HORIZONTAL, VERTICAL AND ROCKING FLUID-ELASTIC RESPONSE AND DESIGN OF CYLINDRICAL LIQUID STORAGE TANKS

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SUMMARY

Fluid-elastic behaviors with horizontal, vertical and rocking movement of an ideally cylindrical liquid storage tank are discussed, and an experimental verification on model tanks and simplified procedures for analyzing each type fluid-elastic response to earthquakes are presented.

INTRODUCTION

Seismic problems of liquid storage tanks have come to the attention of engineers in Japan as well as in other countries. Various viewpoints should be considered in the seismic evaluation of cylindrical liquid storage tanks. First of all fluid-elastic interaction between a contained liquid and a flexible tank wall is one of the most important phenomena.

In earlier days a tank was analyzed assuming that it is rigid (Refs. 1, 2) or not exactly taking the interaction into account (Refs. 3, 4). However, recently many studies on the fluid-elastic response analysis to earthquakes have been reported since Edwards (Refs. 5 ∿ 10). Some researchers (Refs. 11 ∿ 14) have discussed uplift at the base and cos $n\theta$ vibrational modes along the circumference. Also previously Kana (Ref. 15) pointed out the possibility of parametric resonance when a cylindrical tank was excited longitudinally.

This paper will deal with the fluid-elastic hehaviors of an ideally cylindrical tank. Non-linear effects such as the base uplift and so on will not be considered, but the detailed evaluation of so basic hehaviors has still to be carried out. The authors will present their methodology and experimental verification, and will propose simplified procedures of analysis for the design practice.

THEORETICAL TREATMENT

Fundamental Equations

Let us consider a cylindrical tank containing a liquid as shown in Fig. 1. In order to apply the finite element method, the following primary-complementary mixed variational functional is derived (Ref. 16):

$$L(u, v, w, \eta, \phi) = \int_0^{2\pi}\int_0^H \frac{mh}{2}(\dot{u}^2+\dot{v}^2+\dot{w}^2)\, a d\theta dz - \pi(u, v, w) - \int_0^{2\pi}\int_0^a\int_0^H \frac{\rho}{2}\,\mathrm{grad}^2\phi\, r d\theta dr dz + \int_0^{2\pi}\int_0^H \rho\dot{w}\phi\, a d\theta dz + \int_0^{2\pi}\int_0^a \left(\rho\dot{\eta}\phi - \frac{\rho g}{2}\eta^2\right) r d\theta dr \quad (1)$$

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in which u, v, w are the displacements of the tank, η and ϕ the elevation change and the velocity potential of the liquid, m and ρ the mass density of the tank and the liquid, g the acceleration of the gravity force and π the strain energy of the shell (including that of the base foundation in the case of rocking movement).

The assumptions are as follows:

(1) The flow of the liquid is inviscid, incompressible and irrotational.
(2) The displacements are infinitesimal.
(3) The base uplift is neglected.
(4) The shell is ideally cylindrical.

Earthquake inputs are expressed in the following form. In the case of horizontal and rocking movement,

$$\left.\begin{array}{l} v = -\overline{X}\sin\theta \\ w = \overline{X}\cos\theta \end{array}\right\} \text{ at } z = 0 \tag{2}$$

When the tank is fixed at the base, the boundary condition u = 0 at z = 0 is added. In the case of vertical movement,

$$\left.\begin{array}{l} u = \overline{Z} \\ v = w = 0 \end{array}\right\} \text{ at } z = 0 \tag{3}$$

Finite Element Formulation

Let the degrees of freedom composed of the tank displacements, the elvation change and the velocity potential be δ, η and ϕ respectively in applying the finite element method. The equations are as follows:

$$\begin{bmatrix} M + S_1H^{-1}S_1^T & S_1H^{-1}S_2^T \\ \text{sym.} & S_2H^{-1}S_2^T \end{bmatrix} \begin{Bmatrix} \ddot{\delta} \\ \ddot{\eta} \end{Bmatrix} + \begin{bmatrix} K + Kr & 0 \\ \text{sym.} & Kf \end{bmatrix} \begin{Bmatrix} \delta \\ \eta \end{Bmatrix} = -\ddot{\overline{X}} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \text{ or } -\ddot{\overline{Z}} \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix} \tag{4}$$

in which $S_1H^{-1}S_2^T$ represents the coupling effect between the shell deformation and the sloshing, but the effect is negligible, so we have only to consider δ for short-period components of earthquake waves (Ref. 6)*. $S_1H^{-1}S_1^T$ does the added mass of the liquid to the tank. Kr does the rotational spring stiffness of the base foundation.

