SOIL STRUCTURE INTERACTION EFFECTS ON THE SEISMIC BEHAVIOR OF CYLINDRICAL LIQUID STORAGE TANKS

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SUMMARY

The influence of soil flexibility on dynamic behavior of a slab-supported cylindrical liquid storage tank is investigated. Solutions from both the frequency-dependent as well as the frequency-independent standpoints are obtained. It is concluded that if the base excitation frequency is small compared to the first natural frequency of the rigid-based system the soil-structure interaction is significant.

BACKGROUND

The problem of response of slab-supported liquid storage containers subject to seismic excitation has been of concern for many years. In 1957 G.W. Housner, (Ref. 1), studied the hydrodynamic pressures developed when such tanks are subjected to ground motions and presented results in the form of equivalent masses together with their locations for representing force and moment effects on the rigid tank due to liquid motion. A more recent treatment was offered by A.S. Veletsos and J.Y. Yang, (Ref. 2). In 1978 T. Balendra and W.A. Nash, (Ref. 3), presented a finite element analysis of an elastic domed cylindrical liquid storage tank subject to base excitation. Comprehensive shake-table tests of broad cylindrical liquid storage tanks were carried out by D.P. Clough, (Ref. 4), in 1977 and additional tests of tall, narrow tanks were reported by A. Niwa, (Ref. 5), in 1978. In 1980 M.A. Haroun, (Ref. 6), carried out forced vibration tests on three full-scale water storage tanks to determine their dynamic characteristics. More recently Y. Goto and T. Shirasuna, (Ref. 7), studied the dynamics of cylindrical tanks completely embedded in relatively soft ground. None of the above investigatios of slab-supported tanks have considered soil-structure interaction effects. The purpose of the present work is to consider this effect.

ANALYSIS

Let us consider the vertical cylindrical storage tank shown in Figure 1a. The tank is assumed to consist of N ring-like elements with the mass of each ring being concentrated as its geometric center as shown in Figure 1d. Further, let $\ddot{u}_g$ be the ground acceleration; h_i be i-th nodal point's height; m_i be the mass of the i-th nodal point; x_i be the displacement of

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the i-th point relative to the base; I_i be the mass moment of inertia of the i-th element about its axis of rotation; x_0 be the base translation relative to the ground motion; m_0 be the mass of the footing; ϕ be the base rotation; $u_i = u_g + x_0 + \phi h_i + x_i$ be the total displacement of the i-th nodal point; [M], [C], [K] be the mass, damping, and stiffness matrices respectively for the system when it is resting on a rigid foundation; and H(t), M(t) be the interaction effects between the base and the structure relating shear force and moment respectively.

The equations of motion for dynamic equilibrium of the N nodal points are:

$$[M]\, \ddot{\underline{u}}_i + [C]\, \dot{\underline{x}}_i + [K]\, \underline{x}_i = \underline{0} \qquad (1a)$$

where i = 1, 2, .. N

The equation of motion of the structure-foundation system as a whole, in translation, is:

$$(m_1\ddot{u}_1 + m_2\ddot{u}_2 + .. + m_N\ddot{u}_N) + m_0(\ddot{u}_g + \ddot{x}_0) + H(t) = 0. \qquad (1b)$$

A comparable equation may be written for rotation; this involves the I_i. These equations are then cast into matrix form.

With the assumption of a massless disk on the elastic half-space the interaction forces can be expressed as:

$$\begin{Bmatrix} H(t) \\ M(t) \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{x}_0(t) \\ \ddot{\phi}(t) \end{Bmatrix} + \begin{bmatrix} \overset{*}{c}_{11} & \overset{*}{c}_{22} \\ \overset{*}{c}_{21} & \overset{*}{c}_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}(t) \\ \dot{\phi}(t) \end{Bmatrix} + \begin{bmatrix} \overset{*}{k}_{11} & \overset{*}{k}_{12} \\ \overset{*}{k}_{21} & \overset{*}{k}_{22} \end{bmatrix} \begin{Bmatrix} x_0(t) \\ \phi(t) \end{Bmatrix} \qquad (2)$$

