

ASSESSMENT OF ATC-3 FOR STEEL STRUCTURES
BASED ON OPTIMIZATION ALGORITHM

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SUMMARY

A computer program ODSEWS-2D-II (Optimum Design of 2-Dimensional Steel Structures for Static, Earthquake, and Wind Forces - Version II) developed at the University of Missouri-Rolla was used to design several structural systems. This program can be used for the analysis and design of trusses, braced and unbraced frames subjected to various types of static loads and dynamic forces including ATC-03-6 provisions. The design technique is based on the optimality criterion resulting from a uniform distribution of strain and kinetic energy of the constituent members of a system. Sample examples are provided to show some deficiencies of the ATC-03-6 requirements.

INTRODUCTION

Ever since ATC-03-6 (Ref. 1) has been published, a considerable amount of research effort has been expended by both the academic and practicing communities. This effort has basically emphasized the logic of provisions (Ref. 2), comparative analyses of a structural system for various code provisions (Ref. 3), and the conventional design of typical systems. It is well known in indeterminate structural analysis that conventional design and analysis are based on the member stiffnesses assumed. If the assumed stiffnesses are misjudged, the design cannot be improved regardless of the number of analytical cycles and the sophistication of the computer programs. Consequently, for a structural system with different sets of given stiffnesses, various response behaviors can be observed. The reliable design should be based on optimum design procedures, which are based on mathematical programming from which an economical and serviceable structure can be obtained. The optimum design results should satisfy a set of constraints, such as displacements, stresses, frequencies, buckling loads, and member sizes as well as the dynamic forces recommended in the code provisions. Thus, the stiffness redistribution in a system can be mathematically determined according to the constraint and loading requirements. It is worthwhile to mention that by using an optimum design computer program, one can accelerate the design process and consequently reduce the design time. Apparently, the benefits are inherently increased.

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Two computer programs called ODSEWS-2D-II (Ref. 4) and ODSEWS-3D (Ref. 5) have been developed at the University of Missouri-Rolla for the analysis and design of 2-D and 3-D structures respectively. This paper is based on ODSEWS-2D-II for which the displacement method and consistent mass technique are used in structural formulation with consideration being given to the P-Δ effect. The structural systems can be trusses, braced and unbraced frames. The braced systems may have single-, double-, K-, and eccentric-bracings and the seismic inputs can be one- and two-dimensional; one-dimension is horizontal, two-dimension is horizontal coupled with vertical. The dynamic forces may be separated into three categories: a) seismic excitations at the base, b) dynamic forces applied at the structural nodes, and c) wind forces acting on the structural surfaces. The seismic excitations are derived from the following: 1) actual earthquake records, 2) response spectra, including those of Newmark, Seed, and Hausner, 3) U.S. Uniform Building Code (Ref. 6), 4) Chinese Seismic Building Code (Ref. 7), and 5) ATC-03-6 including the equivalent lateral forces with and without soil-structural interaction and the modal analysis with and without soil-structural interaction.

OPTIMIZATION ALGORITHM

Optimality Criteria - The Lagrange equation of Eq. 1 is used for Kuhn-Tucker conditions as shown in Eq. 2.

$$L = W_T + \sum_{j=1}^m \lambda_j h_j, \quad \frac{\partial W_T}{\partial \delta_i} + \sum_{j=1}^m \lambda_j \frac{\partial h_j}{\partial \delta_i} = 0, \quad (1,2)$$

in which λ_j corresponds to the j^{th} constraint h_j , W_T is an objective function, and δ_i is the design variable of member i . Equation 2 may be rewritten as

$$- \left(\sum_{j=1}^m \lambda_j \frac{\partial h_j}{\partial \delta_i} \right) / \frac{\partial W_T}{\partial \delta_i} = 1 \quad (3)$$

which is the basis for the optimality criterion method.

