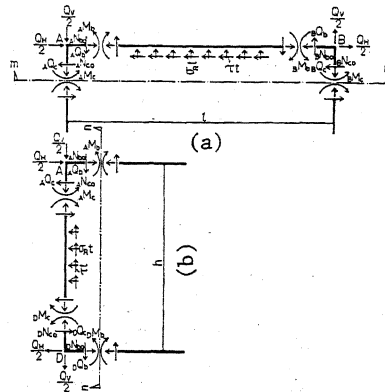


Fig. 4 Equilibrium of Forces Acting on the Sections (a) for Beam (b) for Column of Surrounding Frame

Now, failure of a surrounding frame is considered to be shearing failure of beam end. Therefore, shearing force at a beam end A is expressed with the shearing strength \bar{Q}_b of beam to introduce the following relation:

$$\bar{Q}_b = -A Q_b = A N_{co} + \frac{Q_v}{2} \dots \dots \dots (6)$$

where

- $A Q_b$: shearing force of beam at a joint A, \bar{Q}_b : shearing strength of beam

Considering Eq. (6), Eq. (3) gives:

$$Q_H = p \cdot \sigma_y l + \frac{1}{\alpha} \left(\bar{Q}_b + \frac{A M_c + B M_c}{l} \right) + A Q_c + B Q_c \dots \dots \dots (7)$$

Vertical section n-n of a surrounding frame as shown in Fig. 3(b) is subsequently taken in the vicinity of joints A, D in order to consider shearing failure of the column. An equilibrium equation for the beam is formulated and solved in the following:

$$Q_V = p_s \sigma_s l h + \frac{1}{\beta} \left(\bar{Q}_c - \frac{A M_b + D M_b}{h} \right) - A Q_b - D Q_b \dots \dots \dots (8)$$

where

- Q_b (Q_b): shearing force at a joint A(D) of beam
 $A M_b$ (M_b): end moment at a joint A(D) of beam
 A (D) β : ratio of the distance between a point D and a position of the center of all constraining reaction acting on the column to the height of a story
 \bar{Q}_c : shearing strength of column

Multiplying Eq.(8) by l/h and using the relation $Q_H = Q_V \cdot l/h$, the following equation is obtained:

$$Q_H = p_s \sigma_s l + \frac{1}{\beta} \left(\bar{Q}_c - \frac{A M_b + D M_b}{h} \right) \frac{l}{h} - (A Q_b + D Q_b) \frac{l}{h} \dots \dots \dots (9)$$

Assumption of the Ultimate Bending Moment Distribution of a Surrounding Frame

Originally, such a problem as shown in Fig. 3(b) is statically indeterminate with high degrees of redundancy and equations of equilibrium conditions for themselves are not sufficient for the solution. For the exact ultimate bearing strength, elasto-plastic incremental analysis of a statically indeterminate frame should be applied. However, we aim here to calculate the ultimate bearing strength by paying attention on stress situation at the ultimate condition. The following items of assumption for the analysis are established based on conventional theories and experimental studies. (1) Bending moment M of surrounding frame is taken to be the sum of bending moment M' of the constraining reaction only and the bending moment M'' resulted from horizontal force. (2) The ratio of the bending moment M'' to the bending moment M' is determined by the condition that bending moment M of a surrounding frame is zero at a loading point B of the tensile external force in the diagonal direction and at a point D (Ref. 1,2). (3) The distribution of the constraining reaction is uniform and namely, $\alpha = \beta = 0.5$. According to the items (1) and (2) of the above assumption, a equilibrium system of a surrounding frame F is divided into that of a surrounding frame F' under symmetrical stress due to constraining reaction only shown in Fig. 5(b) and that of a surrounding frame F'' under anti-symmetrical stress due to horizontal force shown in Fig. 5(c). These subsystems are in a state of equilibrium and therefore, the equilibrium of the original system is considerably simplified. Analysing a surrounding frame F' , end moments of a column and a beam at a point A are obtained as follows:

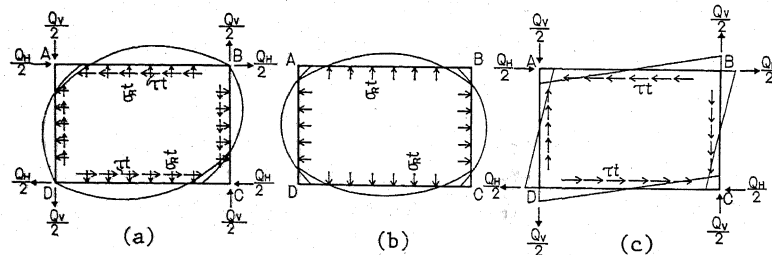


Fig. 5 The Ultimate Bending Moment Diagram of Surrounding Frame
 (a) Bending Moment (M) of Surrounding Frame
 (b) Bending Moment (M') of Surrounding Frame F' Subjected to Constraining Reaction Only
 (c) Bending Moment (M'') of Surrounding Frame F'' Subjected to Horizontal Force

