OPTIMUM DESIGN OF STEEL AND REINFORCED CONCRETE 3-D SEISMIC BUILDING SYSTEMS

K. Z. Truman (I)
F. Y. Cheng (II)
Presenting Author: K. Z. Truman

SUMMARY

This paper presents a general procedure for optimum design of 3-dimensional building systems, subjected to interacting ground motions with various constraints. Numerical examples show the rapid convergence to an optimal solution and the significance of the interacting ground motions on the design results. The significance of the interacting ground motions will be explored with respect to the optimal design displacements and stiffness.

INTRODUCTION

The research project,sponsored by the National Science Foundation, granted to the University of Missouri-Rolla is to develop an optimization technique for 3-dimensional building systems and to study the effects of 3-dimensional interacting ground motions with respect to the optimal design of various structural systems. The computer program INRESB-3D (Ref. 1) has been modified in order to perform elastic optimization. This optimization program, ODSEWS-3D, can load a structure with static loads, dynamic forces, and the seismic excitation including the ATC-06-3 (Ref. 2) code provisions.

The design assumes each floor to be rigid in its horizontal plane while being flexible in the vertical planes. This characteristic allows each structure to be represented with one vertical degree of freedom at each structural node and one rotational along with two translational degrees of freedom for each floor. P-Δ effects and external stiffness are considered in the stiffness formulation. The constituent members of the building systems can be steel beam-columns, beams and braces and/or reinforced concrete beam-columns and flexural panels. The element degrees of freedom are: braces can have axial deformation, the beams can have torsional and major-axis bending, the flexural panels can have axial deformation and major-axis bending, and the beam-columns can have

(I) Ph.D. Candidate of Civil Engineering, University of Missouri -Rolla, Rolla, Missouri, USA and Research Engineer of Civil Engineering, Washington University, St. Louis, Missouri, USA

(II) Professor of Civil Engineering, University of Missouri -Rolla, Rolla, Missouri, USA

torsional, minor-axis, and major-axis bending as well as axial deformations.

OPTIMIZATION TECHNIQUE

Optimality Criteria

The optimality criterion is derived from the Lagrangian function:

$$L(\lambda,\delta) = B(\delta) + \sum_{j=1}^{m} \lambda_j g_j \tag{1}$$

where λ_j is the Lagrangian multiplier for the j^{th} constraint, m is the total number of constraints, g_j is the j^{th} constraint (the difference between the actual response and the prescribed upper or lower limit response), and $B(\delta)$ is the objective function. The objective function is:

$$B(\delta) = \sum_{i=1}^{n} \gamma_i \, V_i(\delta_i) \tag{2}$$

in which B is the total structural weight or cost, γ_i is the weight or cost per unit volume for element i, $V_i(\delta_i)$ is the volume of element i, δ is the vector of primary design variables, and n is the total number of structural elements. The cost per unit volume can include approximate costs for construction such as erecting steel, forming concrete, and designing details. A more extensive objective function can be found in (Ref. 3). Primary design variables, δ, are the geometric quantities chosen to represent the different types of elements (i.e., beam-columns use the major-axis moment of inertia). Secondary design variables are all other geometric properties defining the element's cross section. Global design variables are primary design variables used to represent one or more elements through the use of variable linking. By applying the Kuhn-Tucker necessary conditions with linking of the design variables, the optimality criterion becomes:

$$T_i = \frac{\sum_{j=1}^{m} \lambda_j \left(\sum_{k=1}^{p} \frac{\partial g_j}{\partial \delta_k} \right)}{\left(\sum_{k=1}^{p} \frac{\partial B}{\partial \delta_k} \right)} = 1 \qquad i = 1,n_1 \tag{3}$$

where p is the number of elemental primary design variables linked to one active global design variable, and n_1 is the number of independent global design variables. Passive elements are those elements which are forced to have their primary design variables equivalent to a maximum or minimum size (side constraints).

