NONLINEAR DYNAMIC ANALYSIS OF CYLINDRICAL TANKS WITH IMPERFECT CIRCULAR SECTION CONTAINING LIQUID

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SUMMARY

The dynamic response of thin elastic cantilevered cylindrical tanks partially filled with liquid related to horizontal excitations is studied.

The circumferential section of the tank is not a perfect circle and the radius is represented by a Fourier series. Fundamental equations are based on Novozhilov's shell theory for the description of tank motion, which considers the nonlinear strain-displacement relations, and a classical potential flow theory for the coupled hydrodynamic forces.

Numerical results show the effects of modes of a high order to the oval deformations of circular section.

INTRODUCTION

In the design of circular cylindrical shell tanks containing liquid, it is necessary in many cases to know the interaction of the shell with the liquid to earthquake excitations. This problem has been studied by many investigators and most previous analytical work has been based on the assumption which only the first mode of circumferential wave-perfect circle, has been excited by horizontal ground motion because of the orthogonality of the circumferential displacement functions. However D.P.Clough and A.Niwa have presented in 1978 (Ref. 1) that significant out-of-round displacements have been observed in tanks and the base section axial stresses associated with the out-of-round displacement of the fixed-base tall tank are about one-fourth the magnitude of the basic nondistorting stress component. They have indicated initial imperfections of the tank geometry as the cause.

To explain this experimental results, authors (Ref. 2) and A.S.Veletsos et al. (Ref. 3) have respectively presented the analytical study in 7th WCEE in which the radius of tank has been expanded in a Fourier series to consider the coupling of the first mode with modes of a high order. But it has been clear from these results that responses corresponding to experimental results could not been obtained by the analysis using linear shell theory.

Then in this report, the coupling phenomenon of circumferential wave modes of a high order related to liquid-tank system with imperfect circular section are formulated and solved analytically by a nonlinear shell theory and a harmonic balance method (Ref. 4,5). Fundamental equations are based on Novozhilov's shell theory , which considers the nonlinear strain-displacement relations, and a classical potential flow theory for the coupled

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hydrodynamic forces. Substituting these equations to the Lagrange's equations and applying harmonic balance method, these equations are reduced to a system of nonlinear simultaneous algebraic equations with respect to the components of circumferential wave mode for each order. Solving these equations, the stationary response solutions to harmonic excitations are derived.

DERIVATION OF NONLINEAR GOVERNING EQUATIONS

The System Considered

The tank is thin and elastic shell and contains liquid. The coordinate system of a free-fixed cylindrical shell to be investigated is as shown in Fig.1. The radius of the shell is represented by the following Fourier series because of the imperfect circular section.

$$a(\vartheta) = a_0 + \sum_{j=1}^{J} a_j \cos j\vartheta + \sum_{j'=1}^{J'} a_{j'} \sin j'\vartheta \tag{1}$$

where a_0 is the radius of the shell with perfect circular cross section and a_j , $a_{j'}$ are the coefficients of the initial imperfection.

In this study, restricting the equations given by Novozhilov to the circular cylindrical shell, the strain-displacement relations take the form (Ref. 6).

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_\theta = \frac{1}{a}\left(\frac{\partial v}{\partial \vartheta} + w\right) - \frac{z}{a^2}\left(\frac{\partial^2 w}{\partial \vartheta^2} - \frac{\partial v}{\partial \vartheta}\right) + \frac{1}{2}\left(\frac{1}{a}\frac{\partial w}{\partial \vartheta} - \frac{v}{a}\right)^2$$

$$\gamma_{x\theta} = \frac{1}{a}\frac{\partial u}{\partial \vartheta} + \frac{\partial v}{\partial x} - \frac{z}{a}\left(2\frac{\partial^2 w}{\partial x \partial \vartheta} - \frac{\partial v}{\partial x}\right) + \frac{\partial w}{\partial x}\left(\frac{1}{a}\frac{\partial w}{\partial \vartheta} - \frac{v}{a}\right) \tag{2.a-c}$$