FLUID-ELASTIC RESPONSE TO EARTHQUAKES

Horizontal Response

Modal Analysis. Using the natural modes, we can obtain the following equation (see Ref. 13):

$$P = P_0 + \sum_{i=1}^{\infty} P_{1i} + \sum_{j=1}^{\infty} P_{2j} \tag{5}$$

in which P is the total dynamic pressure, P_0 the impulsive pressure, P_{1i} the fluid-elastic pressure and P_{2j} the convective pressure. For usual short-period earthquakes the last one of the right-sided terms is negligibly small

* An example of fluid-elastic coupling associated with sloshing can be found between a contained liquid and a floating roof (Ref. 17).

compared with the first and second ones since the natural period of sloshing is considerably long. Thus

$$P = P_0(z)\,\ddot{\bar{X}} + \sum_{i=1}^{\infty} P_{1i}(z)\,\ddot{A}_i \tag{6}$$

in which $\ddot{A}_i$ is the relative acceleration of the ith mode.

Usually the first modal response is predominant over the others. Then

$$P \approx p_0(z)\,\ddot{\bar{X}} + P_{11}(z)\,\ddot{A}_1 \tag{7}$$

$\ddot{A}_1$ is determined by the natural period and the damping ratio. The range of the fundamental natural period calculated under the assumption of fixed base is shown in Fig. 2 for actual oil storage tanks (Ref. 18).

Experimental Verification. Some experiments were conducted to verify the above-mentioned phenomenon. Fig. 3 illustrates results of the authors' test, which indicates good agreement between the experiment and the theory.

Simplification. For practical use approximate procedures are needed. Detailed analysis of the fluid-elastic interaction yields the following simplified equations. For the natural period of the first mode (Ref. 18),

$$T_b = \frac{2}{\lambda}\sqrt{\frac{W}{\pi g E h_{1/3}}} \tag{8}$$

in which $\lambda = 0.067\,(H/D)^2 - 0.30\,(H/D) + 0.46$ provided that $0.15 \leq H/D \leq 2.0$, W is the total weight of the liquid, E the Young's modulus of the tank material and $h_{1/3}$ the wall thickness at one-third of the height H. For the dynamic pressure (Ref. 19),

$$P_{max} = p_0(z)\,|\ddot{\bar{X}}|_{max} + p_{11}(z)(\alpha_h - |\ddot{\bar{X}}|_{max}) \tag{9}$$

in which α_h is the maximum absolute horizontal acceleration.

From Eqn. 9 the base shear and the base overturning moment are expressed as follows:

$$Q \approx W_0\,|\ddot{\bar{X}}|_{max} + W_1(\alpha_h - |\ddot{\bar{X}}|_{max}) \tag{10}$$

$$M \approx W_0 H_0\,|\ddot{\bar{X}}|_{max} + W_1 H_1(\alpha_h - |\ddot{\bar{X}}|_{max}) \tag{11}$$

These results coincide well with recent Housner et al. (Ref. 9) as shown in Fig. 4.

Vertical Response

Similarly to the above-mentioned horizontal response, the following simplified equations are derived (Ref. 20). For the fundamental natural period,

$$T_v = \frac{2}{\lambda_v}\sqrt{\frac{W}{\pi g E h_{1/3}}} \tag{12}$$

in which $\lambda_v = \sqrt{I_1(\pi H/4D)/I_0(\pi H/4D)}/2.356$.