Recursive Relationship and Lagrandian Multipliers

All elements which are not controlled by the side constraints must be sized in a manner which is consistent with the optimality criterion. This is accomplished by using a linear recursion equation:

$$\beta_i^{\eta+1} = \left(1 + \frac{1}{r}(T_i - 1)\right)\beta_i^{\eta} \qquad i = 1,n_1 \tag{4}$$

in which η is an iteration indice, r is a convergence control parameter, and β_i is the i^{th} active primary design variable. The linear recursive relationship is used in the derivation of a set of simultaneous equations used to find the approximate Lagrangian multipliers for a given set of active constraints. These equations with linking included are:

$$-rg_j(\delta) + \sum_{i=1}^{q}\left(\sum_{k=1}^{p}\frac{\partial g_j}{\partial \delta_k}\right)\beta_i - r\sum_{i=q+1}^{n_1}\left(\sum_{k=1}^{p}\frac{\partial g_j}{\partial \delta_k}\right)(\Delta\beta_i)$$

$$= \sum_{\ell=1}^{m}\lambda_\ell \sum_{i=1}^{q}\frac{\left(\sum_{k=1}^{p}\frac{\partial g_j}{\partial \delta_k}\right)\left(\sum_{k=1}^{p}\frac{\partial g_\ell}{\partial \delta_k}\right)}{\left(\sum_{k=1}^{p}\frac{\partial B}{\partial \delta_k}\right)}\beta_i \tag{5}$$

where $(q-n_1)$ is the number of passive global design variables, and λ_ℓ are the m Lagrangian multiplers to be found.

Sensitivity Analysis

The objective function gradient can be determined in closed form; whereas, the constraint gradients must be determined numerically. Static displacement and stress gradients are determined by the virtual load technique, and the frequency constraint gradients are found from an equation derived by taking the partial derivative of the free vibration equation premultiplied by the eigenmode (Ref. 3).

Dynamic displacement gradients are found by taking the partial derivative of the modal analysis equation:

$$\frac{\partial\{X\}}{\partial\delta_i} = \frac{\partial[\Phi]}{\partial\delta_i}\{Q\} + [\phi]\frac{\partial\{Q\}}{\partial\delta_i} \tag{6}$$

in which $[\Phi]$ is the matrix of the eigenvectors and $\{Q\}$ is the vector of modal participation factors. The eigenvector gradients can be found from the equations for the partial derivative of the undamped, free vibration solution:

$$\left[[K_T]-\omega_j^2[M_T]\right] \frac{\partial\{\phi_j\}}{\partial\delta_i} = -\left[\frac{\partial[K_T]}{\partial\delta_i} - \omega_j^2 \frac{\partial[M_T]}{\partial\delta_i}\right]\{\phi_j\} + \frac{\partial\omega_j^2}{\partial\delta_i}[M_T]\{\phi_j\} \qquad (7)$$

which requires the inversion of a singular matrix. These singularities are treated similarly to that of eliminating the degrees of freedom associated with supports for a structural analysis. The support degrees of freedom are set to zero, and the appropriate rows and columns of the stiffness matrix are eliminated. Similarly, one or more, for repeating eigenvalues, of the eigenvector gradient components can be set to zero which eliminates these rows and columns and creates a nonsingular matrix. For example, if the eigenvector of the j^{th} mode is determined such that the eigenvector always has a unit value for this first component, the change in this first component will, with respect to a change in any design variable, be zero (i.e., $\partial\phi_j/\partial\delta_k=0$). By eliminating the row(s) and column(s) associated with the zero gradient components, a nonsingular matrix is found which will provide the gradients of the eigenvector components. Dynamic stress gradients are calculated using a psuedo load technique since Eq. 6 provides each dynamic displacement gradient. (Ref. 3)

Scaling

Any time a constraint is violated, or there are no active constraints, scaling is performed to force the building system to reach an active constraint. Scaling is performed by multiplying the primary design variables by a factor which is the largest ratio of actual to limiting response. These factored primary design variables are then used to determine the new secondary design variables. Since the secondary design variables are generally not linearly dependent upon the primary design variable, scaling will not be a linear process.