Objective Functions - Objective functions are the functions to be optimized. The most common relates the elements to the total weight of the structure as

$$W_T = \sum_{i=1}^n \gamma_i \Delta_i \quad (4)$$

in which W_T is the total weight, γ_i the specific weight for element i , Δ_i the volume as a function of the primary design variable for element i , and n the number of structural elements. Other objective functions can be expressed in terms of costs (Ref. 4) as

$$\text{COST} = C_s \gamma \Sigma A_{is} L_{is} + 0.00916 \gamma \Sigma A_{is}^{0.79} L_{is} + 0.72 C_{sc} \gamma \Sigma Z_{isb}^2 \left[\frac{10800}{k_{isb}} + 0.6 \left(\frac{Z_{isb}}{I_{isb}} \right)^2 \right]$$

$$\begin{aligned}
& + C_{sw} \gamma \sum_{i=1}^n \frac{z_{isb}^2}{I_{isb}} (0.73 + 0.0026 \frac{z_{isb}^2}{I_{isb}} + 0.064 \frac{z_{isb}}{I_{isb}^{0.5}}) + C_{sp} \sum_{i=1}^{n_1} \left[\left(\frac{9.4 I_i / z_i}{2.09 - 0.812 (z_i^4 / I_i^3)} \right. \right. \\
& + \left. \frac{4.5 I_i}{z_i} \right) L_i + \sum_{ir=1}^{n-n_1} (4 L_{ir} A_{ir}^{1/2}) \left. \right] + 8.52 \alpha \gamma N C_{c1} [1 - e^{-15.25 a_{max}}] (0.06556 a_{max} \\
& + 0.0043) \sum_{i=1}^{story} C_i \quad (5)
\end{aligned}$$

in which the six terms respectively correspond to the cost of members, cost of extra size of members, connection cost of beams, welding cost of beams, painting cost of members, and repair cost resulting from seismic damages. The notations in Eq. 5 are C_s = steel price, γ = density of steel, i = number of members, A = cross-sectional area of members, L = member length, C_{sc} = connection material price, z = plastic modulus of a member, I = moment of inertia of a member, isb = number of beams, k = dimensionless factor, C_{sw} = welding price, C_{sp} = painting price, n_1 = total number of beams and columns, ir = bracing members, α = percentage of total construction cost, C_{c1} = total construction cost, N = life expectation of structure, a_{max} = maximum normalized ground acceleration, and C_i = drift coefficient.

Primary and Secondary Design Variables - In Eq. 3, δ_i represents the primary design variable of the element i . For example, a column has four geometric properties that describe the moments of inertia about the major and minor axes, the torsional moment of inertia, and the cross-sectional area. The primary design variable used for columns, beams, and flexural panels is the moment of inertia about the major axis. The secondary design variables include any properties required other than the primary design variable. $\partial h_j / \partial \delta_i$ involves the derivative of structural stiffness with respect to the design variable. The derivative of the stiffness of each member with respect to the primary design variable will be based upon the chain-rule involving the secondary design variables. For simplicity, it is common to express explicitly the relationship between the primary and secondary variables in empirical forms (Ref. 8).

Constraint Functions - The optimization algorithm is based on the energy distribution of the constituent members of a system for which various constraint functions must be first established. Then the optimality criteria can be obtained through the energy distribution based on recursion procedures for stiffness redistribution. The individual constraint functions are briefly described as follows:

a) Flexibility Constraint Function for Static or Dynamic Loads - The flexibility constraints of a system result from the displacement limitations of either certain nodes or all the nodes. The displacement constraint function may be expressed by using the following virtual work at any nodal point:

$$h_j = \{Q_j\}^T \{r\} \quad (6)$$

in which $\{Q_j\}$ is the load vector with a unit value for the j th direction

and zero values for the others, and $\{r\}$ is the vector of generalized displacements attributable to the static load, $\{R\}$. By letting $\{q_j\}$ be a vector of the generalized displacements attributable to $\{Q_j\}$ and differentiating h_j of Eq. 6 with respect to the design variable, δ_i , one obtains

$$\frac{\partial h_j}{\partial \delta_i} = -\{q_j\}^T \frac{\partial [K_T]}{\partial \delta_i} \{r\}. \quad (7)$$

