STABILITY OF LIQUID STORAGE TANKS
UNDER EARTHQUAKE EXCITATION

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SUMMARY

Based on potential flow theory for incompressible, inviscid fluid flow in conjunction with the response spectrum approach, the maximum pressure distribution in the deformable liquid filled cylindrical tank is calculated. The coupled fluid-structure-interaction problem is solved in a semi-analytical manner. By applying this maximum pressure distribution to the shell incrementally the loss of stability (mainly caused by axial compressive membran forces near the base) is considered by a non-linear finite element analysis. Particularly the influence of shape imperfections on the stability is investigated in the non-linear numerical calculation.

INTRODUCTION

Elastic buckling of the shell at the base (shell crippling) is - beside of other phenomena as elephant footing, cracking, overturning ... - one of the possible damage mechanisms of cylindrical liquid storage tanks under earthquake excitation. The shell crippling during earthquake may be considered as a local instability due to concentrated axial membrane forces in the base area of the circular shell, caused by the overturning moment resulting from the dynamically activated fluid pressure.

Let us consider cylindrical shells filled with liquid, fixed at the bottom and free at the top. The height of the shell is L, the height of the liquid is H ($H \leq L$) and R denotes the radius of the circular shell. The earthquake is characterized by its response spectra and the corresponding maximum ground acceleration A, whereby only unidirectional horizontal excitation of a rigid base is taken into account in this paper.

INTERACTION OF THE SHELL AND THE LIQUID

There are many papers dealing with the interaction of the tank wall and the liquid filling. The earlier ones are treating the shell to be rigid (Ref. 1). But it could be shown that the deformability of the tank may have a tremendous influence on the distribution of the dynamically activated pressure (Ref. 2) and, hence, on the stability of the shell.

By applying the response spectrum method (Ref. 3, p. 389 ff.) the maximum pressure distribution relevant for the buckling analysis can be

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estimated with high accuracy. This approach requires the knowledge of the free vibration modes of the complete interacting system: shell and liquid.

Let us consider a cylindrical shell which at the time t experiences a horizontal bottom displacement, $u_b(t)$, at the axial coordinate x=0 in the angular direction $\varphi=0$. The common displacement may be split into a parallel displacement of the shell and in vibrations with the relative displacements - in relation to the basecircle - $w(x,\varphi,t)$, $v(x,\varphi,t)$. w stands for the relative radial displacement, v for the relative tangential displacement. ω in the following means the circular frequency according to the first eigenmode. Here we have to take into account that during horizontal ground acceleration in perfect circular cylindrical shells only eigenmodes with the circumferential wave number m=1 are being activated (Ref. 4). Some recent experimental investigations (Refs. 5, 6) show, however, that in real tanks due to unavoidable imperfections also eigenmodes with m>1 contribute to the coupled fluid-shell-motion. This fact is of significant importance especially in the treatment of vibrations of concentric shells with rather narrow liquid columns inbetween them (Ref. 6). For the sake of simplicity this paper only deals with the first of the eigenmodes with m=1, a procedure which seems justified for many practical applications. (The action of higher eigenmodes with m=1 is negligible in many cases; on the other hand, for the higher eigenmodes we can proceed in an analogous way and superimpose in accordance with the superposition rule.)

As shown in Ref. 7 the viscosity and compressibility effects are negligible with respect to the stability of the shell. Thus the potential flow theory is applicable in conjunction with time dependent boundary conditions due to the vibration of the tank wall; see Ref. 2. Since the response spectrum method is applied only to free vibrations, natural frequencies and corresponding mode shapes of the liquid filled shell need to be calculated. Hence the time dependence is harmonic and the problem leads to usual eigenvalue calculations.

The dynamically activated pressure distribution can be divided into three parts:

- the convective fluid pressure, p_{bc}, due to surface sloshing,
- the impulsive fluid pressure contribution, p_{bi}, attributed to the parallel displacement of the fluid column, $u_b(t)$, i.e. the rigid body motion of the tank wall,
- a further impulsive fluid pressure contribution, p_d, activated by the relative motion of the tank wall, i.e. the wall deformation, with respect to the rigid base circle.

