RADIATION DAMPING OF CYLINDRICAL LIQUID STORAGE TANK RESTING ON ELASTIC BODY

S. Yamamoto (I)
K. Kawano (II)
N. Shimizu (III)
S. Umebayashi (IV)
M. Yamagata (IV)
Presenting Author: S. Umebayashi

SUMMARY

This paper presents an evaluation for the equivalent modal damping ratio due to radiation damping of cylindrical liquid storage tank resting on elastic soil. Tanks of various sizes are modeled into equivalent mass-spring models by taking into account the dynamic coupling between liquid and shell plate. The soil is assumed as a semi-infinite elastic body. Parametric studies are carried out for this equivalent model. The validity of this model is studied by the 3-dimensional FEM with energy transmitting boundaries and by observation results of actual tank.

INTRODUCTION

An appropriate evaluation of the effect due to dynamical soil-structure interaction is important in aseismic design. We consider the structure-foundation-soil vibration model shown at the top in Fig. 1. At the free surface ground motion F_F which is amplified from the bedrock ground motion F_R by the existence of surface soil layer is generally observed.

However the actual incident ground motion F_B on a foundation differs from the far field free surface ground motion because of the dynamical soil-structure interaction (in general $F_B<F_F$). Moreover vibrational characteristics of actual structures differ from those of structures built in rigid half space.

The vibration model for analysis of soil-structure interaction is categorized into three types as shown in (a) - (c) of Fig. 1.

Fig. 1 (a) models the reduction of incident ground motion due to the effect of soil-structure interaction. Fig. 1 (b) models the equivalent mass-spring-dashpot vibration model under the assumption that the incident ground motion is equal to the far field free surface ground motion. Fig.1 (c) shows the structure-foundation-soil total system as a vibration model. Fig. 1 (c) is the only model to treat the exact dynamical interaction. However, since this model is manpower-and- computer-intensive, it is unsuitable for aseismic design.

Existing aseismic design provisions are being enacted by tacitly taking into account the model of Fig. 1 (b).

This paper attempts to evaluate radiation damping of cylindrical storage

---

(I) Manager , CHIYODA Chemical Eng'g & Construction Co., Ltd.
Yokohama, Japan
(II) Section manager, ditto.
(III) Senior engineer, ditto.
(IV) Engineer , ditto.
(V) Engineer , ditto

tank shown in Fig. 2 by modelling it in an equivalent mass-spring model as shown in Fig. 1 (b). The validity of this equivalent model is comfirmed by FEM analyses and observations of earthquake responses in actual tank.

NOMENCLATURES

B	: Effective width of foundation for rocking motion
C_T	: Viscous damping of cylindrical tank
C_H	: Viscous damping of soil for horizontal motion
C_R	: Viscous damping of soil for rocking motion
D	: Diameter of cylindrical tank
E	: Young's modulus of shell plate of tank
H	: Height of equivalent mass from upper surface of foundation
H_{fi}, H_{fe}	: H when bottom pressure is included, or is excluded
H_B	: Thickness of foundation
H_ℓ	: Height of liquid
heq.	: Critical damping ratio of cylindrical tank due to radiation damping
I_B	: Moment of inertia about the center of upper surface of foundation
i	: Imaginary unit, $i^2 = -1$
K_T	: Equivalent spring of cylinder storage tank
K_H, K_R	: Spring of soil for horizontal motion, or for rocking motion
m_T	: Equivalent mass of cylindrical tank
m_B	: Mass of foundation
m_ℓ	: Mass of liquid
R	: Radius of cylindrical tank
R_B	: Radius of foundation
T_f	: First natural period of cylindrical tank built on rigid half space by taking into account the coupling effect between liquid and shell plate
T'	: First natural period of cylindrical tank-soil interaction system
t	: Average thickness of shell plate of tank
u_g	: Ground displacement
V_s	: Shear wave velocity of soil
y_T	: Horizontal displacement of cylindrical tank relative to foundation
y_B	: Horizontal displacement of foundation relative to foundation
ρ_ℓ, ρ_B	: Density of liquid, or of foundation
ρ_s	: Density of soil
Γ_2	: Gamma function of 2nd order
ρ	: Density of tank shell
θ	: Rocking angle of foundation
ω	: Angular frequency
ν_s	: Poisson's ratio of soil