Assumed Displacement Modes of Shell

If we expand the displacements of Eqs.(2.a-c) into Fourier series, the coupled terms between the different harmonic numbers take place. Then, the decomposition of the equation of motion associated with the classical normal modes can not be applied. However, if the effects of nonlinear terms are assumed to be small, the coupled displacements can be approximated by the linear combinations of normal modes obtained by the analyses of linear free vibrations for the circular cylindrical shell with imperfect section. Then, the displacements may be expressed as

$$u(x,\vartheta,t) = \sum_{(mn)=1}^{M'N'} u_{(mn)}(x,\vartheta)\eta_{(mn)}(t)$$

$$v(x,\vartheta,t) = \sum_{(mn)=1}^{M'N'} v_{(mn)}(x,\vartheta)\eta_{(mn)}(t)$$

$$w(x,\vartheta,t) = \sum_{(mn)=1}^{M'N'} w_{(mn)}(x,\vartheta)\eta_{(mn)}(t) \tag{3.a-c}$$

where $\eta_{(mn)}(t)$ is the (mn)-th orthogonal generalized coordinate. $u_{(mn)}$, $v_{(mn)}$, $w_{(mn)}$ in Eqs.(3.a-c) are given by the combinations of normal modes based on the Rayleigh-Ritz method. That is, $u_{(mn)}$, $v_{(mn)}$, $w_{(mn)}$ are expressed as

$$u_{(mn)}(x,\vartheta) = \sum_{j=1}^{M'}\sum_{i=1}^{N'} \cos j\vartheta \frac{df_i(x)}{dx} C_{ji}^{(mn)}$$
$$v_{(mn)}(x,\vartheta) = \sum_{j=1}^{M'}\sum_{i=1}^{N'} \sin j\vartheta f_i(x) B_{ji}^{(mn)}$$
$$w_{(mn)}(x,\vartheta) = \sum_{j=1}^{M'}\sum_{i=1}^{N'} \cos j\vartheta f_i(x) A_{ji}^{(mn)} \qquad (4.a\text{-}c)$$

where $C_{ji}^{(mn)}$, $B_{ji}^{(mn)}$, $A_{ji}^{(mn)}$ are generalized coordinates, which are obtained as the eigen vectors for the linear vibration analysis of the system, $f_i(x)$ is the flexural vibration mode of a cantilevered beam.

If the problem is restricted to the shell whose cross section is a perfect circle, the displacements can be fixed to one circumferential wave mode because of the orthogonality with respect to the circumferential mode.

Boundary Conditions of Shell-Liquid System

Internal liquid of the shell is assumed to be nonviscous, irrotational and imcompressible. Based on the linear potential flow theory, the velocity potential induced by free vibration of the circular cylindrical shell-liquid system is obtained.

Representing the boundary value problem between the liquid and the shell by the velocity potential Φ; the following relations are given.

$$\nabla^2\Phi(x,r,\vartheta,t) = \left(\frac{\partial^2}{\partial x^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\vartheta^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\right)\Phi = 0 \qquad (5)$$

i) $\frac{\partial\Phi}{r\partial\vartheta}(x,r,0,t) = \frac{\partial\Phi}{r\partial\vartheta}(x,r,\pi,t) = 0$ ii) $\frac{\partial\Phi}{\partial r}(x,0,\vartheta,t) = 0$

iii) $\frac{\partial\Phi}{\partial x}(0,r,\vartheta,t) = 0$ iv) $\frac{\partial\Phi}{\partial t}(h,r,\vartheta,t) = 0$

v) $\frac{\partial\Phi}{a\partial r}(x,a,\vartheta,t) = \dot{W} = \sum_{(mn)=1}^{M'N'} w_{(mn)}(x,\vartheta)\dot{\eta}_{(mn)}(t) + \dot{Z}(t)\cos\vartheta$ (6.a-e)

The solution of these equations are given by (Ref. 7) and the internal hydrodynamic pressures can be obtained from the linear Bernoulli equation.