$${}_AM_c' = -{}_AM_b' = -(\tau tl - p_s \sigma_s tl) \frac{\mu^2 + k}{12(1+k)} \cdot \frac{h^2}{l} \dots \dots \dots (10)$$

where

${}_A^M({}_A^M)_b'$: end moment at a point A of a column (beam) of a surrounding frame F' being subjected to the constraining reaction
 k : ratio of stiffness of beam to that of column
 $\mu = l/h$: ratio of the distance between the center lines of the columns to that of the beams

On the other hand, the following relation is obtained from the item (2) of the assumption:

$${}_AM_c = 2{}_AM_c', {}_BM_c = 0 \dots \dots \dots (11) \quad {}_AM_b = 2{}_AM_b', {}_DM_b = 0 \dots \dots \dots (12)$$

Substituting Eq.(10) into Eq.(11) and Eq.(12), the following equations are derived:

$${}_AM_c = -(\tau tl - p_s \sigma_s tl) \frac{\mu^2 + k}{6(1+k)} \cdot \frac{h^2}{l}, {}_BM_c = 0 \dots (13) \quad {}_AM_b = (\tau tl - p_s \sigma_s tl) \frac{\mu^2 + k}{6(1+k)} \cdot \frac{h^2}{l}, {}_DM_b = 0 \dots (14)$$

For the shearing force supported by a column and a beam due to Eq.(7) and Eq.(9), only a surrounding frame F'' can be taken into consideration because a surrounding frame F' is symmetrically stressed and the horizontal force becomes zero as a whole. Therefore,

$${}_AQ_c + {}_BQ_c = 2{}_AQ_c' \dots \dots \dots (15) \quad {}_AQ_b + {}_DQ_b = 2{}_AQ_b' = -2{}_AQ_c' \frac{h}{l} \dots \dots \dots (16)$$

where

${}_A^Q({}_A^Q)_b'$: shearing force at a point A of a column (beam) of a surrounding frame subjected to the horizontal force

On the otherhand, the item (2) of the assumption yields:

$${}_AQ_c' = -\frac{{}_AM_c' + {}_DM_c'}{h} = -\frac{2{}_AM_c'}{h} = -\frac{2{}_AM_c'}{h} \dots \dots \dots (17)$$

where

${}_A^M({}_A^M)_D'$: end moment at a point A(D) of a column of a surrounding frame subjected to the horizontal force

Making use of Eq.(17), Eq.(10), Eq.(13) and Eq.(15) are substituted into Eq.(7) and Eq.(10), Eq.(14) and Eq.(16) into Eq.(9) to obtain Eq.(18) and Eq.(19), respectively:

$$Q_H = p_s \sigma_s tl + 2\bar{Q}_b \cdot \frac{\mu(3\mu(1+k) + \mu^2 + k)}{3\mu^2(1+k) + \mu^2 + k} \dots \dots (18) \quad Q_H = p_s \sigma_s tl + 2\bar{Q}_c \cdot \frac{3\mu(1+k) + \mu^2 + k}{3(1+k) + \mu^2 + k} \dots \dots (19)$$

Eq.(18) is the ultimate strength formula of the quake-resisting wall of beam shear-failure type and Eq.(19), that of column shear-failure type.

Correction of Shearing Strength by the Axial Force

\bar{Q}_b and \bar{Q}_c in Eq.(18) and Eq.(19) are considered to be the sum of shearing strength without the axial force and corrected value by the axial force. Namely,

$$\bar{Q}_b = \bar{Q}_{bo} - \eta({}_AN_{bo} + N_b) \dots \dots \dots (20) \quad \bar{Q}_c = \bar{Q}_{co} - \eta({}_AN_{co} + N_c) \dots \dots \dots (21)$$

where

\bar{Q}_b (\bar{Q}_c): shearing strength of a beam (column) without the axial force
 ${}_A^N({}_A^N)_{bo}$ (${}_A^N({}_A^N)_{co}$): the axial force at a joint A of a beam (column) of a surrounding frame
 N_b (N_c): the additional axial force of a beam (column) of a surrounding frame F
 η : correction factor of shearing strength by the axial force

N_{bo} in Eq.(20) is the sum of the axial force N'_{bo} of a beam at a joint A of a surrounding frame F' subjected to the constraining reaction only shown in Fig. 5(b) and the axial force N''_{bo} of a beam at a joint A of a surrounding frame F'' shown in Fig. 5(c). Therefore, the following equation is obtained as seen in Fig. 6.