This leads to the following general equations of motion including soil-structure interaction effects:

$$[\overset{*}{M}]\, \ddot{\underline{X}} + [\overset{*}{C}]\, \dot{\underline{X}} + [\overset{*}{K}]\, \underline{X} = -\, \underline{F}\, \ddot{u}_g \qquad (3)$$

where, for example $[\overset{*}{M}]$ is a matrix having in its elements [M], the h_i, the m_i and the I_i. Thus, $[\overset{*}{M}]$, $[\overset{*}{C}]$, $[\overset{*}{K}]$ are the mass, damping, and stiffness matrices respectively of the soil-structure system, $\underline{X}$ and $\underline{F}$ are the displacement and load vectors of the same system, and the c's and k's are the impedance coefficients of the foundation medium. These coefficients actually depend upon Poisson's ratio, the shear modulus, and the mass density of the half-space. Also, they are usually frequency-dependent. Approximate frequency-independent values of the c's and k's have been suggested by several authors. In the present work we investigate behavior of the system in (a) the frequency dependent domain, and (b) the frequency independent domain. In (a) modal analysis is not applicable since the system does not possess classical normal modes. Even in (b) the damping matrix cannot be diagonalized under the same transformation that diagonalizes both the mass and the stiffness matrices. Many approximate methods have been advanced to counter these difficulties.

To determine H(t) and M(t) in the frequency dependent domain the elastic half-space is subjected to steady-state harmonic motion. Equation (2) then yields the H(t) and M(T) as functions of base translation and rotation due to steady state harmonic motion as well as impedance functions involving the dimensionless frequency, together with the translational and rotational stiffness associated with the base slab. These impedance functions are to be found in the work of A.K. Chopra and J.A. Gutierrez, (Ref. 8). In the frequency dependent domain these were evaluated by A.S. Veletsos and Y.T. Wei, (Ref. 9) for a wide range of parameters. Thus, in this domain, for a ground acceleration $\ddot{u}_g(t) = Ue^{i\omega t}$ any response variable may be expressed in the form $x(t) = \overset{*}{\underline{x}}(\omega)e^{i\omega t}$ etc. This leads to

$$[A]\ \overset{*}{\underline{X}} = -\ \overset{*}{F}\ U \tag{4}$$

where $[A] = \left[[K(\omega)] - \omega^2[M] + i\omega\ [C(\omega)]\right]$; $\overset{*}{\underline{F}} = \underline{\mathbf{F}}$ as in Eq. (3); (5)

$$\overset{*}{\underline{X}} = \begin{Bmatrix} \overset{*}{\underline{x}}(\omega) \\ x_0(\omega) \\ \overset{*}{\emptyset}(\omega) \end{Bmatrix}$$; U is the peak value of the ground acceleration.

To determine H(t) and M(T) in the frequency-independent domain we employ approximations to obtain

$$\begin{Bmatrix} H(t) \\ \\ M(t) \end{Bmatrix} = \begin{vmatrix} c_x & 0 \\ & \\ 0 & c_\emptyset \end{vmatrix} \begin{Bmatrix} \dot{x}_0 \\ \\ \dot{\emptyset} \end{Bmatrix} + \begin{bmatrix} k_x & 0 \\ & \\ 0 & k_\emptyset \end{bmatrix} \begin{Bmatrix} x_0 \\ \\ \emptyset \end{Bmatrix} \tag{6}$$

where the k_i and c_i are functions of the radius of the base slab, the elastic parameters of the foundation, and the total mass moment of inertia of the structure plus base slab about the rocking axis at the base. As has been suggested by the results of previous authors, the off-diagonal terms have a negligible effect on the dynamics of the system and are thus neglected. Thus, in the frequency-independent domain the equations of motion may be put in the form:

$$[\overset{*}{M}]\ \ddot{\underline{X}} + [\overset{*}{C}]\ \dot{\underline{X}} + [\overset{*}{K}]\ X = -\underline{F}\ \ddot{u}_g \tag{7}$$

If the system has N degrees of freedom, Eqs. (4) and (7) represent (N+2) simulataneous equations. The two extra unknowns are the base translation and base rotation.

