EARTHQUAKE RESPONSE OF SEA-BASED STORAGE TANKS

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SUMMARY

An effective method for the seismic analysis of submerged underwater storage tanks resting on a horizontal seabed to horizontal earthquake excitations is presented. This approach follows general hybrid-finite element techniques. A comprehensive and predictive computer program for use in such a tank response analysis has been developed for engineering applications. The effects of fluids surrounding and inside the tank are studied. It is demonstrated that these effects have significant impact on the tank response analysis.

INTRODUCTION

Few published works address directly the subject of seismic response of flexible, submerged, underwater storage tanks. The closest ones are those of Takayama (Ref. 1), Tung (Ref. 2) and Helou (Ref. 3). Takayama considered seismically-induced transient waves of oil/water inside rigid cylindrical tanks. Tung treated the problem of water motion surrounding a rigid, submerged cylinder under horizontal ground motion. Helou extended these works to that of a flexible, submerged circular cylindrical tank filled with oil and water.

However, all these works assume small amplitude waves and simple tanks to make available analytic or semi-analytic solutions. For tanks of more complex geometry, numerical techniques must be resorted to. Previously, the finite element method was used by Liaw and Chopra (Ref. 4.) to study the seismic response of a flexible axisymmetric intake tower, surrounded by water. But, the water inside the tower is treated as a lumped mass adding to that of the tower exploiting the slenderness of the tower. Mei *et al.* (Ref 5.) considered the seismic response of flexible dams and offshore structures in which compressibility of fluid is introduced (but there are no internal waves). Here, the use of the finite element method for exterior water motion was enhanced by the adoption of an analytic super-element.

In this study, we formulate and develop a computer code to analyze the seismic response of flexible underwater storage tanks. The tank in question is axisymmetric and rigidly attached to the ocean floor; it is completely filled with oil and water. The ocean floor inside and outside the tank is assumed to be level, and only horizontal ground motion is considered.

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FORMULATION

Let the axisymmetric tank of Figure 1 be filled with oil and water (of densities ρ_o and ρ_w, respectively), with respective heights of h_o and h_w, and let the coordinate system be as shown in Figure 1. Discretize the tank into toroidal finite elements with quadrilateral (vertical) cross-section. The equation of motion of the tank structure can then be written

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{\ddot{f}_h\} - \{P_S\} \tag{1}$$

where [M], [C] and [K] are the mass, damping and stiffness matrices of the system, respectively. $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ are the acceleration, velocity and displacement vectors of the nodal circles relative to its base, and $\{\ddot{f}_h\}^T = (1,0,-1,1,0,-1,\ldots,1,0,-1)|\ddot{f}_h|$, where $\ddot{f}_h$ is the horizontal ground acceleration. To solve for $\{x\}$, we first need some information on $\{P_S\}$, the hydrodynamic pressure due to the presence of fluids inside and outside of the tank.

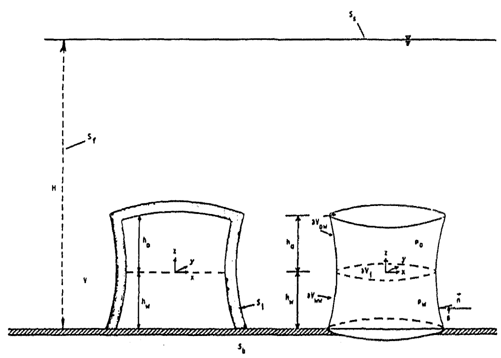

Figure 1. Coordinate system for axisymmetric submerged storage tank.

For interior fluids, the pressure P is solved from the following boundary value problem, based on the usual assumptions of inviscid and incompressible fluids, irrotational motion and small amplitude waves:

$$\nabla^2 P_o = 0 \quad \text{in } V_o \quad \text{and} \quad \nabla^2 P_w = 0 \quad \text{in } V_w \tag{2}$$

$$\left.\begin{aligned} \frac{\partial P_o}{\partial n} &= -\rho_o[(\ddot{f}_h + \ddot{x}) \cdot n] \quad \text{across } \partial V_{ow} \\ \frac{\partial P_w}{\partial n} &= -\rho_w[(\ddot{f}_h + \ddot{x}) \cdot n] \quad \text{across } \partial V_{ww} \end{aligned}\right\} \tag{3}$$

$$\frac{\partial P_w}{\partial z} = 0 \quad \text{along } \partial V_{wb} \quad (z = -h_w) \tag{4}$$

and $P_o = P_w$ along $z = 0$, (5)