in which $[K_T]$ includes the elastic stiffness $[K_S](\Sigma[K_i])$ and the geometric stiffness $[K_G]$. When the design variable is linear and $[K_G]$ is neglected, $\partial[K_T]/\partial\delta_i = [K_i]\delta_i$ and $\partial W_T/\partial\delta_i = \rho_i l_i n_i$, in which ρ_i is the mass density and n_i the linear factor relating A_i to δ_i . For a single constraint, Eq. 3 reveals that the optimum structure for a specified displacement is the one in which the ratio of the average virtual energy density to the mass density is the same for all its members.

The dynamic displacement constraint function can be expressed in a form similar to that of Eq. 7 in terms of dynamic virtual work. However, $\{Q\}$ and $\{r\}$ are in terms of both time, t , and the design variable, δ . The derivation of the constraint function can now be expressed as

$$\frac{\partial h_j}{\partial \delta_i} = \{q_j\}^T \left\{ \omega^2 \frac{\partial [M_T]}{\partial \delta_i} \{r\} - \frac{\partial [K_T]}{\partial \delta_i} \{r\} \right\} \quad (8)$$

which signifies the virtual strain energy combined with kinetic energy of the i th element for linear relationship of the design variables; $[M_T]$ includes the structural stiffness, $[M_S]$, and the nonstructural stiffness, $[M_n]$.

b) Stiffness Constraint Function for Static or Dynamic Loads - The stiffness of the structure can be described by the work caused by the static or dynamic loads, $\{R\}$, multiplied by the static or dynamic displacements, $\{r\}$, in the form of

$$h_j = 1/2 \{R\}^T \{r\} \quad (9)$$

because the product, $\{R\}^T \{r\}$, is an inverse measure of the stiffness. Thus, h_j may be called a measuring function of the stiffness. The stiffness constraints serve to measure the limitations of the stresses. Differentiating Eq. 9 with respect to the design variables yields Eqs. 10a and b for static and dynamic cases respectively.

$$\frac{\partial h_j}{\partial \delta_i} = -1/2 \{r\}^T \frac{\partial [K_T]}{\partial \delta_i} \{r\}, \quad \frac{\partial h_j}{\partial \delta_i} = 1/2 \{r\}^T \left\{ \omega^2 \frac{\partial [M_T]}{\partial \delta_i} \{r\} - \frac{\partial [K_T]}{\partial \delta_i} \{r\} \right\}, \quad (10a, b)$$

which may be similarly interpreted as the average strain energy of the i th element for the static case and the average strain energy combined with the kinetic energy of any element, i , for dynamic case.

c) Constraint Function for Natural Frequency or Buckling Load - The natural frequency or buckling load of any mode $\{n_j\}$ of a structure can be obtained by using the Rayleigh quotient as expressed in Eqs. 11a and b respectively

$$h_j = \frac{\{n_j\}^T [K_T] \{n_j\}}{\{n_j\}^T [M_T] \{n_j\}}, \quad h_j = \frac{\{n_j\}^T [K_S] \{n_j\}}{\{n_j\}^T [K_G] \{n_j\}}. \quad (11a, b)$$

Differentiation of the above with respect to the design variable yields

$$\frac{\partial h_j}{\partial \delta_i} = \frac{\{n_j\}^T \left[\frac{\partial [K_T]}{\partial \delta_i} - \omega_j^2 \frac{\partial [M_T]}{\partial \delta_i} \right] \{n_j\}}{\{n_j\}^T [M_T] \{n_j\}}, \quad \frac{\partial h_j}{\partial \delta_i} = \frac{\{n_j\}^T \left[\frac{\partial [K_S]}{\partial \delta_i} \right] \{n_j\}}{\{n_j\}^T [K_G] \{n_j\}}. \quad (12a, b)$$

