A NEW FORMULATION OF OPTIMUM ASEISMIC DESIGN USING FUZZY MATHEMATICAL PROGRAMMING

Kohei Furukawa (I)
Hitoshi Furuta (II)
Presenting Author: K. Furukawa

SUMMARY

This paper aims to derive a new formulation of optimum aseismic design by introducing the concept of fuzzy sets theory. By using fuzzy mathematical programming, the optimum aseismic design can be reduced to a problem of maximizing the degree of satisfaction. A new design method is proposed, in which the fuzzy mathematical programming and the dynamic optimality criterion method are combined. Several design examples are presented to demonstrate the efficiency and applicability of the design method developed herein.

INTRODUCTION

Success in structural design depends on how to balance such contradictory alternatives as economy and safety. In order to establish a rational design method, many studies of the optimum structural design have been presented most of which reduce the design problem to a problem of minimizing the weight or cost of the structure subjected to the constraints of safety. However, the formulation posesses two problems; one is that structural design problems should be essentially dealt with as one of multi-objective problems. In any structural design, it is desirable to decrease the displacements and stresses as well as the reduction of weight. The other is concerned with the design conditions employed there; the allowable stresses or displacements and the design acceleration of earthquake may not be always rationally determined from the viewpoint of optimum aseismic design. This paper aims to discuss these two problems by use of the concept of fuzzy sets theory (Ref. 1).

At first, representative uncertain factors involved in the earthquake resistant design are found out, whose membership functions are determined from the engineering judgement of designer or the experimental and observated data. Similar treatment is performed in the determination of the membership functions of allowable design values. Next, the dynamic optimality criterion under earthquake loading is described for the minimum weight design subjected to frequency constraint (Ref. 2). A new method of optimum aseismic design is proposed by combining the fuzzy mathematical programming and the dynamic optimality criterion. This method can deal with all of economy, safety and design conditions as objectives. Then, the optimum asesmic design can result in a problem of maximizing the degree of satisfaction. Several design examples are presented to demonstrate the efficiency and applicability of the design method developed herein.

(I) Associate Professor, Faculty of Engineering, Yamaguchi Univ., Ube, Japan

(II) Research Associate, Faculty of Engineering, Kyoto Univ., Kyoto, Japan

UNCERTAINTIES IN EARTHQUAKE RESISTANT DESIGN

There exist a number of uncertain factors in the earthquake resistant design. The most uncertain factors are the intensity of earthquake and the allowable values of stresses and displacements. In this study, the design acceleration of earthquake and the allowable displacement are considered as the representatives of the uncertain design conditions.

Design Acceleration of Earthquake

Design acceleration of earthquake should be detemined from the magnitude and the epicentral distance. Here, the earthquake acceleration, $\ddot{Z}$, is estimated as follows, based on the past earthquake records in Japan (Ref. 3).

$$\ddot{Z} = a \cdot 10^{bM} \cdot (D + 10)^{c} \qquad \cdots\cdots\cdots\cdots\cdots \qquad (1)$$

in which a, b, c are constants, M is the magnitude and D is the epicentral distance. Membership functions of M and D are defined by functions shown in Figs. 1 and 2, which are subjectively specified based on the past earthquake records and engineering judgement. Fig. 1 implies that the magnitudes greater than M_2 are sufficiently large from the viewpoint of the earthquake resistant design, but the values less than M_1 are insufficient to guarantee the safety. Fig. 2 also implies that the epicentral distances less than D_1 are satisfactory, while the values greater than D_2 are insufficient to guarantee the safety.

By using these membership functions, the membership function of the design acceleration of earthquake can be calculated, as shown in Fig. 3. The calculation is performed, using a=34.1, b=0.308 and c=-0.925 in Eq. (1), M_1=7.0 and M_2=8.6 for the magnitude and D_1=20 km and D_2=100 km for the epicentral distance. The values of a, b and c are estimated from the statistics of past earthquakes in Japan, whereas M_1, M_2, D_1 and D_2 are determined through the engineering judgement.