wherein the suffixes o and w refer to quantities pertaining to oil and water, respectively. Or, equivalently, we can consider the following variational functional in P_o and P_w:

$$F(P_o,P_w) = \int_{V_o} \frac{1}{2\rho_o} (\nabla P_o)^2 \, dV_o + \int_{\partial V_{ow}} P_o[(\ddot{\vec{f}}_h + \ddot{\vec{x}}) \cdot \vec{n}] \, dA_{ow}$$

$$- \int_{\partial V_i} \frac{1}{2\rho_o} (P_w - P_o) \frac{\partial P_w}{\partial z} \, dA_i$$

$$+ \int_{V_w} \frac{1}{2\rho_w} (\nabla P_w)^2 \, dV_w + \int_{\partial V_{ww}} P_w[(\ddot{\vec{f}}_h + \ddot{\vec{x}}) \cdot \vec{n}] \, dA_{ww}$$

$$- \int_{\partial V_i} \frac{1}{2\rho_o} (P_w - P_o) \frac{\partial P_o}{\partial z} \, dA_i \ , \qquad (6)$$

whose stationarity gives an equivalent system of equations to (2)-(5).

For the discretization of the tank system with toroidal elements and bilinear interpolation functions, using cylindrical global coordinates and natural local coordinates, we can represent equation (6) in the following form:

$$F(\{P_o\},\{P_w\}) = \{P_o\}^T[K_{V_o}]\{P_o\} + \{P_w\}^T[K_{V_w}]\{P_w\}$$

$$+ \{P_o\}^T[0] \ (\{\ddot{f}_h\}+\{\ddot{x}\}) + \{P_w\}^T[W] \ (\{\ddot{f}_h\}+\{\ddot{x}\})$$

$$+ \{P_o\}^T[K_{io}]\{P_o\} + \{P_w\}^T[K_{iw}]\{P_w\}$$

$$+ \{P_o\}^T[K_{ow}]\{P_w\} + \{P_w\}^T[K_{wo}]\{P_o\} \ . \qquad (7)$$

Here the [K]'s are global stiffness matrices and [0] and [W] are global matrices assembled for the corresponding element matrices and

$$\{\ddot{x}\}^T = (\ddot{x}_{1r},\ddot{x}_{1z},\ddot{x}_{1\theta},\ddot{x}_{2r},\ddot{x}_{2z},\ddot{x}_{2\theta},\ldots,\ddot{x}_{Nr},\ddot{x}_{Nz},\ddot{x}_{N\theta}) \ .$$

Let NO and NW be, respectively, the number of elements in $\{P_o\}$ and $\{P_w\}$. Since $P_o = P_w$ at the interface, from the stationarity of F, we have a set of NO + NW - (number of interfacial nodes) equations

$$2[K_{V_o}]\{P_o\} + 2[K_{io}]\{P_o\} + [K_{ow}]\{P_w\} + [K_{wo}]^T\{P_w\}$$

$$= - [0] \ (\{\ddot{f}_h\} + \{\ddot{x}\}) \ , \qquad (8)$$

and

$$2[K_{V_w}]\{P_w\} + 2[K_{iw}]\{P_w\} + [K_{ow}]^T\{P_o\} + [K_{wo}]\{P_o\}$$

$$= - [W] \left(\{\ddot{f}_h\} + \{\ddot{x}\}\right) . \tag{9}$$

If we define $\{P_{ow}\}$ to be the vector of interior nodal pressure distributions, ordered according to the global nodal number sequence (allowing no repetition on interface), the matrix equations of equation (9) can be combined to give:

$$[K_{INT}] \{P_{ow}\} = - [OW] \left(\{\ddot{f}_h\} + \{\ddot{x}\}\right) . \tag{10}$$

In the case of a rigid tank, $\{\ddot{x}\} = 0$ and (10) can be readily solved for the interior pressure distribution $\{P_{ow}\}$. For flexible tanks, (10) is used as input to the general equations of motion of the tank structure.

