

INELASTIC CYCLIC BEHAVIOR OF TUBULAR MEMBERS  
IN OFFSHORE STRUCTURES

W. F. Chen (I)  
H. Sugimoto (II)  
Presenting Author: W. F. Chen

SUMMARY

The Finite Segment Method combined with approximate expressions representing the moment-axial force-curvature as well as axial force-moment-axial strain responses of a beam-column segment is found to be a valid and effective method for inelastic cyclic analysis of tubular members and frames. In this paper, the proposed procedure has been applied successfully in the analysis of offshore tubular members and frames in the post-buckling range as well as under cyclic and reversed loading conditions.

INTRODUCTION

Several methods have been developed in the past to predict the post-buckling and cyclic behavior of beam-columns and frames. These include finite element models, physical models, numerical integration methods, and combined phenomenological methods.

The method proposed here is based on a concept similar to the finite element method combined with the tangent stiffness approach, using approximate moment-axial force-curvature and axial force-moment-axial strain expressions to represent the cross sectional properties of a beam-column segment, or the so-called Finite Segment Method (Ref. 2). So far, this method had not been successfully used to solve the beam-column problem with post-buckling or post-maximum branch. In this method, the actual beam-column is physically replaced by an assembly of finite segments. As a result, the beam-column can now be formulated and solved approximately in terms of the behavior of these segments. This approach may be considered as a physical interpretation of the finite difference method as applied numerically to solve differential equations. This method can be applied for general structural systems.

GENERALIZED STRESS-STRAIN RELATIONSHIPS

The generalized stress and generalized strain relationships required in an elastic-plastic beam-column analysis are moment-axial force-curvature ( $M-P-\phi$ ) and axial force-moment-axial strain ( $P-M-\epsilon_0$ ) relations. Recently, Han and Chen (Ref. 3) proposed the refined curve-fitting expressions for tubular sections with constant axial force or constant moment loading paths (so-called constant curves). In this study, these closed form expressions are adopted

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I Professor and Head of Structural Engineering, Purdue University, West Lafayette, Indiana, U.S.A.

II Formerly Research Assistant, Purdue University, West Lafayette, Indiana, U.S.A.

for an arbitrary loading path. However, some modifications in the expressions of tangent axial and bending modulus reduction factors,  $\beta_1$  and  $\beta_2$ , and of unloading and reloading branches are necessary so that they can be applied for an arbitrary loading path.

Curve-fitting expressions for M-P- $\phi$  of the loading branch may be expressed symbolically,

$$m = h(p, \phi) \quad (1)$$

where  $m = M/M_y$ ,  $p = P/P_y$  and  $\phi = \Phi/\phi_y$  are normalized by their corresponding initial yield quantities.

The tangential plane at an arbitrary state of  $p$  and  $\phi$  can be calculated by differentiating Eq.(1) i.e.,

$$\beta_2 = \frac{dm}{d\phi} = \frac{\partial m}{\partial \phi} + \frac{\partial m}{\partial |p|} \frac{d|p|}{d\phi} \quad (2)$$

Since there are an infinite number of tangent values, the tangent stiffnesses required in the analysis are therefore dependent on the value of  $d|p|/d\phi$  or possibly the finite step  $\Delta|p|/\Delta\phi$ . Unfortunately, the value of  $\Delta|p|/\Delta\phi$  can not be determined unless we know the subsequent new state of configuration of deformation. In order to apply this method for analysis, it is therefore necessary to go through some iteration process.

Curve-fitting expressions for P-M- $\epsilon_0$  relations of the loading branch can also be expressed symbolically as

$$\epsilon_0 = n(m, p) \quad (3)$$

The tangential plane for an arbitrary state of  $p$  and  $m$  can be calculated by differentiating Eq. (3) with respect to  $\epsilon_0$ .

$$\beta_1 = \frac{dp}{d\epsilon_0} = \frac{1}{\frac{\partial \epsilon_0}{\partial p} + \frac{\partial \epsilon_0}{\partial |m|} \frac{d|m|}{dp}} \quad (4)$$

For both the unloading and reloading branches, the doubly symmetric assumption is adopted.