Primary Recursion - The primary recursion relationship is based on the optimality criteria shown in Eq. 1 for resizing members. As a simple illustration, consider a system consisting of c beam-columns and b bracings. The total number of elements is $n = b + c$. The relationship between the primary and the secondary design variables of the beam-column is assumed to be $A = CI_X P$, and the bracing has only the primary design variable of the cross-sectional area, A . Thus, the Lagrange equation can be expressed as

$$L = \sum_{i=1}^c \rho_i^1 C_i I_{xi}^{Pi} + \sum_{i=1}^b \rho_i^1 I_i A_i - \sum_{j=1}^m \lambda h_j. \quad (13)$$

Let the design variables I_{xi} and A_i be denoted by δ_i , then a general expression can be written as $\delta = \Lambda \alpha$ in which Λ is called the scaling factor and α the relative design variable. For beam-columns, let $\mu_i = \delta_i \partial h / \partial \delta_i$, $\mu_i' = \Lambda \mu_i$, $\tau_i = P_i W_i = \rho_i^1 C_i P_i \alpha_i^{Pi} \Lambda^{Pi}$, and $\tau_i' = \tau_i / (\Lambda^{Pi} P_i)$, then the recursion relationship between cycles $v+1$ can finally be obtained by using an iterative procedure, for p active constraints, as

$$(\alpha_i \Lambda)_{v+1} = \alpha_i v \left[\sum_{j=1}^p C_j \frac{1}{P_i^{v+1}} \left(\frac{1}{P_i} \right)^{P_i^{v+1}} \left(\frac{\mu_i'}{\tau_i'} \right)^{\frac{1}{P_i^{v+1}}} \right]_v \quad (14)$$

in which $C_j = \sum_{i=1}^{n_1} [P_i W_i / \delta_i (\partial h_j / \partial \delta_i)]$, and n_1 = number of active elements. Other approximate approaches are to select the largest number of μ_i' / τ_i' for each loading condition and each active constraint (Ref. 5) or to use the least square method for determining the Lagrange multipliers (Ref. 8).

Secondary Recursion - The secondary recursion is used to achieve better optimal results after a local optimal has been obtained. This results from the fact that the primary recursions are coupled with the scaling of the constraints. The secondary recursion is based on the gradients of the active constraints.

SAMPLE EXAMPLES AND OBSERVATIONS

Fifteen-Story One-Bay Frame - The 15-story, one-bay steel frame shown in Fig. 1 is designed according to the equivalent lateral force procedures in Chapter 4 of ATC-3. With regard to the recommendations in the provisions, the design is based on the response modification factor, $R = 8$, the deflection amplification factor, $C_d = 5.5$, and the drift constraint,

$\Delta_a = 0.015 h_{sx}$, in which h_{sx} is the story height below x (details in Case 1 of the next example). In calculating the seismic forces, the coefficients of the effective peak acceleration, A_a , are changed as 1, 3, 5, and 7, and those of the effective peak velocity, A_v , are changed as 3, 5, and 7 for three soil profile types. The optimum structural weights of the 36 design cases are shown in Fig. 2. The seismic design coefficient, C_s , is shown in Fig. 3, and the fundamental periods of the design results are shown in Fig. 4. Note that all the periods in Fig. 4 are greater than the approximate building period, T_a , as well as $1.2 T_a$, which is the upper bound that is supposed to be used in determining the lateral seismic forces as indicated in ATC-3: "The fundamental period of the building, T , in Formula 4-2, $C_s = 1.2 A_v S / (RT^{2/3})$, may be determined based on the properties of the seismic resisting system in the direction being analyzed and the use of establishing methods of mechanics assuming the base of the building to be fixed but shall not exceed $1.2 T_a$." Because T in Formula 4-2 becomes a constant of $1.2 T_a$, the seismic design coefficients vary only when A_v and S change. Figure 3 includes the C_s obtained from Formula 4-3 of $C_s = 2.5 A_a / R$ at $A_a = 1$ and 3, $A_v = 5$ and 7 for both soil types 2 and 3. But Formula 4-3 does not need the period T . Based on the optimum design results, sophisticated mechanics need not be used to find the period because it is $1.2 T_a$ governing the design. Apparently, further studies will be necessary to verify whether the equation $T_a = C_T h_n^{3/4}$, in which C_T is equal to 0.035, and h_n is the height of the building, for steel frames is realistic.