For the dynamic pressure,

$$P_{v,\ max} \sim PH \{ \frac{z}{H} |\ddot{Z}|_{max} + 0.811 \cos \frac{\pi z}{2H} (\alpha_v - |\ddot{Z}|_{max}) \} \tag{13}$$

in which α_v is the maximum absolute vertical acceleration.

Rocking Response

Whole and Partial Rocking. There are two types of base foundation for liquid storage tanks. One is with a concrete slab which is frequently used for a high-pressurized tank, and the other with compacked soil mound mainly for a oil storage tank. These types of base foundation are reduced into the two rocking models; 'whole rocking' and 'partial rocking' in Fig. 5. Each rotation spring rigidity can be estimated as follows:

$$k_r = \begin{cases} \frac{\pi D^4}{64} k_s & \text{for whole rocking} \\ \frac{\pi b D^3}{8} k_s & \text{for partial rocking} \end{cases} \tag{14}$$

in which k_s is the Winkler's spring coefficient and b the effective width of the foundation.

Calculated fundamental fluid-elastic natural period of each model is shown in fig. 6 as a ratio to that of horizontal fluid-elastic vibration for the fixed base. The increase of the ratio of the height to the diameter decreases the difference between both.

Simplification. The above-mentioned natural period of coupling between fluid-elastic and rocking is approximated as follows:

$$T_{br} = \sqrt{T_b^2 + T_r^2} \tag{15}$$

in which T_b is given in Eqn. 8, and

$$T_r = \frac{2}{\lambda r} \sqrt{\frac{W(D/2)^2}{\pi g k_r}} \tag{16}$$

λr is shown in Fig. 7. The error of T_{br} in Eqn. 15 is one to two percent.

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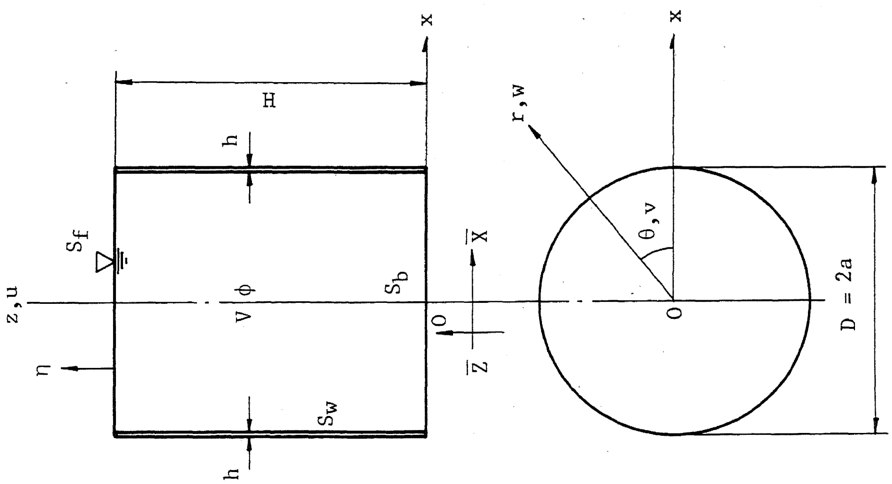