Furthermore, the effects of the frequency of occurrence, the ground condition, the importance of the structure and the uncertainties in modelling and simplification can be included in the determination of design acceleration of earthquake. Then, the design acceleration $\ddot{Z}_D$ can be written as follows, based on the specifications for highway bridges of Japan Road Association (Ref. 3).

$$\ddot{Z}_D = k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot \ddot{Z} \qquad \cdots\cdots\cdots\cdots\cdots \qquad (2)$$

in which k_1, k_2, k_3 and k_4 denote the coefficients of region, ground condition, importance and dynamic response, respectively. To simplify the problem, the design acceleration of earthquake is evaluated herein by Eq. (1). This simplification means that these coefficients are not fuzzy but crisp numbers, which are assumed as $k_1 = k_2 = k_3 = k_4 = 1$. In the case of $k_1 - k_4$ being fuzzy coefficients, Eq. (2) can be calculated according to Ref. 4.

Allowable Displacement

The allowable displacement is treated as a representative of the design constraints with respect to structural safety. Determination of the allowable displacement is based on the subjectivity of engineers, while the allowable stress is specified in the form of the characteritic value obtained from the experimental results. Therefore, the ambiguity or uncertainties involved in the allowable displacement may be greater than that of the allowable stress. Here, a membership function shown in Fig. 4 is used for the allowable displacement (Ref. 5). This figure implies that the values less than δ_1 are satisfactory, but the values greater than δ_2 are insufficient to guarantee the safety.

DYNAMIC OPTIMALITY CRITERION

Stiffness of a structure under earthquake loading can be related to the eigenvalue, λ, which can be evaluated by using Rayleigh quotient,

$$\lambda = x^t K x / x^t M x \qquad (3)$$

in which x is the dynamic displacement vector, M is the mass matrix and K is the stiffness matrix. The minimum weight structure whose eigenvalue, λ, equates to the given allowable eigenvalue, λ_a, can be obtained from the stationary value of the following Lagrangian function,

$$L = \sum A_i l_i \rho_i + \frac{1}{\nu}(\lambda - \lambda_a) \qquad (4)$$

in which ν is the undetermined Lagrangian multiplier and A_i, l_i, ρ_i are the cross sectional area, length and unit weight of the i-th element, respectively. Since the influence of the variation of sectional areas on the dynamic displacement is sufficiently small, the stationary value can be obtained as

$$\frac{\partial L}{\partial A_i} = l_i \rho_i + \frac{1}{\nu} \frac{2 x^t K_i x - \lambda x^t M_i x}{A_i \cdot x^t M x} = 0 \qquad (5)$$

in which M_i and K_i are the mass and stiffness matrices of the i-th element. Replacing $-\nu$ by ν_i, Eq. (5) becomes

$$\nu_i = \frac{2 x^t K_i x - \lambda x^t M_i x}{A_i l_i \rho_i \cdot x^t M x} \qquad (6)$$

Thus, the optimality criterion for dynamic loading problems can be determined from the condition that the values of ν_i are equivalent for all the elements.

DESIGN FORMULATION

By expressing the constraints with respect to design acceleration of earthquake and displacement in terms of fuzzy sets, the earthquake resistant design problem can be written as follows:

$$\begin{aligned} &\text{Minimize} \quad W = \sum A_i \, l_i \, \rho_i \\ &\text{Subject to} \quad \ddot{Z} \gtrsim \ddot{Z}_a \\ &\qquad\qquad\quad \delta \lesssim \delta_a \end{aligned} \quad \cdots\cdots\cdots\cdots \quad (7)$$

in which the symbol $\gtrsim$ ($\lesssim$) denotes approximately large (small) which means a fuzzy relation. The subscript, a, designates the allowable value. An attempt is made to reduce eq. (7) to a problem of fuzzy mathematical programming (Ref. 6 and 7). In general, the fuzzy mathematical programming problem can be expressed as

$$\mu_F(A^*) = \sup_{A \in A_a} [\, \mu_G(A) \wedge \mu_C(A) \wedge \mu_D(A) \,] \quad \cdots\cdots\cdots \quad (8)$$

in which A_a and A^* are the allowable design domain and the optimum value of A, respectively. The symbol $\wedge$ denotes minimum and μ_F, μ_G, μ_C and μ_D are the membership functions of the decision set $\underset{\sim}{F}$, the goal set $\underset{\sim}{G}$, the constraint set $\underset{\sim}{C}$ and the design condition set $\underset{\sim}{D}$, respectively. In order to express Eq. (7) as Eq. (8), the objective function W is transformed into a function of satisfaction μ_G defined on [0,1].

$$\mu_G = W_0 \,/\, W \quad \cdots\cdots\cdots\cdots\cdots \quad (9)$$

in which W_0 is the least weight in the allowable design domain. The membership function of μ_G is shown in Fig. 5. Fig. 6 illustrates the design space of the problem expressed by μ_G, μ_C and μ_D. The optimum point, which is the solution of eq. (8), is obtained at the intersecting point of the three curved surfaces μ_G, μ_C and μ_D. Mostly, Eq. (8) is solved by a method based on the concept of α-level set. In this study, the following new formulation is developed by introducing the optimality criterion into the fuzzy mathematical programming.