The eccentricity, which is measured from the center of the bay to the outside of the bay and induced by the resultant of the seismic forces and the vertical loads at the foundation-soil interface, is 5.025 ft (1.532 m), 6.025 ft (1.836 m), and 7.545 ft (2.300 m) for $A_a = 7$ and $A_v = 7$ corresponding to soil types 1, 2, and 3 respectively. These numbers are less than $L/4 = 7$ ft (2.134 m). The largest stability coefficient ($\theta = P_x \Delta / (V_x h_{sx} C_d)$), in which Δ is the design story drift, V_x the seismic shear force acting between level x and $x-1$ for all the design cases is 0.015, which is much less than the upper bound of 0.1.

Fifteen-Story Two-Bay Frame - For a more general set of conditions, one additional bay is added to the structure of Fig. 1 as shown in Fig. 5. This case is designed for four cases: 1) ATC-03 equivalent lateral forces for $A_a = 7$, $A_v = 7$, and soil type 3 with $C_d = 5.5$, $R = 8$, allowable story drift $\Delta_a = 0.015 h_x \leq \delta_{xi} - \delta_{xi-1}$ and $\delta_{xi} = C_d \delta_{xei}$ in which δ_{xei} is the actual deflection at the i th floor from analyses; 2) ATC-03 modal analysis procedures with the same data as given in case 1, 3) UBC Code with an allowable displacement of $0.005 h_x$; and 4) Chinese Code on the basis of $0.7 a_{max}/T$ in the spectrum. The drift and displacement constraints are not specified in Ref. 8 or steel structural specifications (only specified for reinforced concrete system Ref. 9). An allowable drift is employed on the basis of $\Delta_a = 0.015 h_x / C_d \leq \delta_{xri} - \delta_{xei-1}$ as in the ATC-03 requirements. Among a number of design results, the structural weights are 118.5k (527.088 kN), 98.57k (442.382 kN), 81.43k (362.201 kN), and 132.89k (591.095 kN) for cases 1, 2, 3, and 4 respectively. The Chinese Code requires the heaviest, which, however, could be different if various drift requirements were to be assumed. The shear envelopes are shown in Fig. 6. The increasing envelope associated with UBC is due mainly to $0.07 TV$ (T = natural period, V = base shear) required at the top floor.

The design results of Case 1 are controlled by 1.2 Ta for the most required forces in the procedure. One may consequently conclude that no eigenvalues need be calculated for this structure associated with other Av's, Aa's, and soil types. It may be worthwhile mentioning that the eccentricity is 5.664 ft (1.726 m) and the stability coefficient is 0.0194, which apparently should be smaller for other Av's and Aa's. The above examples are used to illustrate various cases of parameter studies. It is apparent that the optimization methods within the computer programs can be used for engineering practice in order to accelerate the design process and to evaluate different layouts of a structural plan. They can also be used for academic research in the areas of various code provisions, comparative studies of seismic inputs including the influence of interacting ground motion, parameter studies of the stiffness distribution of a typical system, and others. These studies can be conducted with and without risk.

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REFERENCES

1. ATC-03-6, Applied Technology Council, 1978; Amendments to ATC-03-6, National Bureau of Standards, 1982.
2. Harris, J. T., Fenves, S. J., and Wright, R. N., NBS Technical Note 1100, 1979.
3. Prendergast, J. D. and Choi, C. D., Technical Report M-175, 1976. Dept. of Army, Champaign, Ill.
4. Cheng, F. Y. and Juang, D. S., Optimum Design of 2-D Steel Structures for Static, Earthquake, and Wind Forces Including ATC-03-6, UMR Report No. 83-101 for NSF, 1983.
5. Truman, K. Z. and Cheng, F. Y., "Optimum Design of Steel and Reinforced Concrete 3-D Seismic Building Systems", Proceedings of the 8th WCEE, 1984, U.S.A.
6. Uniform Building Code, International Conference of Building Officials, 1982 Edition.
7. Chinese Seismic Code Provisions for Industrial and Civil Structures (in Chinese), TJ 11-78, 1979.
8. Cheng, F. Y., Srifuengfung, D., and Sheng, L. H., ODSEWS -- Optimum Design of Static, Earthquake, and Wind Steel Structures, Rept. NTIS, PB81-232738, National Science Foundation, January 1981.
9. Chinese Code Provisions for Multi-Story Reinforced Concrete Structures (in Chinese), JZ 102-79, 1980, Beijing, China.