In the frequency dependent domain, a solution of (4), in the absence of damping, was obtained directly through a matrix inversion method for the case of steady-state base excitation. Once the frequency response function $\underline{X}(\omega)$ is known, the response to arbitrary base excitation can be determined through use of Fourier transform methods together with the Fast Fourier Transform.

In the frequency-independent domain, neglecting damping, a solution of (7) for the steady-state case is readily found and matrix inversion can be avoided if the modal matrix [Q] of the system is available. For arbitrary ground motion the solution can be obtained by using Duhamel's integral.

An approximation to the stiffness matrix can be obtained by considering the cylindrical tank as a cantilever beam of constant rigidity and having N nodal points. First, the flexibility matrix is obtained by use of virtual work, and its elements f_{ij} are found to be

$$f_{ii} = (1/3EI)h_i^3$$

$$f_{ij} = (1/3EI)h_i^3 + (1/2EI)h_i^2(h_j-h_i) \qquad (8)$$

$$f_{ji} = f_{ij}$$

where i = 1, 2, .., N ; j = i, i+1, i+2, .., N ; E is the modulus of elasticity of the cantilever beam; I is the bending moment of inertia of the cantilever beam; and N is the total number of the nodal points. If R2 and R1 denote the outer and inner radii of the cylinder respectively, $I = (\pi/4)(R2^4-R1^4)$. The stiffness matrix may now be determined by taking the inverse.

For the mass matrix, the mass of the i-th element is m_i = RO1(V_i) where $V_i = \pi(R_2^2 - R_1^2)H1$, RO1 is the mass density of the tank material, H1 = HH/N and HH is the total height of the cantilever beam.

For a liquid filled tank, the stiffness matrix is identical to that described above for the empty tank. For the mass matrix, we employ the concept of lumped masses,(Ref. 1). Figure 1 indicates the approach wherein the dynamic forces exerted by the fluid are represented by a fixed mass M_o moving with the tank walls together with an equivalent simple oscillator of mass M_1 having the same period as the first mode of the fluid. This equivalent system is considered to exert the same horizontal force and overturning moment on the tank walls as does the original system. Using the values of the k_i from (Ref. 9) and introducing the additional variables.

$$k_5 = 2(k_1k_4 + k_2k_3k_6) \text{ and } k_6 = S_a/\ddot{u}_g \text{ max} \qquad (9)$$

where S_a represents an acceleration spectrum corresponding to the period T, we have the maximum overturning moment M_b

$$M_b = \frac{1}{2} k_5 M h \ddot{u}_g \max \tag{10}$$

where M is the total liquid mass and h is liquid depth.

Let us consider a mass (M_{ad}) such that its centroid will coincide with the liquid centroid and it will produce the same overturning moment due to $\ddot{u}_g$ max. Thus,

$$M_{ad} \ddot{u}_g \max (h/2) = \frac{1}{2} k_5 M h \ddot{u}_g \max = k_5 M \tag{11}$$

Thus, k_5 determines the percent of the total liquid mass to be employed in the analysis so that instead of using the model in Figure 1b the model shown in Figure 1c can be used as long as the additional mass M_{ad} from (11) is used. If n denotes the total number of ring-shaped elements below the liquid surface the additional mass for one such element will be $m_{ad} = M_{ad}/n$. For those elements above the liquid level the mass is the same as for the empty tank discussed earlier, but for those below the surface the mass is $m_{it} = m_i + m_{ad}$.