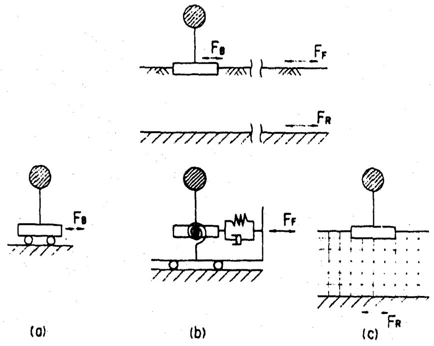
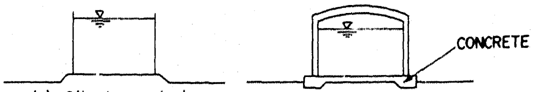


Fig. 1 Structure-foundation-soil vibration model



(a) Oil storage tank (b) Single slab without pile

Fig. 2 Cylindrical strange tank

EQUIVALENT MASS-SPRING MODEL

Type (a) or (b) in Fig. 2 are object of this study in which the bottom plate of tank or the foundation slab directly contacts soil and is not supported by piles. Tank-foundation-soil system of type (a) or (b) is modeled into the equivalent mass-spring model as shown in Fig. 3.

The equations of motion of this mechanical model are written as Eqs. (1) - (3).

$$m_B(\ddot{y}_B + \ddot{u}_g) + C_H \dot{y}_B + K_H y_B = -m_T(\ddot{y}_T + \ddot{u}_g) \text{ ----(1)}$$

$$m_T(\ddot{y}_T + \ddot{u}_g) + C_T \dot{u}_1 + K_T u_1 = 0 \text{ ----------------(2)}$$

$$I_B \ddot{\theta} + C_R \dot{\theta} + K_R \theta = -m_T H(\ddot{y}_T + \ddot{u}_g) \text{ -----(3)}$$

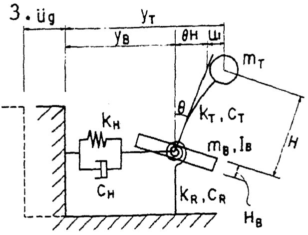


Fig. 3 Equivalent mechanical model

Denoting displacement vector by $\{v\} = \{y_T, y_B, \Theta\}^T$, the equations of motion can be rewritten as Eq. (4).

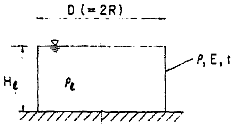
$$[M]\{v\} + [C]\{v\} + [K]\{v\} = -[M]\{f\}\ddot{u} \text{ ------(4)}$$

$$\{f\} = \{1, 1, 0\}^T \text{ -------(5)}$$

$$[M] = \begin{bmatrix} m_T & & 0 \\ & m_B & \\ \text{sym.} & & I_B \end{bmatrix} \text{--(6)} \quad [C] = \begin{bmatrix} C_T & -C_T & -C_T H \\ & (C_T + C_H) & C_T H \\ \text{sym.} & & (C_R + C_T H^2) \end{bmatrix} \text{--(7)} \quad [K] = \begin{bmatrix} K_T & -K_T & -K_T H \\ & (K_T + K_H) & K_T H \\ \text{sym.} & & (K_R + K_T H^2) \end{bmatrix} \text{--(8)}$$

Denoting displacement vector by $\{v\}$ $\{v\} = \{v_1\}e^{i\omega t}$ the eigen-value equation is obtained as Eq. (9).

$$(\lambda^2[M] + \lambda[C] + [K])\{v\} = 0 \text{ ----(9)}$$



Modeling of Tank

Fig. 4 Cylindrical tank fixed on base

As shown in Fig. 4, the tank is assumed to be a cylindrical shell fixed on a base. A number of numerical analyses were carried out by using finite element method in which the dynamic coupling between liquid and shell plate is taken into account (Ref.1, Ref.2). From the results of these numerical analyses, equivalent mass m_T, spring k_T and height H in the simplified mechanical model as shown in Fig. 5 are given by the following approximate formulae.