Let us define the new set $\underset{\sim}{E}$ which consists of $\underset{\sim}{C}$ and $\underset{\sim}{D}$. In practical, Eq. (8) can be expressed as follows (Ref. 8):

$$\begin{aligned} &\text{Maximize} \quad \mu_G(A) \\ &\text{Subject to} \quad \mu_E(A) - \mu_G(A) \geq 0 \end{aligned} \quad \cdots\cdots\cdots\cdots \quad (10)$$

For the simple case as Fig. 6, the optimum solution of Eq. (10) is obtained when the following condition is satisfied:

$$\left| \mu_E(A) - \mu_G(A) \right| \longrightarrow 0 \quad \cdots\cdots\cdots\cdots\cdots \quad (11)$$

Thus, the optimum criterion for the fuzzy mathematical programming can be derived, that is, the value of μ_E is equal to the value of μ_G. Now, the optimum aseismic design problem can be newly formulated by using the following two optimality criteria:

$$\nu_i \longrightarrow \nu_0 \quad (\nu_0 : \text{constant}) \quad \text{for all elements} \quad \cdots\cdots\cdots \quad (12)$$

$$\left| \mu_E(A) - \mu_G(A) \right| \longrightarrow 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (13)$$

The optimality criterion of Eq. (12) provides the optimum areas when the design acceleration of earthquake and the allowable displacement are fixed, while the optimality criterion of Eq. (13) gives the optimum design values for the acceleration of earthquake and the allowable displacement. This method enables to determine both the optimum areas and the optimum design conditions simultaneously. The design problem considered is solved through an iterative procedure. This procedure is as follows:

Step 1 : Assume initial values $A^{(1)}$ and $\mu_E^{(1)}$, where k=1.

Step 2 : Obtain the values of $\ddot{Z}^{(k)}$ and $\delta_a^{(k)}$ from $\mu_E^{(k)}$. By using $\ddot{Z}^{(k)}$, carry out the structural analysis and get $\nu_i^{(k)}$. Obtain the improved $A^{(k+1)}$ under the constraint of $\delta_a^{(k)}$ by the dynamic optimality criterion method.

Step 3 : Calculate $\mu_G^{(k)}$ from $A^{(k+1)}$ and let $\varepsilon^{(k)} = \mu_E^{(k)} - \mu_G^{(k)}$.

Step 4 : If $|\varepsilon^{(k)}| \leqq \varepsilon_1$ and all ν_i are equal, then go to Step 6, otherwise go to Step 5.

Step 5 : Replace $\mu_E^{(k+1)} = \mu_E^{(k)} - r^{(k)}\,\varepsilon^{(k)}$ and return to Step 2 with k = k+1. Note that $r^{(k)}$ should be selected as $0 \leqq \mu_E^{(k+1)} \leqq 1$.

Step 6 : Replace $A^* = A^{(k+1)}$ and $\mu_E^* = \mu_E^{(k)}$. Obtain δ_a^* and $\ddot{Z}^*$ from μ_E^* and terminate the calculation process.

NUMERICAL EXAMPLES

To illustrate the design space, a tower structure example having two design variables is presented as shown in Fig. 7. For simplicity, economy is represented by the total weight and safety is checked by the top displacement of the tower and the design acceleration of earthquake. These parameters are characterized by use of the membership functions shown in Fig. 3, 4 and 5 in which $\delta_1 = 0.4$ m, $\delta_2 = 0.6$ m, $W_0 = 9.42$ ton. Using the design variables A_1 and A_2, the design space of the example is shown in Fig. 8, where the solid lines indicate μ_G and the broken lines indicate μ_E. The optimum point of the problem can be obtained at the intersecting point of μ_E and μ_G where the degree of satisfaction is maximized. This figure shows that the formulation developed herein determines simultaneously the optimum areas and the optimum design conditions such as $\ddot{Z}$ and δ_a. Thus, the formulation is different from conventional optimization concepts in which the design conditions are fixed.