Equation of Forced Vibration

The nonlinear equilibrium equations of motion of the system are obtained from the Lagrange's equations which are given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\eta}_{(qr)}}\right) + \frac{\partial S}{\partial\eta_{(qr)}} + \frac{\partial F}{\partial\dot{\eta}_{(qr)}} = N_{(qr)} \qquad [(qr) = 1,2,\dots,(M'N')] \qquad (7)$$

where T and S are the kinematic and strain energy of the shell, respectively. F is the dissipation function of the shell depending on the viscous damping, $N_{(qr)}$ are the generalized forces produced by the liquid pressures and equivalent damping forces of liquid and $(\cdot)$ shows the derivative with respect to time.

The displacements of the shell, which is subjected to horizontal ground motion Z(t), are represented as follows

$$U(x,\vartheta,t) = u(x,\vartheta,t)$$
$$V(x,\vartheta,t) = v(x,\vartheta,t) - Z(t)\sin\vartheta$$
$$W(x,\vartheta,t) = w(x,\vartheta,t) + Z(t)\cos\vartheta \qquad (8.a\text{-}c)$$

The first term of the right hand side of the above equations are relative displacements of the shell to the ground and expressed as Eqs.(3.a-c). These displacements are used for the representation of the forces and energies in Eq.(7).

NONLINEAR EQUATIONS OF MOTION AND THESE SOLUTIONS

For the system ignoring the nonlinear strains, the frequency equations are derived from Eq.(7) and expressed as

$$[[K] - \Delta[[M_S] + [M_L]]]\{Q\} = 0 \qquad (9)$$

where $\Delta = (1-\nu^2)\rho_s a^2\omega^2/E$, ω are, respectively, nondimensional and dimensional natural frequency of the shell-liquid system, $[M_S]$, $[K]$ are the mass matrix and stiffness matrix of the shell, $[M_L]$ is the added mass matrix of the shell-liquid system which is obtained from the generalized force associated with the elastic displacement w. $\{Q\}$ is an eigen vector and has $(3\times M\times N)$ dimensions, in which M, N are respectively the number of terms expanded in the circumferential and axial directions;

$$\{Q\} = [C_{11}, C_{12}, \cdots, C_{1N}, C_{21}, \cdots, C_{2N}, \cdots, C_{M1}, \cdots, C_{MN}, \\ B_{11}, B_{12}, \cdots, B_{1N}, B_{21}, \cdots, B_{2N}, \cdots, B_{M}, \cdots, B_{MN}, \\ A_{11}, A_{12}, \cdots, A_{1N}, A_{21}, \cdots, A_{2N}, \cdots, A_{M1}, \cdots, A_{MN}]^T \qquad (10)$$

Frequency Equation of Nonlinear System

Using the (mn)-th eigen vector of Eq.(9) into the Eqs.(4.a-c) and substituting the above displacements and forces in Eq.(7), the governing nonlinear equations of motion associated with (qr)-th generalized coordinate are presented as follows.

$$\sum_{(mn)=1}^{M'N'} \{Q^{(qr)}\}^T [M_S + M_L]\{Q^{(mn)}\}[\ddot{\eta}_{(mn)}(t) + 2\alpha\dot{\eta}_{(mn)}(t)] + [M_S^{(qr)} + M_L^{(qr)}]\omega_{(qr)}^2 \eta_{(qr)}(t) \\ + \sum_{(m1n1)=1}^{M'N'} \sum_{(m2n2)=1}^{M'N'} K_{(m1n1)(m2n2)}^{(qr)} \eta_{(m1n1)}(t)\eta_{(m2n2)}(t) \\ + \sum_{(m1n1)=1}^{M'N'} \sum_{(m2n2)=1}^{M'N'} \sum_{(m3n3)=1}^{M'N'} K_{(m1n1)(m2n2)(m3n3)}^{(qr)} \eta_{(m1n1)}(t)\eta_{(m2n2)}(t)\eta_{(m3n3)}(t) \\ + [\Gamma_S^{(qr)} + \Gamma_L^{(qr)}]\ddot{Z}(t) = 0 \qquad (11)$$

where $M_S^{(qr)}$ is the generalized mass of the elastic shell and $M_L^{(qr)}$ is the generalized added mass of impulsive pressure produced by the elastic displacement of the shell. $\Gamma_S^{(qr)}$ and $\Gamma_L^{(qr)}$ are the generalized loads of effective inertia force of the shell and the internal liquid, respectively, and $\ddot{Z}(t)$ is ground acceleration and α is the viscous damping coefficient of the shell.