#### INCREMENTAL STIFFNESS APPROACH FOR NUMERICAL SOLUTIONS

The incremental force-deformation relationship for a structural member can be written as a set of differential equilibrium equations

$$[K_T] \{du\} = ([K]_C + [K]_G) \{du\} = \{df\} \quad (5)$$

or, when introducing finite steps

$$[K]_T \{\Delta u\} = ([K]_C + [K]_G) \{\Delta u\} = \{\Delta F\} \quad (6)$$

$[K]_T$  may be called the tangential stiffness of the system. This stiffness is a function of the current configuration or possibly of the entire history of deformation.

Tangent bending stiffness  $\beta_2$  and tangent axial stiffness  $\beta_1$  are assumed to be functions of  $d|p|/d\phi$  and  $d|m|/dp$  while the axial force of a segment,  $p$ , is constant in the geometrical stiffness  $[K]_G$  during loading. Therefore, to trace the gradient of the deformation path, the tangent stiffness matrix  $[K]_T$  must be solved iteratively as mentioned previously. This iteration may be achieved in the following manner (Fig. 1);

1. Set the tangential plane for  $\beta_1$  and  $\beta_2$  based on the current configuration ( $p, m, \phi$ ) for each segment.
2. Calculate the tangential values of  $\beta_1, \beta_2$  based on the previous values of  $\Delta p/\Delta\phi$  and  $\Delta m/\Delta p$
3. Assemble segment stiffness matrix and build structural stiffness matrix,  $[K]_T$
4. Apply load  $\{\Delta F\}$  and solve for  $\{\Delta u\}$
5. Calculate the values of  $\Delta p/\Delta\phi, \beta_2, \Delta m/\Delta p$  and  $\beta_1$  based on a new configuration of each segment
6. Check old and new values of  $\Delta p/\Delta\phi, \beta_2, \Delta m/\Delta p$  and  $\beta_1$
7. Repeat steps (2) to (6) until the specified tolerance is met

As tangential planes of moment-thrust-curvature (M-P- $\phi$ ) and thrust-moment-axial strain (P-M- $\epsilon_0$ ) space surfaces are fixed in the elastic range,  $\beta_1$  and  $\beta_2$  are always unities and no iteration is involved during the loading process.

#### AUTOMATIC LOAD CONTROL

Incremental techniques are characterized by the continuous accumulation of truncation errors for non-linear problems. The accumulated error may be affected by the number of load steps and the value of the incremental load adopted in the process. This shortcoming may be somewhat relieved by an automatic load control method proposed by Bergan et al (Ref. 1), which keeps the rate of error at some level in the computer analysis.

Another practical problem associated with an incremental solution of post-buckling or post-maximum branch like beam-column behavior is the necessary sign change of the incremental load when an extreme point of the load-displacement curve has been reached. In Fig. 2, a typical behavior of an axially loaded column is shown. Generally, the descending branch can be detected by checking the sign of the determinant of the structural tangent stiffness

matrix. Here, the easiest way is to apply this procedure to deal with this situation instead of calculating the determinant of the matrix. With the tangent stiffness matrix described in the previous section, the solution of displacement,  $u$ , will follow the path o-a-b with increasing load,  $\Delta P$ . The path a-b actually represents the negative sign of the determinant of the tangent stiffness matrix. Actual behavior o-a-c can be traced by decrementing the load  $\Delta P$  after detecting the sign change of the incremental displacement,  $\Delta u$ . The sign of  $\Delta u$  is some measure of the positive definiteness of the incremental stiffness. This situation can also be explained by Fig. 3. Solution path o-a-b can be obtained by increasing  $\Delta P$  without enforcing the total equilibrium condition. Actual solution path o-a-c can be traced by decrementing  $\Delta P$  after reaching the extreme point a. If point a is exactly at the extreme point numerically, a solution cannot be obtained because the incremental tangent stiffness is zero. But this situation did not arise in this study.

## NUMERICAL RESULTS

Cyclic behavior of an axially loaded tubular column with constant lateral load  $0.4Q_y$  and an initial imperfection of  $0.1\% L$  is shown in Fig. 4 and is compared with Han and Chen's analytical (Ref. 3) and Sherman's test results (Ref. 7), among others (Ref. 4,5,8). Unloading is set to begin with the same axial shortening of 1.3 inches. Reloading for the present analysis is set to start when the tangent bending rigidity reaches to zero.

Portal Types of Beam-Columns

In Fig. 5, cyclic analysis by the present method of portal type beam-columns with constant axial force of  $P/P_y = 0.16$  is shown with Sherman's test (Ref. 6) and Han and Chen's analytical results (Ref. 3).