The convective fluid pressure, p_{bc}, (sloshing)

For sloshing, the tank can be treated as if rigid. The calculation of the sloshing period, T_j^*, is demonstrated e.g. in Ref. 8 as follows

$$T_j^* = 2\pi\left(\frac{R}{\eta_j\, g \tanh\left(\eta_j \frac{H}{R}\right)}\right)^{\frac{1}{2}}, \quad f_j^* = \frac{1}{T_j^*}, \quad j = 1,2...$$

η_j is the j-th zero of the derivative of the Bessel function J_1 of the first kind (η_1=1.841, η_2= 5.331, η_3=8.536,...). g being the acceleration of gravity. The negligible influence of the deformability of the shell is shown in Ref. 9.

A^*_j, the spectral acceleration value to the frequency, f^*_j, can be taken from a response spectrum for a maximum base acceleration, A, and a small modal damping value (ζ<0.5 %). The extreme distribution of the convective fluid pressure, p_{bc_j}, corresponding to the j-th sloshing mode, can be taken from Ref. 8 and is as follows

$$p_{bc_j} = R\rho_L\, A^*_j \frac{2}{\eta_j^2 -1} \frac{\cos h(\eta_j \frac{H}{R}\xi)}{\cos h\,(\eta_j \frac{H}{R})} \cos\varphi$$

$$= R\rho_L\, \bar{P}_{bc_j}(\xi)\cos\varphi A^*_j, \quad j = 1,2,3 \ldots, \quad \xi = \frac{x}{H}$$

ρ_L is the mass density of the liquid. The actual extreme fluid pressure, p_{bc}, results from a superposition. In most cases only the first sloshing mode is essential.

The rigid tank-impulsive pressure, p_{bi}

As shown in Ref. 8 the contribution to the extreme impulsive pressure distribution attributed to the "rigid" tank motion can be calculated by

$$p_{bi} = AH\rho_L \sum_{j=1}^{\infty} \frac{8}{\pi^2} \frac{(-1)^{\frac{j-1}{2}} I_1(\lambda_j)}{j^2 I'_1(\lambda_j)} \cos(j\frac{\pi}{2}\xi)\cos\varphi \qquad j=1,2,3 \ldots$$

$$= AH\rho_L\, \bar{P}_{bi}(\xi)\cos\varphi$$

with $\lambda_j = j\frac{\pi R}{2H}$

The impulsive pressure due to shell deformation, p_d

The principal idea of the procedure outlined in Refs. 2,7,8 is to assume the mode shape of the tank wall vibrations, f(x), as to be known or approximated in advance. Hence the radial displacement for the considered first mode with m=1 is given by

$$w(x,\varphi,t) = Cf(x)\cos\varphi\sin\omega t.$$

With max $|f(x)|$=1 the factor C represents the maximum relative displacement amplitude which can be calculated from

$$C = \gamma\frac{(a-A)}{\omega^2},$$

γ is the mode participation factor. The spectral acceleration a related to the frequency ω may be taken from the response spectrum with a proper damping value ζ discussed in the following chapter. The frequency results from the following procedure using the added mass approach as outlined in detail in Ref. 8.

The application of the potential flow theory leads to

$$p_d = R\rho_L \sum_{j=1}^{\infty} \frac{2I_1(\lambda_j)}{\lambda_j I'_1(\lambda_j)} \left(\int_0^1 \hat{f}(\xi)\cos(j\frac{\pi}{2}\xi)d\xi\right)\cos(j\frac{\pi}{2}\xi)\cos\varphi$$

$$= R\rho_L\, \bar{p}_d(\xi)\cos\varphi, \quad \lambda_j = j\frac{\pi R}{2H}, \quad j = 1,3,5\ \ldots, \quad f(x) \to \hat{f}(\xi)$$

which, multiplied by $\gamma(a-A)$ leads to the maximum pressure distribution due to the presumed shell wall deformation $Cf(x)\cos\varphi$.

Since the corresponding resultant pressure force per unit height is

$$r(\xi) = C\omega^2\rho_L\bar{p}_d(\xi)R^2\pi$$

the added mass per unit height can be expressed by

$$m'(\xi) = r(\xi)/(C\omega^2\hat{f}(\xi)) = \rho_L\bar{p}_d(\xi)R^2\pi/\hat{f}(\xi)$$

or the added mass density by

$$\rho'(\xi) = m'(\xi)/(2R\pi h(\xi)) = \rho_L\bar{p}_d(\xi)R/2\hat{f}(\xi)h(\xi)$$

with $h(\xi)$ being the thickness of the tank wall dependent on the axial distance from the bottom.