Using (mn)-th eigen vectors, the orthogonality between the fundamental modes is represented as

$$\{Q^{(mn)}\}^T [K]\{Q^{(m'n')}\} = \Delta_{(mn)}\{Q^{(mn)}\}^T [M_S + M_L]\{Q^{(m'n')}\} = \Delta_{(mn)}[M_S^{(mn)} + M_L^{(mn)}] \qquad (mn = m'n') \\ = 0 \qquad (mn \neq m'n') \qquad (12.a\text{-}b)$$

Substituting the above relation into Eq.(11), Eq.(11) can be expressed as

$$\ddot{\eta}_{(qr)} + 2\zeta_{(qr)}\omega_{(qr)}\dot{\eta}_{(qr)} + \omega^2_{(qr)} + [\sum_{(m1n1)=1}^{M'N'}\sum_{(m2n2)=1}^{M'N'} K^{(qr)}_{(m1n1)(m2n2)}\,\eta_{(m1n1)}\,\eta_{(m2n2)} + \sum_{(m1n1)=1}^{M'N'}\sum_{(m2n2)=1}^{M'N'}\sum_{(m3n3)=1}^{M'N'} K^{(qr)}_{(m1n1)(m2n2)(m3n3)}\,\eta_{(m1n1)}\,\eta_{(m2n2)}\eta_{(m3n3)}]/[\,M^{(qr)}_{S} + M^{(qr)}_{L}\,] = -\beta_{(qr)}\ddot{Z}(t) \tag{13}$$

where $\zeta_{(qr)}=\alpha/\omega_{(qr)}$ is the damping ratio of the shell. $\beta_{(qr)}$ is the participation factor of the (qr)-th mode and is given by

$$\beta_{(qr)} = [\,\Gamma^{(qr)}_{S} + \mathbf{\Gamma}^{(qr)}_{L}\,]/[\,\mathbf{M}^{(qr)}_{S} + \mathbf{M}^{(qr)}_{L}\,] \tag{14}$$

<u>Solve the Equation of Motion</u>

The object of this paper is to investigate the coupling phenomena between the modes of high order and a fundamental mode for the circumferential direction. Then, we adopt two displacement modes corresponding to ω_{11} and ω_{31}, which almost correspond to sliding mode and $\cos 3\theta$ mode, respectively. Note that this number 1, 3 do not mean the m=1, 3, because the coupling between the circumferential modes take place in linear free vibration modes due to imperfect section.

The corresponding nonlinear equations of motion to ω_{11}, ω_{31} are derived from Eq.(13) as follows.

$$\ddot{\eta}_{11} + 2\zeta_{11}\omega_{11}\dot{\eta}_{11} + \omega^2_{11}\eta_{11} + \alpha_1\eta^2_{11} + \alpha_2\eta_{11}\eta_{31} + \alpha_3\eta^2_{31} + \alpha_4\eta^3_{11} + \alpha_5\eta^2_{11}\eta_{31} + \alpha_6\eta_{11}\eta^2_{31} + \alpha_7\eta^3_{31} = -\beta_{11}\ddot{Z}(t)$$

$$\ddot{\eta}_{31} + 2\zeta_{31}\omega_{31}\dot{\eta}_{31} + \omega^2_{31}\eta_{31} + \alpha_8\eta^2_{31} + \alpha_9\eta_{31}\eta_{11} + \alpha_{10}\eta^2_{11} + \alpha_{11}\eta^3_{31} + \alpha_{12}\eta^2_{31}\eta_{11} + \alpha_{13}\eta_{31}\eta^2_{11} + \alpha_{14}\eta^3_{11} = -\beta_{31}\ddot{Z}(t)$$