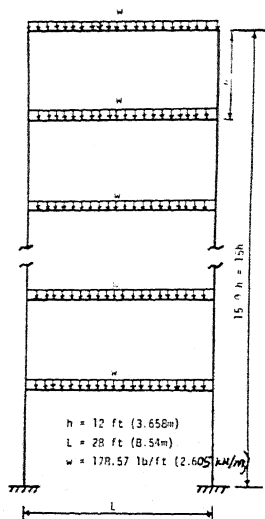


Fig. 1. Fifteen-Story, One-Bay Steel Frame

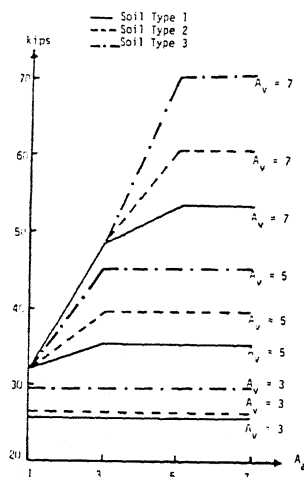


Fig. 2. Optimum Weight vs A_a , A_v , and Soil Types (1 K = 4.448 kN)

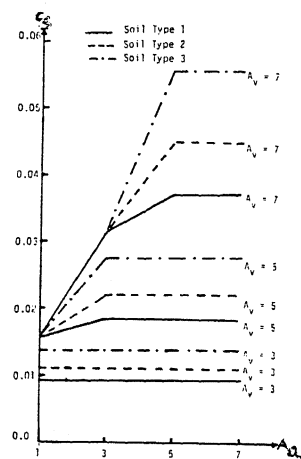


Fig. 3. Seismic Design Coefficient C_s

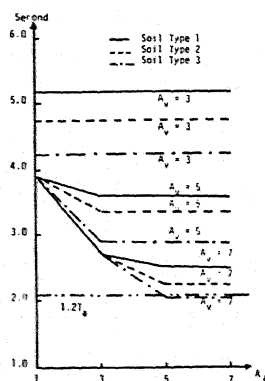


Fig. 4. Fundamental Periods vs A_a , A_v , and Soil Types

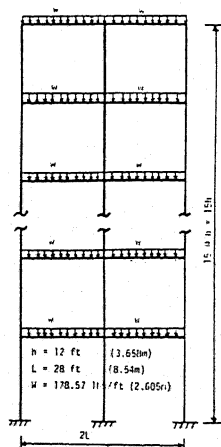


Fig. 5. Fifteen-Story, Two-Bay Steel Frame

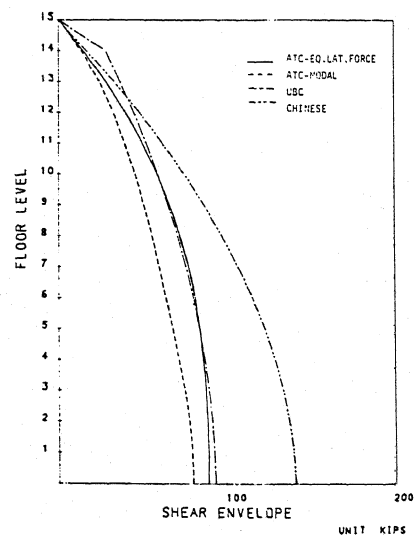


Fig. 6. Shear Envelopes at Optimum Weight of Fig. 5 (1 kip = 4.448 kN)