Now, to get the exterior fluid pressure, consider the same tank submerged in water of depth H as depicted in Figure 1. Under the same assumptions of inviscid and incompressible fluid, irrotational motion, and small amplitude waves, the governing system is:

$$\nabla^2 P_w = 0 \qquad \text{in } V \tag{11}$$

$$\frac{\partial P}{\partial z} = 0 \qquad z = -h_w \tag{12}$$

$$\frac{\partial P}{\partial n} = - \rho_w \left[(\ddot{\vec{f}}_h + \ddot{\vec{x}}) \cdot \vec{n}\right] \qquad \text{across } S_i \tag{13}$$

$$P = 0 \text{ at } z = H-h_w \tag{14}$$

$$P \to 0 \quad \text{as } r \to \beta. \tag{15}$$

To facilitate discretization, we adopt the hybrid approach of using the available analytic solution for P beyond a few wavelengths away (marked S_f in Figure 1) from the tank, as soon as most of the significant geometrical irregularities have passed by. Using Galerkin's variational technique with Green's theorem, and the matching of numerical and analytic solution for P at S_f, we can represent

$$\int_V T_k(\nabla^2 P)dV = 0 = - \int_V (\nabla T_k \cdot \nabla P)\, dV + \int_S T_k \frac{\partial P}{\partial n}\, dS \tag{16}$$

in the equivalent global (finite element "coordinate") form as

$$- [H_V]\{P\} - [Q]^T[A]^{-1}[Q]\ \{P\} - [B]\left(\{\ddot{f}_h\} + \{\ddot{x}\}\right) = 0 . \tag{17}$$

Using equations (10) and (17), we are now ready to solve equation (1). An effective way to treat axisymmetric structures is by expanding the loads and displacements into Fourier series in θ. For the case under consideration, because of the simple nature of the ground excitation, dynamic response analysis of the tank requires the use of only the first term of the Fourier series. Since damping in a structure is usually defined in terms of modal damping ratios, and since in our study the modal superposition method will be used, explicit evaluation of the elements of the damping matrix is not necessary. On the other hand, the procedure to establish the mass and stiffness of axisymmetric structures is well documented in the literature. Now the hydrodynamic pressures act only on the inner and outer surfaces of the tank, thus the elements in $\{P_S\}$ corresponding to non-interfacial nodes are zero. In fact, $\{P_S\}$ has many more null entries: since the pressures act in the direction of the surface normal, all θ-components (circumferential) vanish and if a section of the interface (inner or outer) is cylindrical, the corresponding z-components also vanish. We can, therefore, strip off all non-interfacial modal components from $\{P_{ow}\}$ of (10) and $\{P\}$ of (17), rewrite them in terms of the r,z,θ components and extend them to $\{P_S\}$. At the same time, $[K_{INT}]$ of (10) and $[H_V] + [Q]^T[A]^{-1}[Q]$ of equation (17) are reassembled and combined to yield:

$$[H]\{P_S\} = - [F] \left(\{\ddot{f}_h\} + \{\ddot{x}\}\right) , \tag{18}$$

where [F] incorporates the contributions from [OW] and [B].

If the inversion of [H] can be done efficiently, we can then solve straightforwardly the 2nd order system obtained by substituting (18) into (1):

$$\left([M] - [H]^{-1}[F]\right)\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = - \left([M] - [H]^{-1}[F]\right)\{\ddot{f}_h\} . \tag{19}$$

But, if we want to avoid inverting [H], the modal superposition method commonly used in structural analysis can be utilized. Therein the structural response x, $\dot{x}$ and x are expressed in terms of the eigenvectors (mode shapes $\{\phi\}$) of the undamped structural vibrations (i.e., without fluids). The $\{\phi\}$'s are obtained from the following eigenvalue problem:

$$[K]\{\phi_j\} = \omega_j^2 \, [M]\{\phi_j\} . \tag{20}$$

The displacement $\{x\}$ of the structure can be expressed as a linear combination of the $\{\phi\}$'s as:

$$\{x\} = \sum_{j=1}^{J} \{\phi_j\} \, Y_j , \tag{21}$$

in which the Y_j's are the generalized coordinates. The above expansion is exact if J is equal to the total number of degrees of freedom, 3N, of the structure finite element system. Generally, for earthquake-scale excitations, the responses can be fairly well approximated by using only the first few modes.

Now equation (18) can be written as:

$$[H]\{P_S\} = -[F]\{\ddot{f}_h\} - \sum_{j=1}^{J} [F]\{\phi_j\}\ddot{Y}_j \quad , \tag{22}$$

and by writing $\{P_S\}$ as:

$$\{P_S\} = \sum_{j=1}^{J} \{P_{sj}\}\ddot{Y}_j + \{P_{so}\}\ddot{f}_h \ , \tag{23}$$

our problem reduces to solving

$$([M_j^*] + \{\phi_j\}^T\{P_{sj}\})\ddot{Y}_j + [C_j^*]\dot{Y}_j + [K_j^*]Y_j = -(\{\phi_j\}^T[M]\{L\} + \{\phi_j\}^T\{P_{so}\})\ddot{f}_h \qquad j = 1,\ldots,J \tag{24}$$

without having to invert [H].