(i) Equivalent mass m_T

$$\frac{m_T}{m_\ell} = \exp\left\{-0.3\left(\frac{D}{H_\ell}\right)\right\} - 0.53\exp\left\{-2\left(\frac{D}{H_\ell}\right)\right\} + \left[1.4 - 0.1\left(\frac{D}{H_\ell}\right)\right]\left(\frac{\rho}{\rho_\ell}\right)\left(\frac{t}{R}\right) \text{ --- (10)}$$

(ii) Equivalent spring K_T

$$K_T = m_T\left(\frac{2\pi}{T_f}\right)^2 \text{ ------(11)} \qquad T_f = H_\ell \sqrt{\rho_\ell D/(E \cdot t)} \cdot f_1 \text{ ------(12)}$$

$$f_1 = \left[2.907 + \sqrt{5.625\left(\frac{D}{H_\ell}\right)^2 - 6.75\left(\frac{D}{H_\ell}\right) + 3.43}\right] \Big/ \sqrt{\frac{D}{H_\ell}} \text{ ----------(13)}$$

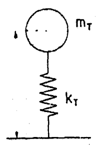


Fig.5 Equivalent mass-spring model fixed on base

(iii) Equivalent height of mass H

$$\frac{m_T H_{fi}}{m_\ell H_\ell} = \left[0.29\left(\frac{D}{H_\ell}\right)^{0.25} + 0.08\Big/\left(\frac{D}{H_\ell}\right)\right]\left[1 + \left(\left\{4.2 - 3.6\left(\frac{D}{H_\ell}\right)\right\} + \left\{4.1 - 0.54\left(\frac{D}{H_\ell}\right) - 0.8\Big/\left(\frac{D}{H_\ell}\right)^{1.1}\right\}\left(\frac{\rho}{\rho_\ell}\right)\right)\left(\frac{t}{R}\right)\right] \text{----} (14)$$

$$\frac{m_T H_{fe}}{m_\ell H_\ell} = \left[0.475 - 0.1567\left(\frac{D}{H_\ell}\right) + 0.01586\left(\frac{D}{H_\ell}\right)^2\right]\left[1 + \left\{2 + 1.1\left(\frac{D}{H_\ell}\right)\right\}\left(\frac{\rho}{\rho_\ell}\right)\left(\frac{t}{R}\right)\right] \text{---------------} (15)$$

Modeling of Soil and Foundation

Soil is assumed to be spring and dashpot obtained from theoretical solutions of rigid circular mass on elastic half space (Ref. 3).

Since the purpose of this study is to estimate equivalent modal damping ratio due to radiation damping, the dashpot of tank C_T is neglected. And also damping regarding rocking motion C_R is considered to negligibly small.

Assuming the distribution of reaction force to be uniform for sway motion and triangular for rocking motion, K_H, C_H and K_R can be expressed as following forms. And total mass of foundation, m_B, and moment of inertia about the axis passing through the upper surface of foundation, I_B, are given as follows.

$$K_H = \frac{2\pi}{2-\nu_s} \rho_s V_s R_B \text{-----} (16) \qquad m_B = \rho_B \cdot \pi R_B^2 \cdot H_B \text{--------------} (19)$$

$$C_H = \frac{\Gamma_2 \pi}{2-\nu_s} \rho_s V_s R_B^2 \text{-----} (17) \qquad I_B = \rho_B \cdot \pi R_B^2 \cdot H_B \left(\frac{R_B^2}{4} + \frac{H_B^2}{3}\right) \text{-----} (20)$$

$$K_R = \frac{\pi}{2} \frac{\rho_s V_s^2}{1-\nu_s}\left[R_B^3 - (R_B - B)^3\right] \text{-----} (18)$$

Assumptions in Numerical Analyses

The variables included in Eq. (9) are listed in Table 1. Some of these variables are taken to be constant based on typical design practice. In the case of an oil storage tank as shown in Fig. 2 (a), the bottom plate of tank is so flexible that dynamic soil-structure interaction would occur in the limited portion near the edge of the bottom plate.

On the other hand, in the case of liquified gas tank shown in Fig. 2 (b), the foundation is assumed to be rigid, thus B/R = 1.0.

Eq. (9) is independent of the value of the radius of foundation R_B. Therefore, from the above mentioned assumptions, fundamental parameters in numerical analyses are found to be D/H_ℓ, V_S, for the oil storage tank and D/H_ℓ, V_S, H_B/R for liquified gas storage tank.