Figure 1 Notations

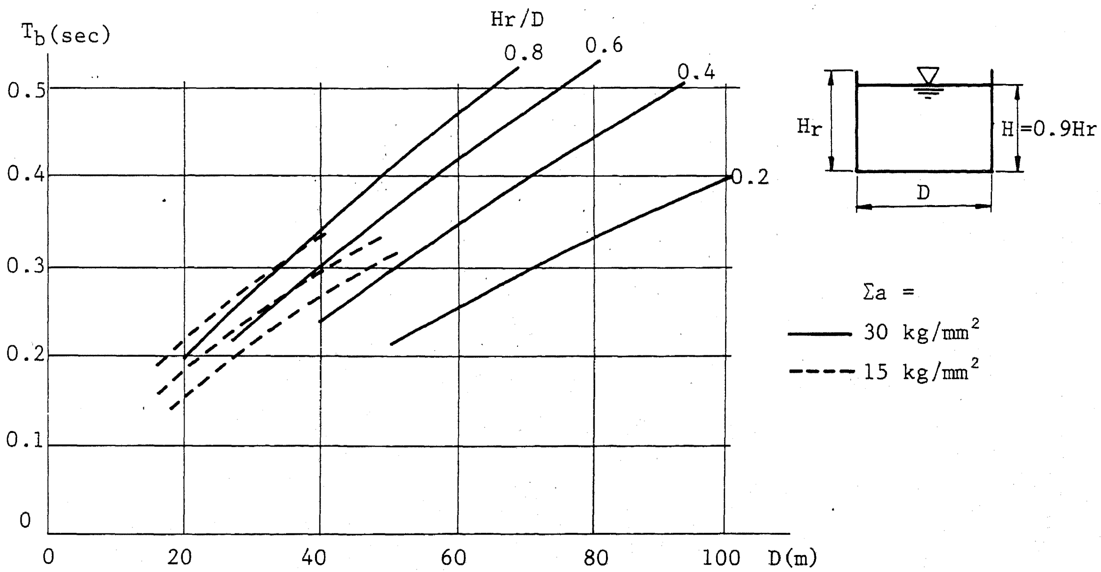


Figure 2 Fundamental Fluid-Elastic Natural Period of Oil Storage Tanks in Japan

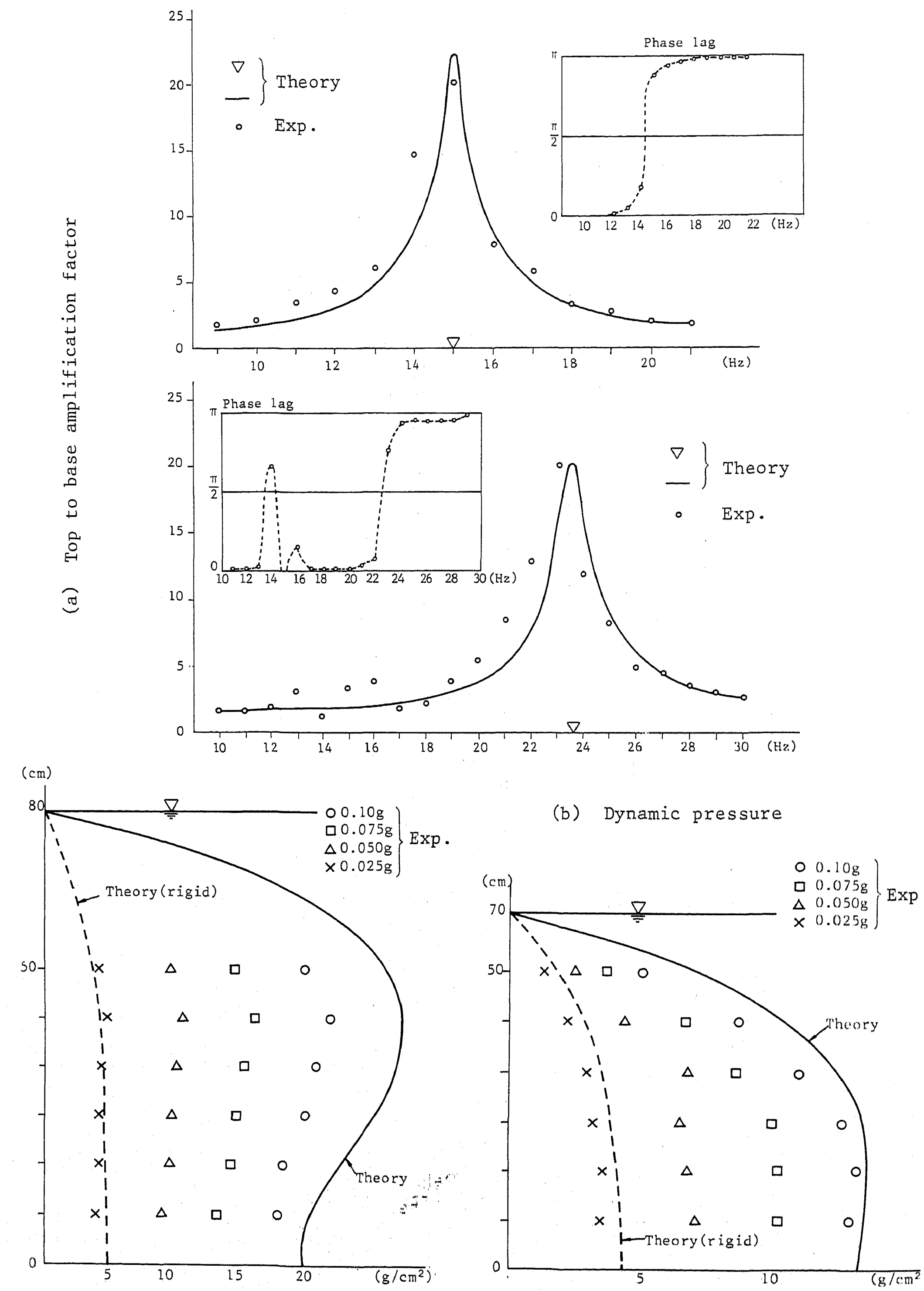