NUMERICAL RESULTS

Consider a tank as described in the FORMULATION having a radius of 10 ft at the base and 10 ft in height. Assume that the oil and water layers are of the same thickness. Let the inclination angle of the tank wall from the vertical axis be β. Using $\rho_o = 53.69$ and $\rho_w = 62.43$ lb/ft^3, and considering a harmonic excitation of frequency $\omega = 10$ rad/s, the amplitude of pressure distributions along the side wall at $\theta = 0°$, for $\beta = 0°$, 15° and 30° have been calculated and are presented in Figure 2. A simple finite element discretization with two 10-element columns is used for these calculations with the element size being thinner near the fluids interface. (Calculations were also performed using ten 10-element columns and the results are basically the same.) Note that the surface wave effects are included in the computation which accounts for the difference in oil and water pressure at $z = 0$. Such difference, however, is seen to be slight and can generally be neglected. A typical run for a 20-element grid requires less than 2 s CPU time on a CDC Cyber 176 computer.

Now assume the same tank is submerged in water of 20 ft in depth. Use a 45-element mesh for the exterior water and take five terms for the far field series solution, we then calculated the pressure distributions on the tank wall at $\theta = 0°$ for $\omega = 10$ rad/s. The pressure is converted to the MKS unit to compare with the results obtained by Tung (Ref. 2) with good agreement. A typical run for the 45-element idealization is of the order of 1.5 s CPU time on a CDC Cyber 176.

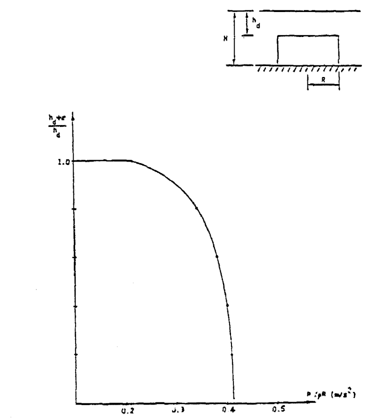

Figure 2.
Pressure distribution at inner wall (θ = 0°) of a submerged inclined tank.

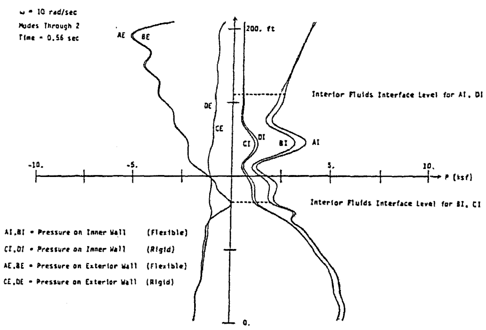

Figure 3.
Pressure distribution at tank wall (θ = 0°) of a Khazzan Dubai tank submerged underwater.

Finally, we consider a so-called Khazzan Dubai tank submerged under 240 ft of water; that is, the tank is an inverted "funnel" with a 135 ft radius base and a roof which is a portion of a 180-ft radius hemisphere; a conical transition connects the roof to a 15-ft radius shaft whose top is 35 ft from the water surface. The tank shell is taken to be uniformly 1 ft thick, made of steel with modulus of elasticity 4.3 x 10^6 ksf, and having Poisson's ratio of 0.27 and mass density of 0.01519 $k \cdot s^2/ft^4$. Five terms are used for the far field series solution whereas the harmonic excitation frequency is chosen as 10 rad/s. The resultant pressure distribution for modes through 2 at the inner and outer tank wall (at θ = 0°) is presented in Figure 3 (curves AI, BI, AE, BE) for interior oil-water interface level at 84.5 ft and at 155 ft (from ocean floor). The tank shell's fundamental vibration frequencies are ω_1 = 20.7 rad/s and ω_2 = 88.3 rad/s. Surface effects are ignored for both the internal waves and exterior waves. The typical CPU time for a run of this realization is on the order of 3.5 s on a CDC Cyber 176. On a VAX-11/750 it takes about 2 minutes, but with proper adaptation to using an array processor in solving the eigenproblem, the time can be significantly reduced. Also presented in Figure 3 are the pressure curves (CI, DI, CE, DE) for a rigid Khazzan Dubai tank of the same kind. It is seen that the flexibility of the tank shell plays a significant role in the tank's dynamic response.

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