(15.a-b)

For the case of the shell with perfect circular section, the integrations of the components $K^{(qr)}_{(m1n1)(m2n2)}$, $K^{(qr)}_{(m1n1)(m2n2)(m3n3)}$ in Eq.(13) become zero identically if the following relations among the indices of Eq.(13) are not satisfied.

$$K^{(qr)}_{(m1n1)(m2n2)} \quad : \quad |m1 \pm m2| = q$$

$$K^{(qr)}_{(m1n1)(m2n2)(m3n3)} \quad : \quad |m1 \pm m2| = |m3 \pm q| \tag{16.a-b}$$

In this case, Eqs.(15.a-b) are reduced to the following equations.

$$\ddot{\eta}_{11} + 2\zeta_{11}\omega_{11}\dot{\eta}_{11} + \omega^2_{11}\eta_{11} + \alpha_4\eta^3_{11} + \alpha_5\eta^2_{11}\eta_{31} + \alpha_6\eta_{11}\eta^2_{31} = -\beta_{11}\ddot{Z}(t)$$

$$\ddot{\eta}_{31} + 2\zeta_{31}\omega_{31}\dot{\eta}_{31} + \omega^2_{31}\eta_{31} + \alpha_{11}\eta^3_{31} + \alpha_{13}\eta_{31}\eta^2_{11} + \alpha_{14}\eta^3_{11} = 0$$

(17.a-b)

SOLUTIONS DERIVED FROM THE HARMONIC BALANCE METHOD

In order to obtain the stationary solutions of Eqs.(15.a-b) and Eqs. (17.a-b), we use the harmonic balance method. Considering the higher harmonic oscillations of two degrees of freedom, the harmonic solutions in the form;

$$\eta_{11} = C_{11}^{(1)}\cos\omega t + S_{11}^{(1)}\sin\omega t + C_{11}^{(2)}\cos 2\omega t + S_{11}^{(2)}\sin 2\omega t$$

$$\eta_{31} = C_{31}^{(1)}\cos\omega t + S_{31}^{(1)}\sin\omega t + C_{31}^{(2)}\cos 2\omega t + S_{31}^{(2)}\sin 2\omega t \qquad (18.a\text{-}b)$$

If the solutions are restricted to the first harmonic oscillation, Eqs.(18.a-b) expressed as

$$\eta_{11} = C_{11}\cos\omega t + S_{11}\sin\omega t$$

$$\eta_{31} = C_{31}\cos\omega t + S_{31}\sin\omega t \qquad (19.a\text{-}b)$$

The difference between the results obtained from Eqs.(18.a-b) and (19.a-b) are so small for the problems which show the a little nonlinearity.

Substituting Eqs.(19.a-b) into Eqs.(15.a-b) and applying the harmonic balance method, nonlinear algebraic equations with respect to coefficients (C_{11}, S_{11}, C_{31}, S_{31}) are derived as follows.

$$(-\omega^2 C_{11} + 2\omega_{11}\omega\zeta_{11}S_{11} + \omega_{11}^2 C_{11}) + \frac{3}{4}\alpha_4(C_{11}^3 + C_{11}S_{11}^2) + \frac{1}{4}\alpha_5(3C_{11}^2C_{31} + S_{11}^2C_{31} + 2C_{11}S_{11}S_{31})$$
$$+ \frac{1}{4}\alpha_6(3C_{11}C_{31}^2 + C_{11}S_{31}^2 + 2S_{11}C_{31}S_{31}) + \frac{3}{4}\alpha_7(C_{31}^3 + C_{31}S_{31}^2) = -\beta_{11}f$$