NUMERICAL EXAMPLES AND RESULTS

Given Conditions

A five story, L-shaped, rigid steel structure shown in Fig. 1a and b is designed for various seismic inputs and external stiffness arrangements as indicated in Table 1. The mass center is located at point A for loads 1-5 and at point B (35" (88.9 cm) to the right and 15" above column 1) for load 6. Each floor has a translational mass of 0.3106 k-s^2/in (54.39 Mg) and a rotational, mass inertia of 16,403 k-in-s^2 (1,853 Mg-m^2). The modulus of elasticity is 30,000 k/in^2 (2.068×10^{11} N/m^2), and the shearing modulus is 11,500 k/in^2 (7.927×10^{10} N/m^2). The seismic inputs are represented by the acceleration response spectrum in Fig. 1c with the components of maximum ground acceleration as summarized in Table 1. The external stiffnesses are applied at the mass centers in order to prevent deflection in that respective direction without preventing rotation. A convergence control parameter of 2.0 was used and the termination criterion was either of three criteria; a weight change of less than 5% between cycles, no more than 20 cycles of optimization, or no more than 25 analyses. A constraint was considered active if the ratio of actual to limiting response was between 0.90 and 1.05. The dynamic displacement constraints were chosen according to ATC recommendations as 0.00625h where h is the height of each floor measured from the ground level. To achieve a practical design, linking was used for which ten global design variables were assumed; one per floor for the columns and one per floor for the beams. Although various wide-flange sections are available in the program, for simplicity, a rectangular cross section with a height to width ratio of 1.5 was used for all elements. Therefore, the equations which relate the secondary to primary design variables are $I_y=0.4444I_x$, $J=1.4444I_x$, and $A=2.8284(I_x)^{0.5}$. All elements were required to have a major-axis moment of inertia no less than 200 in^4 ($8.32 \times 10^{-5} m^4$). The objective function was the total structural weight where the specific weight was 0.490 k/ft^3 (77.0 kN/m^3).

Results and Observations

The results of the optimization are given in Table 1 and Fig. 2. Fig. 2 shows the rapid rate of convergence as all load cases terminate within 4 cycles of optimization. As summarized in Table 1 all cases terminate due to less than a 5% change in weight between cycles. Note that all displacements shown in Fig. 3 and 4 are within or below the stated range for active constraints. The active constraint locations and values along with the passive elements are shown in Table 1. Fig. 5 shows the axial displacements for columns 1 and 7. These displacements are due to the translational effects only. The loads in order of least to largest weight required are 1, 2, 6, 3, 4, and 5. Loads 1 and 2 were expected to produce lighter structures since the y-excitation was less than the x-excitation and the columns are oriented with their major stiffness in that direction. Due to the rigid floor assumption the rotations can significantly affect the final solution, depending upon the location of the mass center with respect to the center of rigidity. Load 6 requires less weight than load 3 since its rigidity center is in the first quadrant. The negative (clockwise) rotation shown in Fig. 6 causes a negative x-displacement at the mass center which helps reduce the x-displacements without additional stiffness. Case 3 provides positive (ccw) rotations but has a mass center near the center of rigidity, therefore having little effect on the x-displacement. This is supported by the nearly equivalent weight and stiffness distribution in Fig. 7, 8, and 9 between loads 3 (multi-component) and load 4 (x-only). This also explains why the single component cases require less weight than their respective externally stiffened cases. Fig. 6 shows cases 2 and 5 as having decreasing rotations between levels 4 and 5 which allows the fifth floor displacements to be non-active. Generally, if the lower floor displacement constraints are active a larger amount of stiffness is required in the lower floors as shown in Fig. 7, (I_{max}=3119 in^4)(129,800 cm^4) and Fig. 8 (I_{max}=1141 in^4) (47,500 cm^4). For a symmetric structure the multi-component loading will produce larger weights than single component loading since the mass center and center of rigidity are identical; whereas, nonsymmetric structures can have a reduction in their weight if the center of rigidity is displaced from the mass center as seen in case 6.

Figs. 7, 8 and 9 show the relative stiffness of the members for the optimal designs. The slope between levels measures the relative change in stiffness between floors for Fig. 7 and 8, and Fig. 9 shows the relative stiffness of beams to columns at each level. From Fig. 7 large columns are required for all loadings at the first level with decreasing sizes from the bottom to the top of the structure. Comparing the levels associated with the active constraints, Fig. 7 shows that the stiffness of the columns below the lowest level active constraint has nearly the identical size with respect to the constrained level. Also, the levels between active constraints have a larger change in stiffness which is needed to allow the next level to displace to the active limit. Fig. 8 shows that beams of similar or larger stiffness compared to level 1 are required for level 2. This is due to the rigid supports and very rigid first story columns. These large first level columns reduce the need for large first level beams, while the added flexibility of level 1 nodal rotations forces the second level to compensate by strengthening with large beams to stop excessive upper level displacements. After the second level the trend is to reduce the beams to the point that all beams at level 5 are passive. From Fig. 9 all loadings except 1 require a weak-beam, strong-column system to limit the deflections. Case 1 requires slightly larger beams than columns at levels 2 and 3 in order to keep the deflections within the constraint limits.