$$N_{bo} = N'_{bo} + N''_{bo} = \frac{1}{2}\sigma_R t h - \frac{1}{2}Q_H + A Q_c' = \frac{1}{2}\sigma_R t h - \frac{1}{2}\tau t l \quad \dots\dots\dots(22)$$

where

N'_{bo} (N''_{bo}): the axial force of a beam at a point A of a surrounding frame F' (F'') subjected to the constraining reaction (horizontal force) only

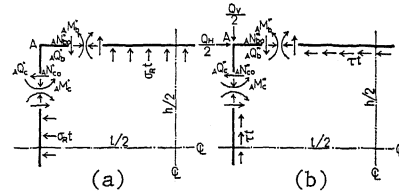


Fig. 6 (a) Equilibrium Situation of Forces Acting on the Section at a Joint A of Surrounding Frame Subjected to Constraining Reaction Only
(b) Equilibrium Situation of Forces Acting on the Section at a Joint A of Surrounding Frame Subjected to Horizontal Force

Similarly, N_{co} in Eq.(20) is expressed in the following as seen in Fig. 6.

$$N_{co} = N'_{co} + N''_{co} = \frac{1}{2}\sigma_R t l - \frac{1}{2}Q_V - A Q_c'' = \frac{1}{2}\sigma_R t l - \frac{1}{2}\tau t h \quad \dots\dots\dots(23)$$

where

N'_{co} (N''_{co}): the axial force of a column at a point A of a surrounding frame F' (F'') subjected to the constraining reaction (horizontal force) only

Using the relation of Eq.(3), Eq.(15), Eq.(16) and Eq.(17), Eq.(22) and Eq.(23) are arranged as follows:

$$N_{bo} = -p_s \sigma_R t l \frac{3(1+k) + \mu^2 + k}{2\{3\mu(1+k) + \mu^2 + k\}} + Q_H \frac{3(1-\mu)(1+k)}{2\{3\mu(1+k) + \mu^2 + k\}} \quad \dots\dots\dots(24)$$

$$N_{co} = -p_s \sigma_R t l \frac{3\mu^2(1+k) + \mu^2 + k}{2\mu\{3\mu(1+k) + \mu^2 + k\}} - Q_H \frac{3(1-\mu)(1+k)}{2\{3\mu(1+k) + \mu^2 + k\}} \quad \dots\dots\dots(25)$$

Substituting Eq.(20) and Eq.(24) into Eq.(18) and also Eq.(21) and Eq.(25) into Eq.(19), the following equations are obtained:

$$Q_H = C_1 \cdot p_s \sigma_R t l + 2 C_2 \cdot (Q_c - \eta N) \quad \dots\dots\dots(26)$$

where for beam shearing failure type,

$$C_1 = \frac{3\mu(1+k)(\mu+\eta) + (1+\mu\eta)(\mu^2+k)}{3\mu(1+k)\{\mu+\eta(1-\mu)\} + \mu^2+k} \quad \dots\dots(27a)$$

$$C_2 = \frac{\mu\{3\mu(1+k) + \mu^2 + k\}}{3\mu(1+k)\{\mu+\eta(1-\mu)\} + \mu^2+k} \quad \dots\dots(27b)$$

$$Q_c = Q_{cs}, N = N_s \quad \dots\dots\dots(27c)$$

where for columnshearing failure type,

$$C_1 = \frac{3\mu(1+k)(1+\mu\eta) + (\mu+\eta)(\mu^2+k)}{3\mu(1+k)\{1-\eta(1-\mu)\} + \mu(\mu^2+k)} \quad \dots\dots(28a)$$

$$C_2 = \frac{3\mu(1+k) + \mu^2 + k}{3(1+k)\{1-\eta(1-\mu)\} + \mu^2+k} \quad \dots\dots(28b)$$

$$Q_c = Q_{cs}, N = N_c \quad \dots\dots\dots(28c)$$

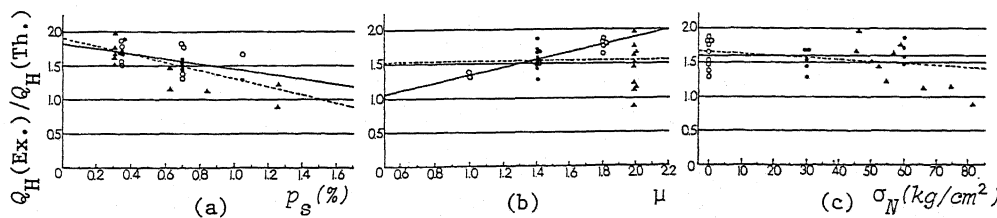
COMPARISON WITH THE RESULT OF THE PRECEDING EXPERIMENTS

The following conditions are placed for choosing test pieces in the preceding experiments in order to compare their results with the ultimate strength formula of Eq.(26). (1) The quake-resisting wall is confined that of a story