$$(-\omega^2 S_{11} - 2\omega_{11}\omega\zeta_{11}C_{11} + \omega_{11}^2 S_{11}) + \frac{3}{4}\alpha_4(C_{11}^2S_{11} + S_{11}^3) + \frac{1}{4}\alpha_5(2C_{11}S_{11}C_{31} + C_{11}^2S_{31} + 3S_{11}^2S_{31})$$
$$+ \frac{1}{4}\alpha_6(2C_{11}C_{31}S_{31} + S_{11}C_{31}^2 + 3S_{11}S_{31}^2) + \frac{3}{4}\alpha_7(C_{31}^2S_{31} + S_{31}^3) = 0$$

$$(-\omega^2 C_{31} + 2\omega_{31}\omega\zeta_{31}S_{31} + \omega_{31}^2 C_{31}) + \frac{3}{4}\alpha_{11}(C_{31}^3 + C_{31}S_{31}^2) + \frac{1}{4}\alpha_{12}(3C_{11}C_{31}^2 + C_{11}S_{31}^2 + 2S_{11}C_{31}S_{31})$$
$$+ \frac{1}{4}\alpha_{13}(3C_{11}^2C_{31} + S_{11}^2C_{31} + 2C_{11}S_{11}S_{31}) + \frac{3}{4}\alpha_{14}(C_{11}^3 + C_{11}S_{11}^2) = -\beta_{31}f$$

$$(-\omega^2 S_{31} - 2\omega_{31}\omega\zeta_{31}C_{31} + \omega_{31}^2 S_{31}) + \frac{3}{4}\alpha_{11}(C_{31}^2S_{31} + S_{31}^3) + \frac{1}{4}\alpha_{12}(2C_{11}C_{31}S_{31} + S_{11}C_{31}^2 + 3S_{11}S_{31}^2)$$
$$+ \frac{1}{4}\alpha_{13}(2C_{11}S_{11}C_{31} + C_{11}^2S_{31} + 3S_{11}^2S_{31}) + \frac{3}{4}\alpha_{14}(C_{11}^2S_{11} + S_{11}^3) = 0 \qquad (20.a\text{-}d)$$

When the cross section of the shell is a perfect circle, the nonlinear coefficients α_7 and α_{12} are equal to zero in Eqs.(20.a-d).

NUMERICAL RESULTS

To show the effectiveness of this procedure, the response functions of shell-liquid system are presented for a model.

The model for a numerical example has the parametric values as shown in Table 1. The radius of the shell with imperfect circular section is $a(\theta)=1828-101.56\cos 2\theta$(cm).

In Table 2 through Table 4, natural frequencies, participation factors and nonlinear coefficients of shell-liquid system are shown, respectively.

Fig.2,3 show the frequency response curves of the maximum displacement w for an empty shell and a shell containing liquid ($h=0.3\ell$), respectively. They are calculated for the amplitude of excitation f=0.3(gal) and $\zeta_{11}=\zeta_{31}=0.03$. In Fig.3, because of η_{31} are excited at the frequency region $\omega/\omega_{11}\doteqdot 1$, and the amplitude of η_{11} is about 1.5 times larger than that for an empty shell, it is presumed that the coupling phenomenon between the circumferential wave modes corresponding to η_{11} , η_{31} are excited at the frequency region near to $\omega/\omega_{11}\doteqdot 1$ by the nonlinearity due to including liquid. In these figures, η_{31} are also excited at the frequency region ω_{31}/ω_{11} , it is presumed that the effects of imperfectness of the circular section, but the effects for the response of first uncoupled circumferential mode η_{11} are a little.

CONCLUSIONS

A procedure considering the imperfectness of the circular section for nonlinear frequency response analysis of the shell-liquid system has been presented.

Numerical results show that the coupling phenomena of circumferential wave modes of a high order have been excited by the effects of internal liquid but the imperfectness of the circular section has not been played an important role.