Table 1. Assumed values and parameters in numerical analyses

		Symbol	Unit	Oil Tank	Liquified Gas
Liquid	Density	ρ_ℓ	$Kgf \cdot s^2/m^4$	102.04	51.02
	Height	H_ℓ	m	D/H_ℓ : Parameter	
Tank	Radius	R	m		
	Density	ρ	$Kgf \cdot s^2/m^4$	801.0	801.0
	Thickness	t	m	$t/R = 1 \times 10^{-3}$	$t/R = 8 \times 10^{-4}$
	Young's Modulus	E	Kgf/m^2	2.1×10^{10}	2.1×10^{10}
Foundation	Radius	R_B	m	——	$R_B/R = 1.1$
	Height	H_B	m	——	Parameter
	Density	ρ_B	$Kgf \cdot s^2/m^4$	——	255.1
Soil	Density	ρ_s	$Kgf \cdot s^2/m^4$	183.7	183.7
	Poisson's Ratio	ν_s	——	0.4	0.4
	Shear Wave Velocity	V_s	m/s	Parameter	
Equivalent Width for Rocking		B	m	$B/R = 0.2$	$B/R = 1.0$

RESULTS OF NUMERICAL ANALYSES BY PROPOSED MODEL

Fig. 6 shows the equivalent modal damping ratio (h_{eq}) of the oil storage tank shown in Fig. 2 (a). It is seen from this figure that h_{eq} takes a maximum value when the value of D/H_ℓ reaches 3.5 and h_{eq} increases monotonously as V_S decreases (or as the stiffness of soil decreases).

Fig. 7 shows the ratio of 1st natural period of the oil tank to that of the fixed base (T'/T_f). The period of soil-structure interaction system (T') becames longer than that of fixed base (T_f), and the value of T'/T_f reaches about 3 when D/H_ℓ is small (or tank is tall).

Fig. 8 - Fig. 10 show h_{eq} of the liquified gas tank shown in Fig. 2 (b) in the case of H_B/R = 0, 0.033 and 0.05 respectively. It can be seen from these figures that h_{eq} decreases with increasing of H_B/R (or increasing mass of foundation). In each figure, it can be seen that h_{eq} takes its maximum value when the value of D/H_ℓ reaches about 2.5, and h_{eq} increases with decrease of V_S as shown in the case of the oil storage tank (Fig. 6).

For both types of tank, h_{eq} has a peak value with respect to D/H_ℓ. The main reason is considered that the effective mass of liquid m_T does not remain constant but decreases with increase of D/H_ℓ.

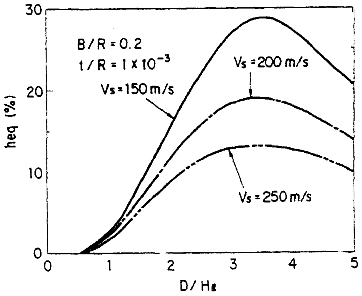


Fig. 6 heq of oil storage tank

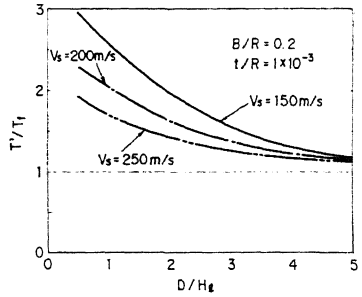


Fig. 7 T'/T_f of oil storage tank

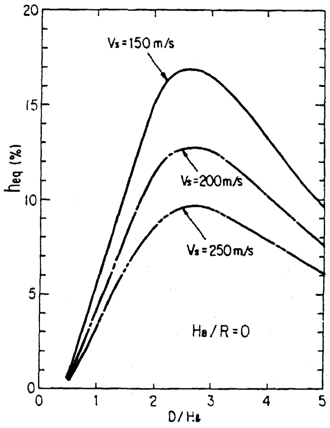


Fig. 8 heq of liquified gas tank (H_B/R = 0)

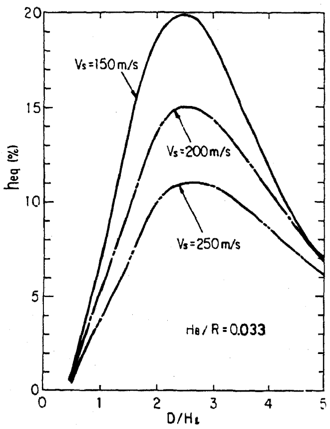


Fig. 9 heq of liquified gas tank (H_B/R = 0.033)