Another numerical example of 5 elements tower structure is shown in Fig. 9, which has the same dimensions as that of Fig. 7 except for the element number. In the calculation, the same membership functions are used for the design acceleration of earthquake, the allowable displacement and the weight. Fig. 10 shows the convergency of the method proposed herein. Although this design problem has 5 design variables, convergency can be achieved after only 12 iterations. In this example, $A^{(1)} = 0.1\text{m}^2$ and $\mu_E^{(1)} = 0.5$ are employed as the initial values.

The top figure of Fig. 10 shows the change of μ_E, μ_G and η. The optimum solution is obtained when μ_E equates μ_G. η represents the deviation of ν_i:

$$\eta = \sqrt{\Sigma\ (\ \nu_i - \overline{\nu}\ )^2 /\ m}\ \ /\ \overline{\nu} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (14)$$

in which $\overline{\nu}$ is the mean value of ν_i. When η is sufficiently small, Eq. (12) is satisfied. In fact, the optimum structure can be gained where η is less than 0.08. Namely, Eq. (12) is satisfied after four iterations. The second figure shows the changes of the top displacement δ_1 and allowable displacement δ_a. Agreement of δ_1 with δ_a proves that the proposed method functions successfully. The third figure shows the change of the design acceleration of earthquake. It can be seen that the design acceleration converges to about 190 gal after 10 iterations.

It is concluded that the proposed method can reach the optimum structure with only a small number of iterations. Furthermore, the method can determine the optimum values such different parameters as weight, design acceleration of earthquake and allowable displacement in a unified manner, and can therefore provide a criterion for the optimum ratio between these parameters.

CONCLUSIONS

In this study, a new formulation of optimum aseismic design is derived by introducing the concept of fuzzy sets theory. By using the formulation, it is possible to treat such the design conditions as allowable stresses or displacements and acceleration of earthquake as well as design variables. The method proposed herein determines simultaneously the rational values of all the above design parameters from the standpoint of maximizing the degree of satisfaction. The method is based on the optimum criteria regarding the earthquake response and the fuzzy mathematical programming. Numerical results confirm that this method has very good convergency. It is evident that the method can be extended to the engineering design problems other than the earthquake resistant design, if their optimality criteria can be established.

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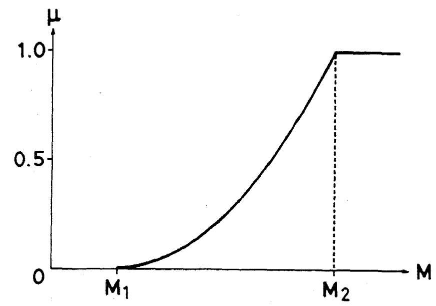