Figure 3 Example of Experimental Verification

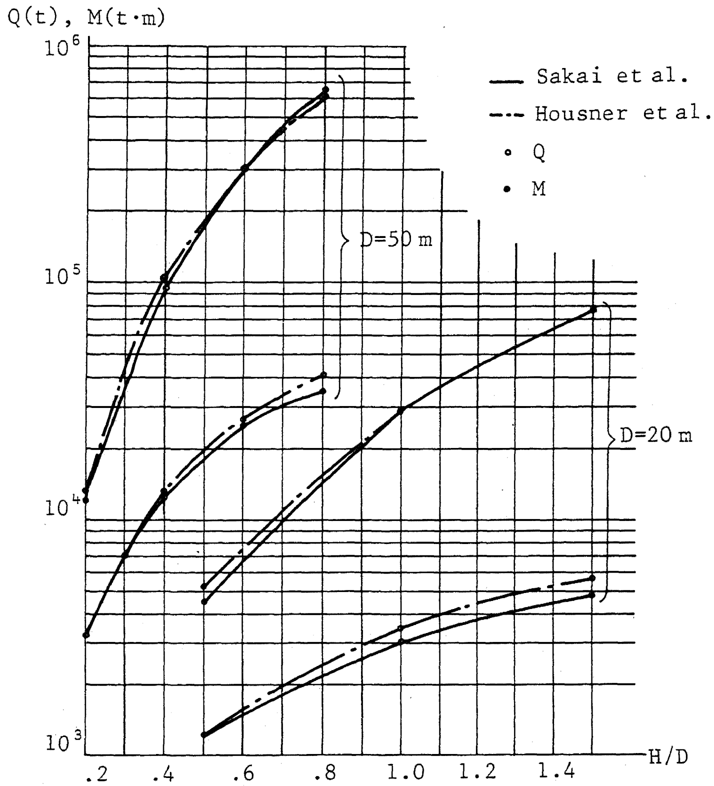
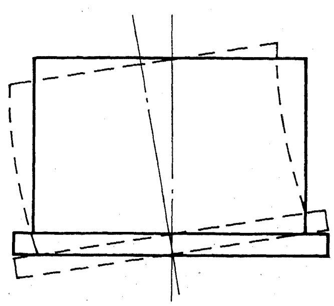
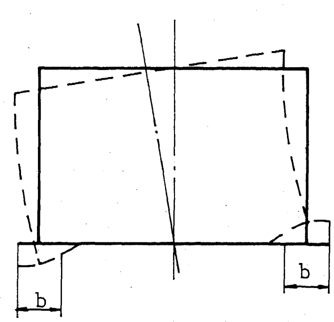