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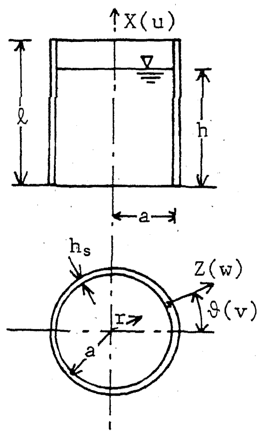

Fig.1 SHELL MODEL

Table 1 NUMERICAL EXAMPLE

a_0(cm)	1828.
ℓ (cm)	1219.
h_s(cm)	2.539
ρ_s(kg/cm^3)	0.008
h (cm)	365.7
ρ_L(kg/cm^3)	0.001
a_2(cm)	-101.56
E(kg/cm^2)	2.1×10^6
ν	0.3

Table 2 NATURAL FREQUENCIES (Hz)

n	m	h=0		h=0.3ℓ	
		IMPERFECT	PERFECT	IMPERFECT	PERFECT
1	1	35.56	34.71	23.60	23.37
	3	17.12	17.13	15.71	15.71

Table 3 PARTICIPATION FACTORS

n	m	h=0		h=0.3ℓ	
		IMPERFECT	PERFECT	IMPERFECT	PERFECT
1	1	-3007.29	-3012.05	-3396.31	-4626.65
	3	68.56	0.0	193.23	0.0

Table 4 NONLINEAR COEFFICIENTS

α	h=0		h=0.3ℓ	
	IMPERFECT	PERFECT	IMPERFECT	PERFECT
α_1	0.986×10^{-4}	0.0	-0.225×10^{-1}	0.0
α_2	0.825×10^{-3}	0.0	-0.543×10^{-1}	0.0
α_3	0.176×10^{-2}	0.0	-0.186×10^{-1}	0.0
α_4	0.951×10^{5}	0.750×10^{5}	0.878×10^{4}	0.587×10^{4}
α_5	0.942×10^{5}	0.606×10^{5}	0.228×10^{5}	0.762×10^{4}
α_6	0.109×10^{6}	0.988×10^{5}	0.224×10^{5}	0.201×10^{5}
α_7	0.360×10^{4}	0.0	0.783×10^{4}	0.0
α_8	-0.153×10^{-1}	0.0	0.180×10^{-1}	0.0
α_9	0.353×10^{-2}	0.0	-0.373×10^{-1}	0.0
α_{10}	0.412×10^{-3}	0.0	-0.272×10^{-1}	0.0
α_{11}	0.341×10^{5}	0.341×10^{5}	0.182×10^{5}	0.182×10^{5}
α_{12}	0.108×10^{5}	0.0	0.235×10^{5}	0.0
α_{13}	0.109×10^{6}	0.988×10^{5}	0.224×10^{5}	0.201×10^{5}
α_{14}	0.314×10^{5}	0.202×10^{5}	0.761×10^{4}	0.254×10^{4}

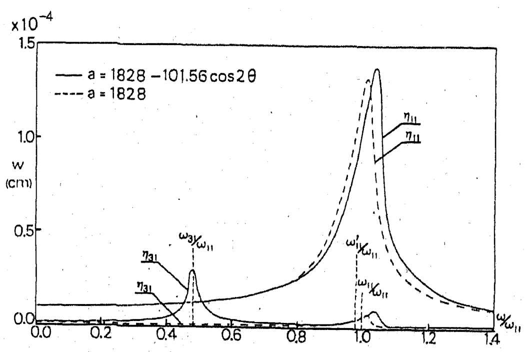

Fig.2 FREQUENCY RESPONSES (h=0)

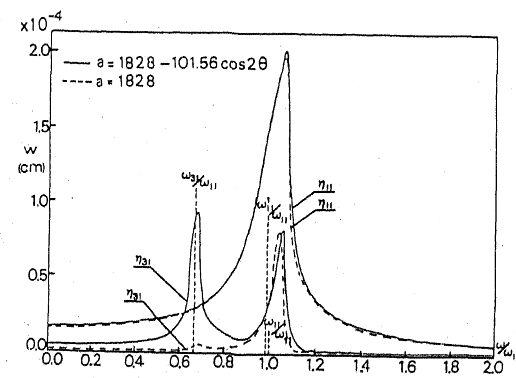

Fig.3 FREQUENCY RESPONSES (h=0.3ℓ)