EXAMPLE

Let us consider a circular cylindrical tank with an inner radius of 2.995 inches, an outer radius of 3.005 inches, a height of 12.0 inches, a mass density of 0.733 10^{-3}lb-sec^2/in^4, and an elastic modulus of 29.0 x 10^6lb/in^2. The base disk to which the tank is attached has a radius of 5.00 inches, a thickness of 1.00 inch, and a mass density of 0.21x10^{-3}lb-sec^2/in^4. The soil medium on which the base disk rests has a mass density of 0.18x10^{-3}lb-sec^2/in^4, a shear modulus of 50,000.00lb/in^2, and a Poisson's ratio of 0.3. The base of the tank is subjected to steady-state harmonic excitation represented by $u_g = 1.0 \cos \omega t$ where ω is determined through use of the foundation stiffness coefficients given in (Ref. 9). Next, the interaction parameters are determined for a number of base frequencies and, for example, for $\omega = 10$, we found $k_5 = 1.237$ and $k_6 = 1.758$ for the case of the above tank being half-filled with water. The k_6 value was determined from the acceleration spectrum of the 1940 El Centro and the period of liquid oscillation T_1 from (Ref.1). The radial displacement at the top of a half-filled tank at the end of a diameter in the direction of the ground motion is shown in Figures 2 and 3. Figure 2 indicates the displacement in terms of dimensionless frequency of excitation as determined by computation in the rigid base case and also with consideration of soil structure interaction in the frequency dependent situation. Figure 3 indicates the response as determined through use of frequency independent analysis. Additional plots for other response variables are shown in (Ref. 10).

CONCLUSIONS

For the tank considered here, if the ratio of the base excitation frequency ω to the first natural frequency of the rigid-based structure ω_1 is greater than approximately 2.5 the effects of soil-structure interaction on deformations are not significant. If, however, this ratio is less than about 2.5 the interaction effect is quite significant particularly when ω is close to one of the natural frequencies of the soil-structure system. Use of the frequency independent domain concept leads to an estimate of dynamic behavior but in general frequency dependence must be employed to obtain more precise engineering results.

REFERENCES

1. Housner, G.W., "Dynamic Pressure on Accelerated Fluid Containers," Bulletin Sesimological Society of America, Vol. 47, No. 1, 1957, pp. 313-347.
2. Veletsos, A.S., and Yang, J.Y., "Earthquake Response of Liquid Storage Tanks," Advances in Civil Engineering Through Engineering Mechanics, ASCE, 1977, pp. 1-24.
3. T. Balendra and W.A. Nash, "Earthquake Analysis of a Cylindrical Liquid Storage Tank with a Dome by Finite Element Method," University of Massachusetts, Department of Civil Engineering. Report to N.S.F., 1978.
4. Clough, D.P., "Experimental Evaluation of Seismic Design Methods for Broad Cylindrical Tanks," Department of Civil Engineering, University of California, Berkeley, Report No. EERC/77-10, 1977.
5. Niwa, A., "Seismic Behavior of Tall Liquid Storage Tanks," Department of Civil Engineering, University of California, Berkeley, Report No. EERC/78-04, 1978.
6. Haroun, M.A., "Dynamic Analyses of Liquid Storage Tanks," Report EERL, 80-04, California Institute of Technology, Pasadena, 1980.
7. Goto, Y., and Shirasuna, T., "Response Behavior of Large and Grouped Underground Tanks During Earthquakes," Earthquake Behavior and Safety of Oil and Gas Storage Facilities, Buried Pipelines, and Equipment, ASME, PVP-Vol. 77, 1983, pp. 47-55.
8. Chopra, A.K., and Gutierrez, J.A., "Earthquake Analysis of Multistory Buildings Including Foundation Interaction," Earthquake Engineering and Structural Dynamics, Vol. 3, pp. 65-77, 1974.
9. Veletsos, A.S., and Wei, Y.T., "Lateral and Rocking Vibration of Footings," Soil Mech. and Foundation Div. Proceedings of ASCE, Sept. 1971, SM9, 1227-1248.
10. Daysal, H., and Nash, W.A., "Soil-Structure Interaction Effects on the Response of Cylindrical Tanks to Base Excitation," Technical Report to the National Science Foundation, University of Massachusetts, Amherst, MA, March, 1982.