and a bay. (2) The method of loading is fundamentally simultaneous loading of tension and compression in two diagonal directions. (3) The case of compression loading in a diagonal direction is divided into simultaneous loading of tension and compression in two directions and the additional external force at a joint. (4) Shear reinforcement ratio of a wall plate is set to be more than 0.2%. (5) Shearing strength of the beam and the column without the axial force is determined with Ohno, Arakawa's formula. (6) Correction factor due to the axial force is set to be 0.1 with reference to a previous study. Test pieces taken in this study satisfying the previous items are twenty-two pieces of simultaneous tensile-compressive loading type in the two diagonal directions and thirteen pieces of compressive loading type in a diagonal direction, namely thirty-five pieces in total (Ref. 3,4).

Fig. 7(a),(b),(c) show the ratio of the experimental value to the theoretical value in terms of shear reinforcement ratio of a wall plate p_s , ratio of the length to the width of a wall plate μ and additional axial stress of a column σ_N , respectively. The solid lines in the figure is obtained with the method of least squares for the test pieces of tensile and compressive loading type and the broken lines, for those of compressive loading type. Increment of shear reinforcement ratio of a wall plate in Fig. 7(a) decreases the difference between the theoretical and the experimental values, as seen from the gradient of solid and broken lines in the figure. About the effect of the ratio of the length to the width of a surrounding frame in Fig. 7(b), the solid line shows that the difference between the theoretical and experimental values increases as the ratio of dimensions of a surrounding frame becomes large, while the broken line stays almost flat. About the effect of additional axial stress of a beam in Fig. 7(c), there is not large difference between the solid and broken lines in the figure and the effect due to the amount of additional stress is not clear.

The theory yields a value of safety side with scattering deviation of two times as much as that of the experiment. Therefore, the deviation is studied with the multiple regression analysis Eq.(29) is the ultimate shearing strength Q_H of a reinforced concrete quake-resisting wall after performing the multiple regression analysis.



- Tensile-Compressive Loading Type without Additional Axial Force on Column
- △ Tensile-Compressive Loading Type with Additional Axial Force on Column
- ▲ Compressive Loading Type with Additional Axial Force on Column

Fig. 7 Comparison of the Ultimate Strength of Quake-Resisting Wall between Theoretical and Experimental Values

- (a) Effect of Shear Reinforcement Ratio of Wall Plate
- (b) Effect of Ratio of Dimensions of Quake-Resisting Wall
- (c) Effect of Additional Axial Stress of Column

$$Q_H = 0.54 C_1 \cdot p_s \sigma_{yt} l + 1.74 \times 2 C_2 (\bar{Q}_o - \eta N) + 0.90 (\text{ton})$$

..... (29)

Theoretical value $Q_H(\text{theory})$ calculated with the ultimate strength formula of Eq.(29) and experimental value $Q_H(\text{experiment})$ are plotted for abscissa and ordinate, respectively in Fig. 8, where $Q_H(\text{experiment})/Q_H(\text{theory})=1.0$ is indicated with a bold line, $Q_H(\text{experiment})/Q_H(\text{theory})=0.8, 1.2$ with fine lines and \circ shows beam shearing failure type, \bullet column shearing failure type. A multiple regression coefficient of this case is 0.976 and 78.6% of all test pieces fall in the error region of $\pm 20\%$.

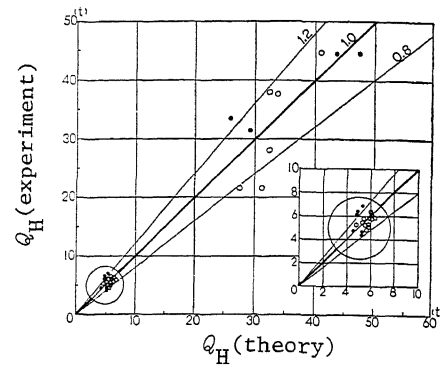


Fig. 8 Comparison between Theoretical Value $Q_H(\text{theory})$ Calculated with the Ultimate Strength Formula and Experimental Value $Q_H(\text{experiment})$

CONCLUSION

The author has proposed in this study a formula of the ultimate shearing strength of a reinforced concrete quake-resisting wall for practical designing by obtaining a theoretical formula based on resistance-failure mechanism and comparing with the preceding experimental result.

In comparison to a conventional formula taking the allowable shearing force of a wall plate and the allowable shearing strength of columns of a surrounding frame for the ultimate strength, the proposed formula is more concretely based on the resistance failure mechanism and more accurate. In addition, possibility of expansion into an unsolved problem of the ultimate strength formula in the case of bending failure of a surrounding frame is also shown. The author is planning as a further study to make the ultimate strength formula of this study more complete with additional data and references.

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