It is seen that all these three curves are relatively close to one another in their loading and unloading branches. After one cycle, test data shows a degradation of lateral load capacity which probably results from local buckling.

### Framed Structures

Figure 6 presents numerical results of axial force-displacement relations for a simple framed structure with the effect of end restraint from beams. The initial load increments in numerical analysis are 0.8, 0.35, 0.3 and 0.35 kips for the analysis of frames with  $L_1$  equal to  $0.5L_2$ ,  $L_2$ ,  $2L_2$  and infinite. The change of the length of beam,  $L_1$  shows clearly the effect of end restraint on the beam-column behavior where  $L_1 = 0$  represents fixed-ended and  $L_1 = \infty$  represents pin-ended columns in axial load and deflection relations. Axial force and deflection curves of descending branches for restrained beam-columns lie between those for pinned and fixed end cases. Decreasing the length of the beams results in a parallel upwards movement of curves in descent branches.

## CONCLUSIONS

The following conclusions can be made.

1. The combination of Finite Segment Method, generalized stress-strain approach, incremental stiffness approach and automatic load control technique is a valid and effective method for computer-based beam-column and frame analysis. It has been applied successfully in the analysis of tubular beam-columns and frames with arbitrary structural systems with members having large values of shape factor, and different end conditions. But there are limitations for this model. These are the limitations on the length of beam-columns with post-buckling branch and on the analysis of the post-fully plastified branches in tensile loading, where no elastic rigidity can be expected.
2. Sudden changes of tangential axial rigidity in small moment regions of adopted curve fitting axial force-moment-axial strain relationships cause convergence difficulties when analyzing short beam-columns where post-buckling behavior is expected ( $KL/r < 70$ ).
3. The concept of variable axial force and moment in setting tangential bending and axial rigidities of the segments gives fair results of beam-column and frame analysis when using an incremental stiffness method.
4. Doubly symmetric assumption of moment-axial force-curvature and of axial force-moment-axial strain relationships in unloading and reloading branches of the sectional properties appears to be satisfactory in predicting test results of beam-columns as a whole.
5. In order to assess the local buckling behavior reported in some post-buckling and cyclic experimental studies, further studies are needed to predict the effects of geometric changes in cross section of pipe on the moment-axial force-curvature-axial strain relationships.

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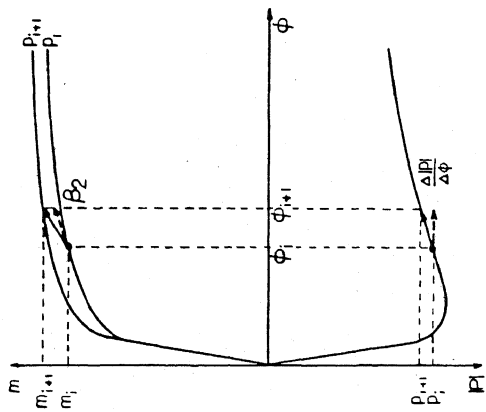


Figure 1. Effect of Variable Axial Force on Bending Rigidity Reduction Factor,  $r_c$

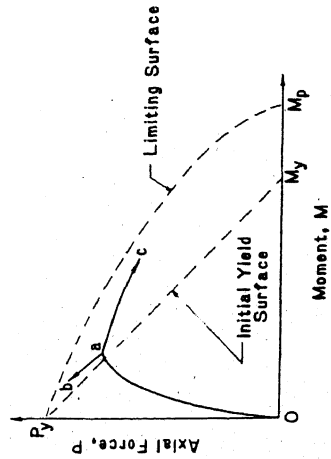


Figure 3. Equilibrium Path of Beam-Column Problem.