With the effective mass density

$$\rho^*(\xi) = \rho_s+\rho'(\xi) \qquad (\rho_s \ldots \text{ mass density of the tank wall})$$

a "dry" shell can act as an approximative substitute of the liquid filled shell as long as vibrations with an eigenmode approximated by $\hat{f}(\xi)$ are considered. Neglecting the follower force effects of the liquid pressure and the influence of the prestress of the tank wall due to pressure loading the solution of the eigenvalue problem of free vibrations of this dry shell substitute (usually using numerical methods as e.g. Ref. 4) renders the fundamental frequency ω and a corresponding mode shape $\hat{\hat{f}}(\xi)$ which in general is not indentical with $\hat{f}(\xi)$. $\hat{\hat{f}}(\xi)$ may be used as an improved approximation of the first mode shape of the liquid filled tank. This procedure leads to the following iteration scheme:

$$\begin{array}{llll}
i=1 & \text{assume: } \hat{f}_i(\xi) \Rightarrow \rho_i^*(\xi) \Rightarrow \omega_i, \hat{\hat{f}}_i(\xi) & \left|\left|\int_0^1 \hat{\hat{f}}_i(\xi)d_\xi \Big/ \int_0^1 \hat{f}_i(\xi)d_\xi\right| - 1\right| < \delta & ? \\
 & \quad\uparrow & \quad\downarrow & \quad\downarrow \\
 & i=i+1 \leftarrow \hat{f}_{i+1}(\xi) \Leftarrow \hat{\hat{f}}_i(\xi) \leftarrow & \quad\text{no} & \quad\text{yes} \\
 & & & \omega, \hat{f}(\xi), \gamma, a
\end{array}$$

As shown in Ref. 8 the mode participation factor may be calculated from

$$\gamma = \frac{\int_0^1 \frac{\bar{p}_d(\xi)}{h(\xi)} d\xi}{\int_0^1 \frac{\hat{f}(\xi)}{h(\xi)} \bar{p}_d(\xi) d_\xi}$$

whereby ρ_s is neglected as against ρ'; i.e. $\rho_s \ll \rho'$ and $\rho^* \approx \rho'$.

The resultant maximum pressure distribution

In addition to the axisymmetric static pressure distribution the individual components of the dynamically activated pressure can be superimposed by application of a proper superposition rule as for example the SRSS method, Ref. 3:

$$p(\xi,\varphi) = \rho_L g(H-\xi)+P(\xi)\cos\varphi$$

with

$$P(\xi) = R_{\rho_L} \{\sum_j |A_j^* \bar{P}_{bc}(\xi)|^2 + |\gamma(a-A)\bar{P}_d(\xi) + A\frac{H}{R}\bar{P}_{bi}(\xi)|^2\}^{\frac{1}{2}}$$

For the example treated in Ref. 8 the individual components and the resultant dynamically activated maximum pressure distribution is shown in Fig. 1.

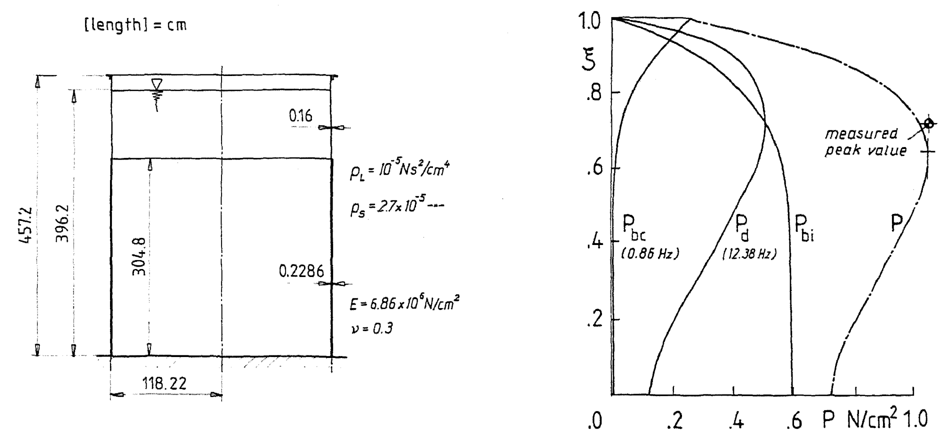

Fig. 1 Calculated resultant maximum dynamic wall pressure

DAMPING OF THE VIBRATION OF THE LIQUID FILLED TANK

There exists only few literature about the damping of the liquid motion in a tank and of the common vibration tank/liquid. Generally it is found that damping both due to friction of the liquid on the wall and the bottom and due to internal friction is very small. Some analytical estimations are given in Ref. 7 and lead to damping ratios $\zeta<0.5\,\%$. The authors recommend 0% for analysis.