Figure 4 Comparison of Base Shear and Overturning Moment



(a) Whole rocking



(b) Partial rocking

Figure 5 Two Types of Rocking Model

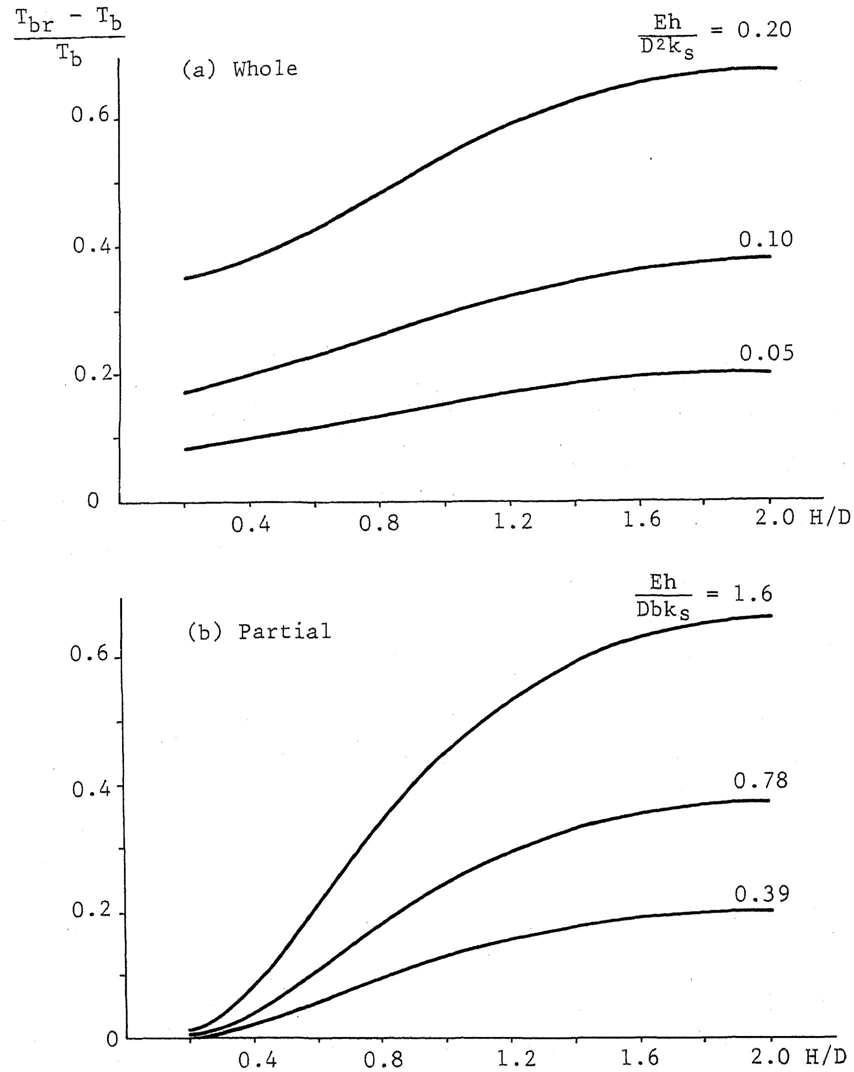


Figure 6 Elongation of Natural Period by Rocking

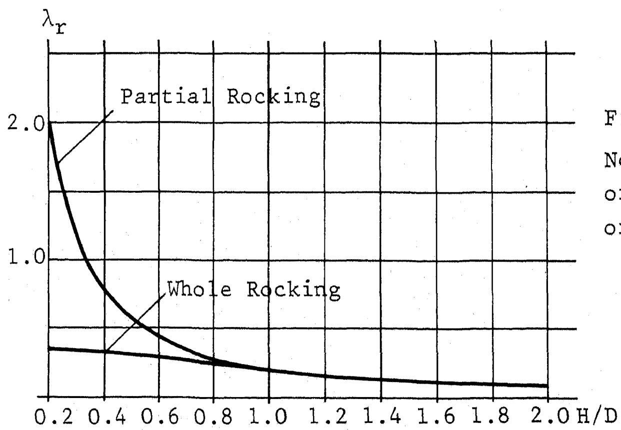


Figure 7
Non-dimensional Parameter of Natrual Period of Rocking