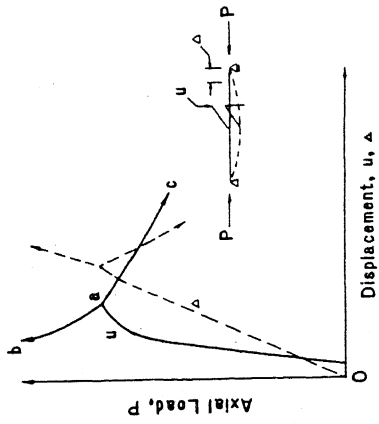


Figure 2. Axial Load - Displacement Relationship for Post-Maximum Regions

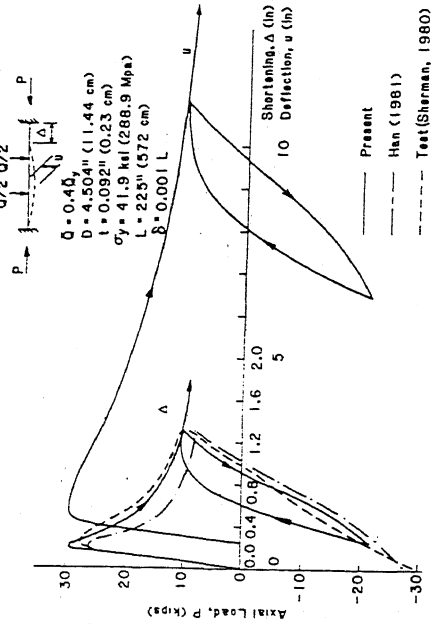


Figure 4. Cyclic Behavior of Fixed-Ended Beam-Column ( $KL/r=72$ ,  $\alpha/\Delta_y=0.4$ )

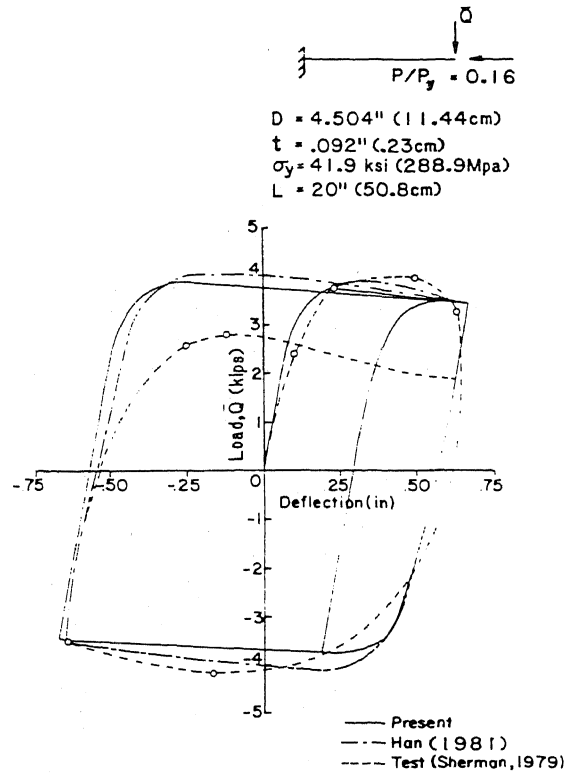


Figure 5. Cyclic Behavior of Portal Type Beam-Column.  
( $K_L = 2.0$ ,  $P/P_y = 0.16$ )

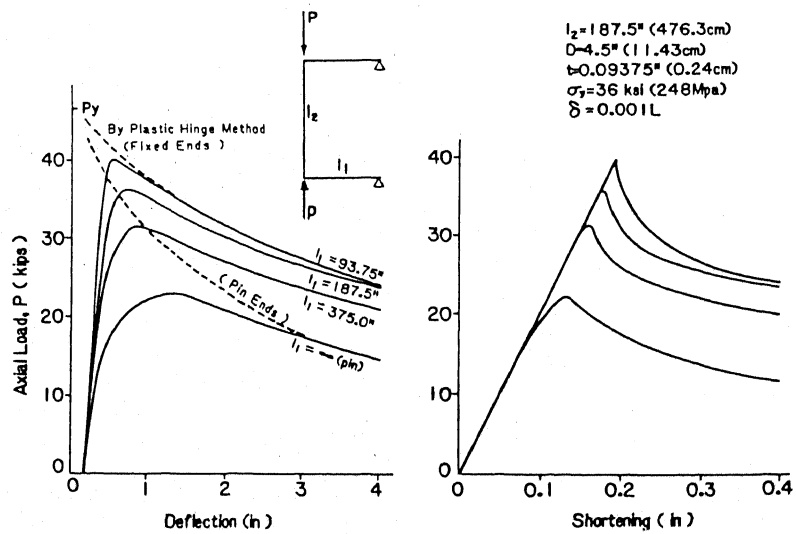


Figure 6. Axial Load - Displacement Relationships for Simple Frame with Different Length of Beams