The damping behaviour of the common vibration tank/liquid was investigated experimentally in the last years. Specifically the experimental results of Kernforschungszentrum Karlsruhe, Refs. 10 and 11, shall be mentioned. It was found that the damping ratios ζ_{empty} and ζ_{full} do not differ significantly. This fact agrees well with the findings reported in Ref. 12 and experiments performed by the authors of this paper, see Ref.13. It leads to the conclusion that beside other damping mechanisms as friction at the clamping the shell is damped hysteretically. In this case the modal damping ratio ζ does not depend on the specific mass μ of the system. In the case of viscous damping ζ is proportional to $\mu^{-1/2}$ and this would decrease ζ significantly for a liquid filled shell due to the apparent specific mass of the liquid. The authors therefore recommend for earthquake analysis ζ values for "dry" structures as given in Ref. 14, p. 102.

STABILITY OF THE EARTHQUAKE LOADED TANK

As shown in Ref. 15, for the current type of tanks the influence of the shell mass forces can be neglected when calculating the stresses activated by the fluid pressure in the cylindrical shell and analysing the stability of the shell, i.e. the stress- and stability-analysis can - as an approximation - be performed for a quasi-static system. The quasi-static stability approach may be justified by the fact that the frequency of the time dependent pressure loading is considerably lower than the fundamental natural frequency of the empty shell. Furthermore the buckling mode is completely different from the vibration mode due to earthquake excitation. Also very recent buckling experiments published in Ref. 6 justify the static approach. Hence, we are considering a cylindrical shell fixed at the base, free at the top, under a static load, $p(\xi,\varphi)$. For a stability analysis of the shell in the classical sense we need the membrane forces n_x, n_φ, $n_{x\varphi}$. According to the membrane theory of perfect circular cylindrical shells, Ref. 9, we get for a pressure distribution:

$$n_\varphi(\xi,\varphi) = R(P(\xi)\cos\varphi + H\rho_L g(1-\xi))$$

In Ref. 8 it is shown that the axial membrane force is given by

$$n_x(\xi,\varphi) = -\frac{H^2}{R}\int_\xi^1 P(\tilde{\xi})\ (\tilde{\xi}-\xi)\,d\tilde{\xi}\cos\varphi$$

The mathematical justification for calculating the membrane forces only by the membrane theory is also given in Ref. 8.

According to the concept of several design standards, as for example Refs. 16 - 18, the safety factor regarding stability limit is calculated by comparing the classical critical axial membrane stress of a perfect circular cylindrical shell under a uniformly distributed axial load, Ref. 19:

$$\bar{\sigma}_x^{cr} = \bar{n}_x^{cr}/h = \frac{1}{\sqrt{3(1-\nu^2)}}\,\frac{Eh}{R},$$

with the maximum value of the axial membrane compressive-stress $n_x(0,0)/h$. In order to take account for imperfections a "knockdown" factor, α, is applied and the safety factor, s, is estimated by

$$s = \alpha\ \frac{\bar{n}_x^{cr}}{n_x(0,0)}$$

with $\alpha = 0.2 \div 0.4$.

Experimental investigations, as for example shown in Refs. 20, 21, lead to the conclusion that this procedure represents a conservative estimation even if no "knockdown" is used to account for imperfections. Also from these experiments it can be concluded that the imperfection is balanced by the nonuniformity effect as discussed at the end of this chapter.