References

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2. Tentative Provisions for the Development of Seismic Regulations For Buildings, ATC-06-3, U.S. Government Printing Office, Washington, D.C., 1978.

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TABLE 1. LOADS AND FINAL RESULTS

Case	Symbol	Load(g) x	Load(g) y	Ext. Stif.	% Δ Wt.	# Opt. Cycles	# of Analyses	Act. Const. (Final)	Lag. Mult.	Passive Elements
1	○	.0	.225	-	0.76	3	8	y5=4.518"	44.10	C5, B5*
2	□	.300	.225	x-dir.	1.74	3	7	y3=2.711"	62.44	C4, C5, B4, B5
3	△	.300	.225	-	0.06	3	4	x5=4.525" x4=3.598"	67.96 54.49	B5
4	■	.300	.0	-	0.23	3	4	x5=4.518" x4=3.485"	75.97 26.86	B5
5	●	.300	.225	y-dir.	0.08	4	9	x4=3.609" x3=2.703"	68.66 52.06	C5, B4, B5
6	▲	.300	.225	-	1.07	3	5	x5=4.564" x4=3.576"	51.50 46.94	B5

* Ci represents all columns on the i^{th} level and Bi represents all beams on the i^{th} level.

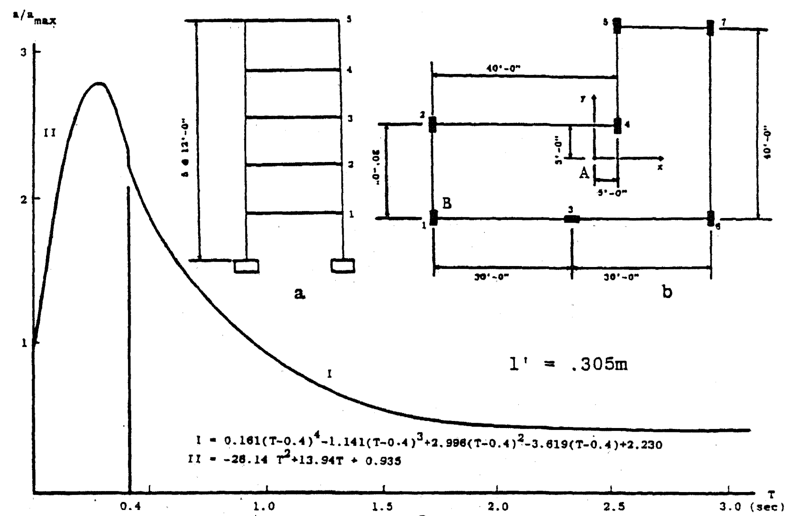

FIGURE 1. a) TYPICAL ELEVATION, b) FLOOR PLAN AND AXES,
c) RESPONSE SPECTRUM (a_{max} = max. ground accleration)