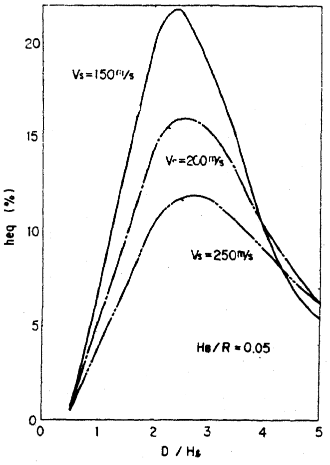


Fig. 10 heq of liquified gas tank (H_B/R = 0.05)

EVALUATION OF PRESENT MECHANICAL MODEL

The present mechanical model is evaluated through finite element analyses and observation in actual tank during earthquakes.

Numerical Analyses by Finite Element Model

Total vibration system with respect to liquid-tank-foundation-soil was modeled by the finite element model shown in Fig. 11. In this model, stiffness of bottom plate and foundation is so large that they can be assumed rigid. In order to take energy dissipation from the side and the bottom of the model into account, energy transmitting boundaries and dashpot are used (5 and 6 in Fig. 11) (Ref. 4).

Radius of tank R and shear wave velocities of 1st layer and 2nd layer, V_{s1}, V_{s2}, are the parameters in this model, and take the values as shown in Table 2. The other conditions are the same as those of liquified gas tank listed in Table 1.

Therefore, it can be considered that this model corresponds to liquified gas tank with rigid foundation resting on stratified semi-infinite elastic space.

Fig. 12 shows equivalent modal damping ratios h_{eq} of this model which are obtained from frequency response function of acceleration at shell plate of tank. In this figure, the values of h_{eq} obtained by the mechanical model are also drawn.

It can be seen from this figure that h_{eq} obtained by FEM analyses coincide considerably well with those of the proposed mechanical model.

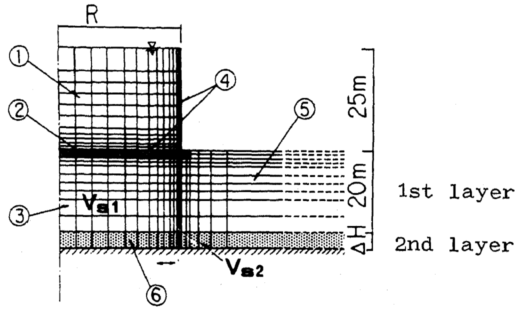


Fig. 11 Finite element model

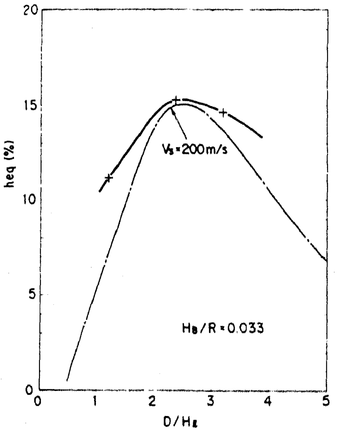


Fig. 12 h_{eq} obtained by FEM in comparison with those of equivalent mechanical model

Table 2 Parameters in FEM analyses

CASE	Shear Wave Velocity Vs (m/s)		Diameter of Tank D (m)	D/Hℓ	heq (%)
	1st. Layer	2nd. Layer			
1	200	500	60	2.4	15.1
2	200	500	30	1.2	11.1
3	200	500	80	3.2	14.6

Observations in Actual Tanks

Earthquake observation have been carried out for a water tank which is located on reclaimed land in Tokyo bay. At this site, the reclaimed deposits about 5m thick overlie about 10m thick sandy alluvium, and there exists as much as a 40m thick soft clayey marine deposits under this sandy alluvium. Typical soil profile and main dimensions of tank are shown in Fig. 13. A pair of strong-motion accelerographs are installed in the upper and lower positions of the shell plate. For eathquakes with more than 5 gal of vertical acceleration, these accelerographs operate simultaneously and record seismic motions automatically on magnetic tape.

We have obtained three available data from the earthquakes listed in Table 3. Transfer functions between the upper and the lower portion of shell plate were determined by processing the obtained data statistically using AR-model (Ref. 2). From these transfer functions modal damping ratios and 1st natural period are determined and shown in Table 3.