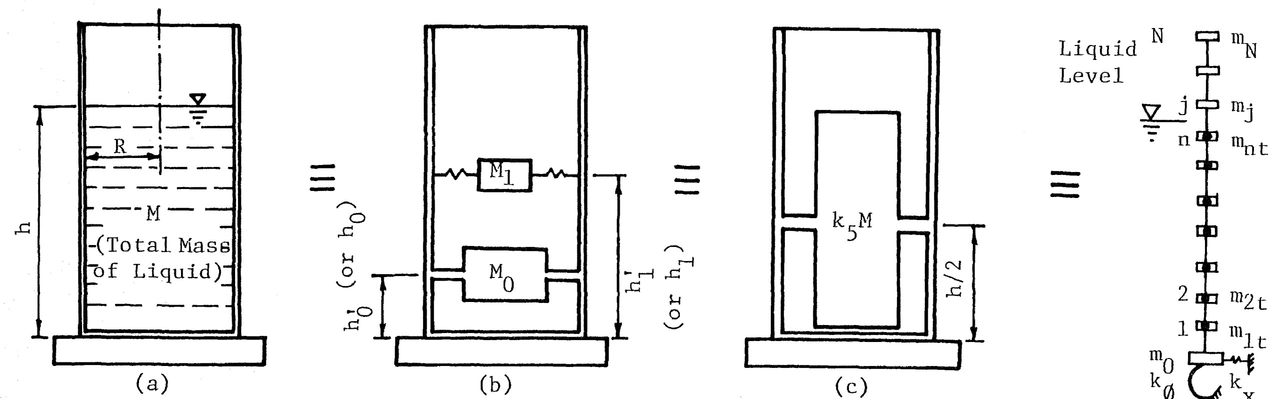

Fig. 1. a) Tank with Liquid b) Equivalent Dynamic Model c) An Equivalent Mass such that it will exert the same Bending Moment at the Base level as System (b) would (d) Distribution of Equivalent Mass k_5M.

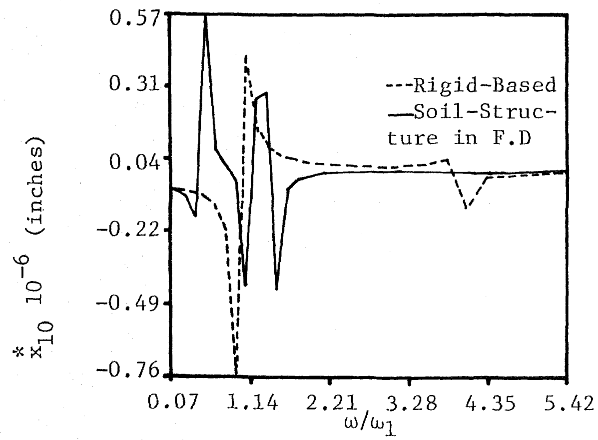

Fig. 2. A comparison between the solutions for rigid-based and soil-structure systems in Frequency Domain concerning $\overset{*}{x}_{10}$. (Half-Filled Tank)

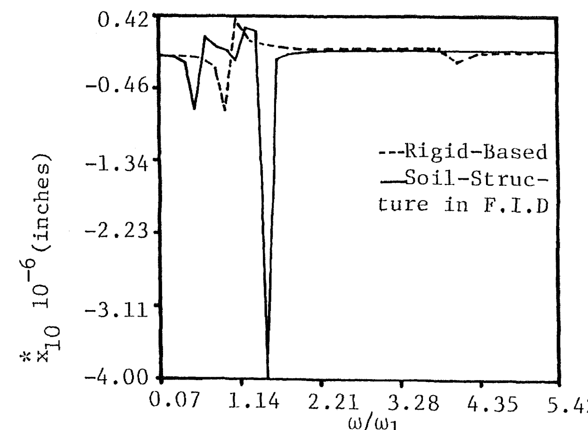

Fig. 3. A comparison between the solutions for rigid-based and soil-structure systems in Frequency Independent Domain concerning $\overset{*}{x}_{10}$.(Half-Filled Tank)