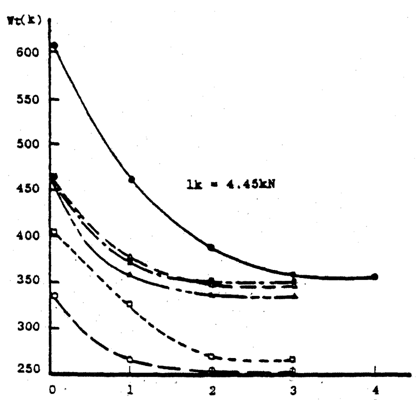

Figure 2. WEIGHT vs. OPT. CYCLES

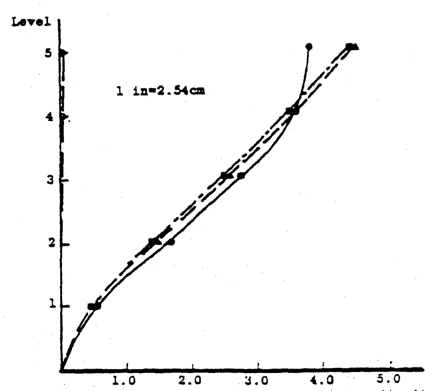

FIGURE 3. X-DISPLACEMENTS

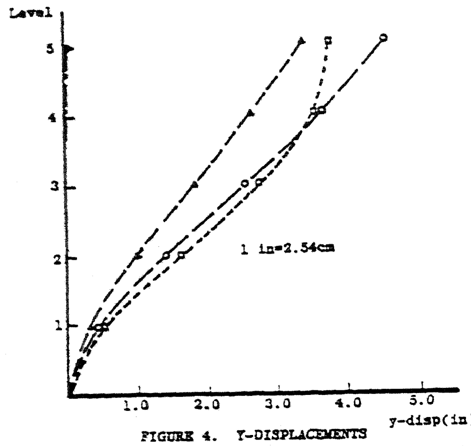

FIGURE 4. Y-DISPLACEMENTS

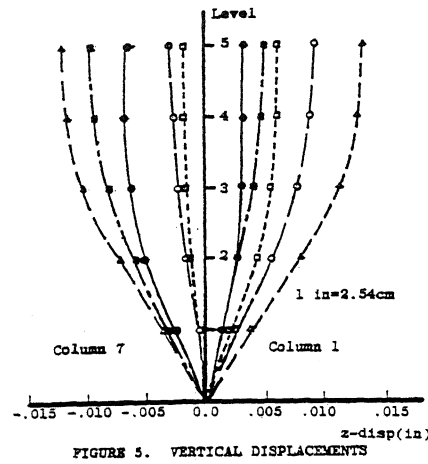

FIGURE 5. VERTICAL DISPLACEMENTS

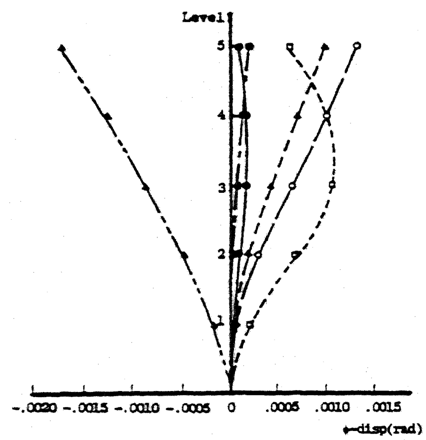

FIGURE 6. ROTATIONAL DISPLACEMENTS

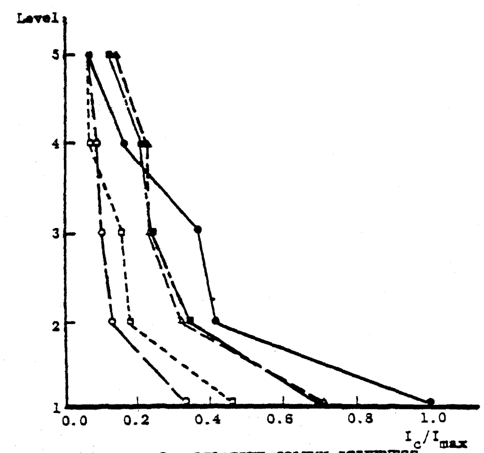

FIGURE 7. RELATIVE COLUMN STIFFNESS

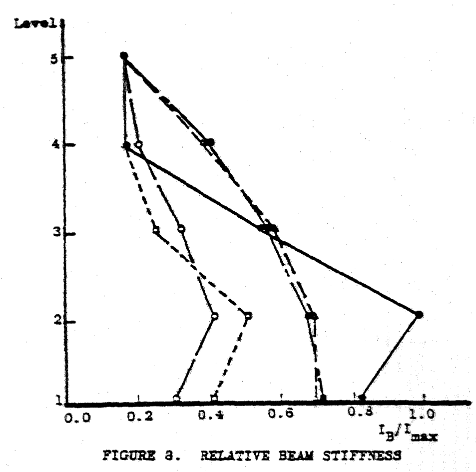

FIGURE 8. RELATIVE BEAM STIFFNESS

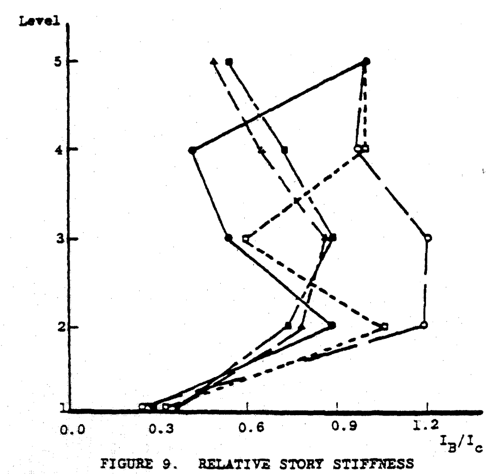

FIGURE 9. RELATIVE STORY STIFFNESS