In Ref. 8 the authors describe a nonlinear finite element stability analysis of the modal tank shown in Fig. 1. The results of this analysis realize the above mentioned experimentally obtained results:

The endurable maximum axial membrane stress is considerably higher than the classical buckling stress:

$\bar{\sigma}_x^{cr} = 8028$ N/cm^2 ($\bar{s} = 3.24$) classical, perfect shell

$\hat{\sigma}_x^{cr} = 11315$ N/cm^2($\hat{s} = 3.725$) nonlinear FE, perfect shell

$\hat{\hat{\sigma}}_x^{cr} = 11062$ N/cm^2($\hat{\hat{s}} = 3.922$) nonlinear FE, imperfect shell

with imperfections leading to

$$\tilde{R}(\xi,\varphi) = R|1+\varepsilon(a_1\xi + a_2\xi^2 + a_3\xi^3)\cos n\varphi$$

$$(\varepsilon = 4.23\times10^{-3},\ a_1 = 2.275,\ a_2 = 0.875,\ a_3 = -3.15,\ n = 4).$$

The safety factor s can be interpreted as being the number by which the maximum ground acceleration of the earthquake under consideration, A, must be amplified in order to cause shell crippling.

Based upon experiments at the EERC in Ref. 21 it is outlined that the compressive axial membrane stresses causing buckling of earthquake loaded liquid filled cylindrical shells must be more than 2.35 times larger than the maximum allowable values specified by the American Petroleum Institute (Ref. 18). This observation agrees with the above mentioned fact of endurable maximum compressive membrane stresses considerably higher than the usual buckling formulas would predict.

The knockdown factor, α, in the standards is based on static buckling tests of small cylinders under uniform axial compression. The axial compressive stress of the earthquake loaded shell is, however, concentrated rather locally in the area of small values of φ and ξ. Hence, a certain stiffening effect may be contributed by the surrounding shell. In addition to this the over-all imperfections are less relevant than the local imperfections near the base which are usually smaller than in the remaining cylinder. Furthermore the axisymmetrical static fluid pressure, p_s, leads (except a very narrow area at the base) to circumferential tensile stresses which give a certain stabilizing effect. These facts are the reason why the standards are rather conservative in case of earthquake loading.

Considerations of the differences between the numerical results of the perfect and the imperfect shell, i.e. ($\hat{\sigma}^{cr}$, $\hat{s}$) - ($\hat{\hat{\sigma}}^{cr}$, $\hat{\hat{s}}$), shows that the critical buckling stress, $\hat{\hat{\sigma}}^{cr}$, of the imperfect shell is smaller than the one of the perfect shell, $\hat{\sigma}^{cr}$. This conforms with the general assumption. However, the safety factor of the imperfect shell, $\hat{\hat{s}}$, is higher than that of the perfect shell, $\hat{s}$. This surprising result has its reason in the circumferential distribution of the axial compressive stress. The imperfections described above decrease the peak value of the axial compressive force in the buckling area (see Fig. 2). Of course, a sin(nφ)-circumferential imperfection distribution - which should be used for security reasons - leads to a decrease of the critical amplification factor, $\hat{\hat{s}}$. This imperfection distribution is now studied by nonlinear finite element calculations.

A further result of the nonlinear finite element stability analysis is the buckling mode shown in Fig. 3 which reveals the local nature of the buckling as can be observed on tanks failed by buckling due to earthquake, see for example Ref. 22.

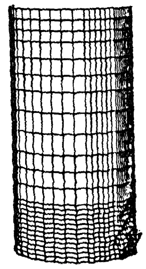

Fig. 3 Buckling mode

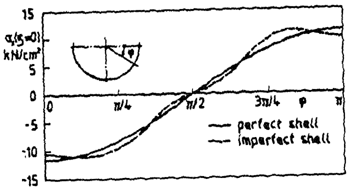

Fig. 2 Circumferential distribution of the axial membrane force at the base circle just before buckling

CONCLUSIONS

A rather simple approach is shown how to calculate the maximum pressure distribution on the tank wall during earthquake. A further development should include the contributions attributed to the vibration modes with m>1, i.e. more than one circumferential waves. The damping value of the liquid filled tank, which is essential in the response spectrum analysis, can be chosen as being the same as the damping ratio of the empty tank. The application of the current design standards based on knockdown factors $\alpha<1.0$ for calculating the critical buckling stress or the safety factor for buckling due to earthquake represents a very conservative approach as long shell crippling (i.e. local elastic buckling) is concerned. Calculations and experiments show that the maximum compressive axial membrane stress leading to shell crippling is higher than the classical bifurcation pressure even if imperfect shells are considered. Finally, the quasi-static stability approach is applicable, dynamic stability analysis need not to be applied for buckling of earthquake loaded liquid storage tanks.

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