Fig. 1 Membership Function of Magnitude

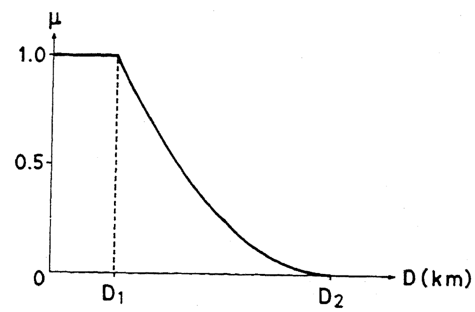

Fig. 2 Membership Function of Epicentral Distance

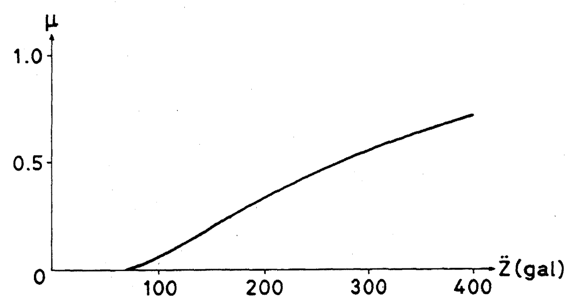

Fig. 3 Membership Function of $\ddot{Z}$ Calculated from M and D

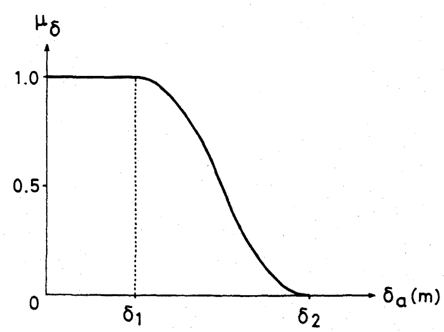

Fig. 4 Membership Function of Allowable Displacement

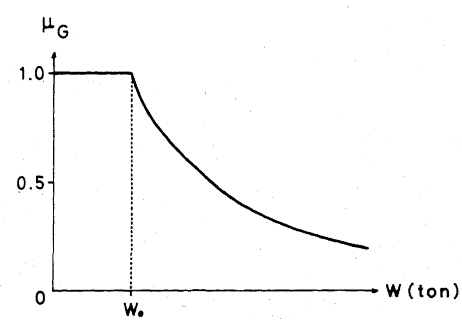

Fig. 5 Membership Function of Weight

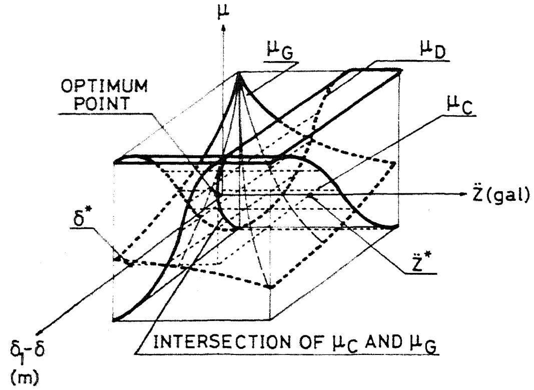

Fig. 6 Design Space Expressed by μ_G, μ_C and μ_D

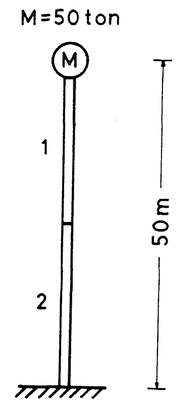

Fig. 7 Tower Model with Two Elements

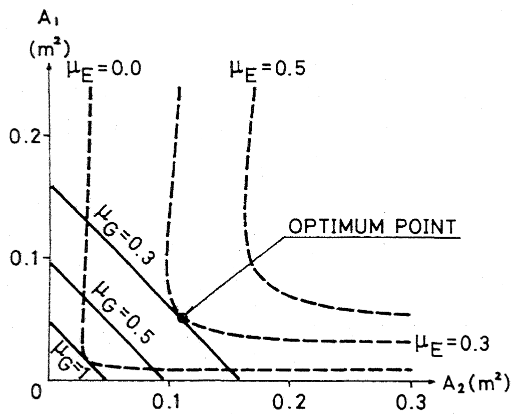

Fig. 8 Design Space Expressed by A_1 and A_2

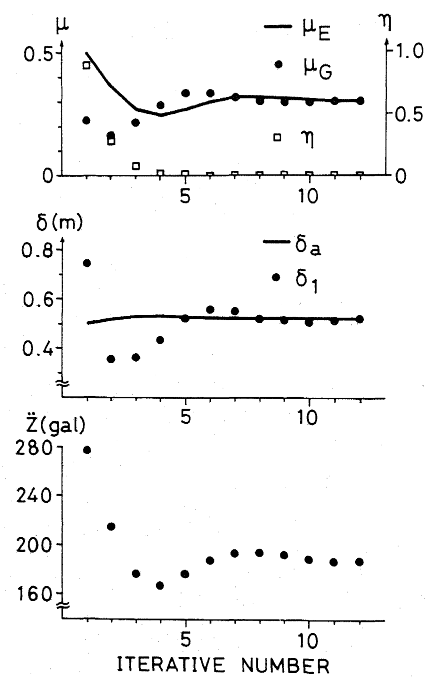

Fig. 10 Convergent Process

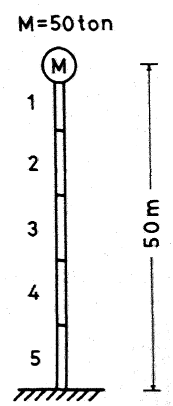

Fig. 9 Tower Model with Five Elements