It can be seen from Table 3 that 1st natural period of the tank is 0.25-0.29 s which is about twice of those of the fixed base, and damping ratios are within a range of 7-9% with one exception.

The value of D/H_ℓ of this tank is about 1.5. It can be seen from Fig. 6 and Fig. 7 that the above mentioned observed values coincide well with the results of present mechanical model. From this point of view, the value of B/R in the case of oil storage tank assumed to be 0.2.

The value of B/R = 0.2 would be much larger than that derived by static theory. This might indicate that the value of B/R with regard to dynamical interaction differs from the static one. At the present time, we can not give any theoretical basis for this, but further study on this would reveal the dynamic behavior of cylindrical storage tanks, especially of oil storage tanks.

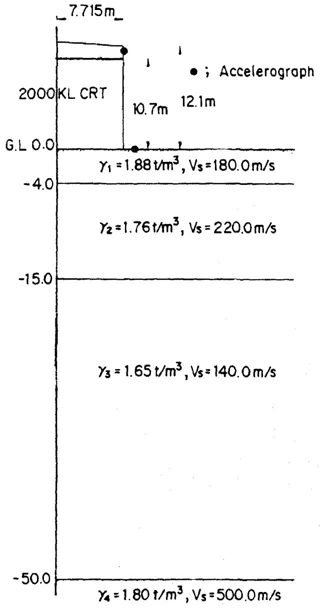


Fig. 13 Main dimensions of water tank and soil profile

Table 3 Characteristics of observed earthquakes, obtained periods and damping ratios

Earthquake			Max. Acceleration Tangential (Gal)		Tank		1st. Natural Period			Damping Ratio (%)
Date, Time	Epicentral Region, Depth	Magnitude M	Tank Base	Tank Top	Liquid Height H_ℓ (m)	Shell Plate t (mm)	Fixed Base T_f (s)	T' (s)	T'/T_f	
23 : 23 July 23, 1982	Off Ibaragi Pref., 40 km	7.0	16.2	23.8	10.80	8.0	0.144	0.270	1.88	9
13 : 33 Aug.12, 1982	Off Izu Isl. , —	5.7	9.8	21.8	9.84	8.0	0.132	0.256	1.94	7
21 : 14 Feb. 27, 1983	Southern Ibaragi Pref. 40 km	6.3	21.5	33.2	10.70	8.0	0.143	0.286	2.00	9

CONCLUSION

The mechanisms of damping for cylindrical liquid storage tank are constituted mainly of hysteresis damping, structural damping and radiation damping. Among these dampings, radiation damping could be expected to be larger than the others, because the large-sized cylindrical tanks directly contact soil with large area.

We presented the equivalent mechanical model shown in Fig. 3, and calculated the equivalent mechanical modal damping ratio due to radiation damping (h_{eq}) with changing the size of tank and stiffness of soil.

From this parameteric study, we obtained the following results.

(1) h_{eq} takes it maximum value at $D/H_{\ell} = 3.5$ for the oil storage tank, and at $D/H_{\ell} = 2.5$ for the liquified gas storage tank.

(2) h_{eq} increases with decreasing of stiffness of soil and with increasing of mass of foundation.

The result (1) is explained chiefly by the nature of effective mass of liquid with respect to D/H_{ℓ}, and it is very interesting for aseismic design of liquid storage tank.

Moreover the validity of this mechanical model was proven by FEM analyses in the case of liquified gas storage tank.

If the value of $B/R = 0.2$ in the proposed equivalent mechanical model, satisfactory explanation can be given for the observation results in actual water tank.

REFERENCES

1. Shimizu, N., Yamamoto, S. and Kawano, K. (1982): "Study on aseismic design of cylindrical storage tanks." - Part 1. Transactions of JSME, Vol.48, No.426, PP.215-228 (in Japanese).

2. Shimizu, N., Yamamoto, S. and Kawano, K. (1982): "Study on aseismic design of cylindrical storage tanks." - Part 2. Transactions of JSME, Vol.48, No.427, PP.328-348 (in Japanese).

3. Tajimi, H. (1968): "Earthquake engineering." Shokokusha, PP.80-90 (in Japanese).

4. Shimizu, N., Yamamoto, S. and Koori, Y. (1977): "Three-dimensional dynamic analysis of soil-structure system by thin layer element method." - Part 2. Transactions of AIJ, No.254, PP.39-48