DYNAMIC BEHAVIOUR OF CABLE-STAYED BRIDGES

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SUMMARY

In the following study, a matrix discrete method based on a finite element idealization is used and a numerical example is presented as a verification of the three-dimensional response analysis.

The analysis has proved that the mode shapes of such structures are complex and damping is an important parameter for achievement of more realistic values of the dynamic response of the structure.

INTRODUCTION

This study deals with the cable-stayed bridges which are treated as non-conventional not only because of their large spans but also due to their unusual structure when compared to the bridges of the traditional type.

According to the new concept of the cable-stayed bridges, they should consist of cables, towers, beams and decks. The static and dynamic analyses of these structures are rather complex due to the variety of the components which are characteristic for these structures. Therefore, a more precise three-dimensional mathematical model should be formulated in order to present the interaction of the structural components. The mathematical model formulated on the basis of the finite element method gives most reliable results with respect to these demands.

Discretization of a concrete cable-stayed bridge with fan configuration has been presented. Thus, the free vibrations and the dynamic response of the bridge in transversal and longitudinal direction have been analysed for such a mathematical model.

ANALYSIS USING LUMPED-MASS MODEL

Generally speaking, the equilibrium equations for a finite element system in motion are nonlinear and can be written as:

$$M\ddot{U} + C\dot{U} + KU = G + R(t) \qquad \ldots (1)$$

If these equations are linearized i.e., if the nonlinear terms G from (1) are not taken into account, the system of equations (1) will be reduced to a system of equations with small amplitudes of vibration with respect to the static equilibrium of the model. In this way, the matrix K which represents the total structural stiffness can be considered constant while the system is in motion.

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The assemblage of the individual element stiffnesses forms the global stiffness matrix:

$$K = \sum_{m} Km \qquad \dots (2)$$

where Km is the stiffness matrix of the m^{th} element.

Three types of finite elements which have been used during the assemblage process are presented in Fig. (1a), (1b) and (1c), where Km^{c} (2 x 2) is the matrix of the flexible element, Km^{b} (12 x 12) is the matrix of the three-dimensional beam element and Km^{d} (24 x 24) is the stiffness matrix of the element obtained by superimposing plate bending and plane stress.

The system of the linearized equations obtained from (1) can be solved by application of the standard mode superposition method. This will be done, first, by computation of the circular frequencies and mode shapes, then by decoupling of the linearized system and finally by solving the decoupled equilibrium equations. In the end, the finite element nodal point displacements are obtained by superposition of the response in each mode.

NUMERICAL EXAMPLE

The theory developed in the previous section has been used to perform mathematical discretization of a cable-stayed bridge with fan configuration and to obtain the dynamic characteristics and the response to the earthquake effect.

The principle characteristics of the bridge - The whole structure is constructed of concrete with two solid post-tensioned edge beams and a reinforced concrete deck. Two concrete towers rise 24 m above the edge beams. Centre to centre distance of edge beams is 10.4 m; central span between the towers is 112 m and end spans, each 56 m.

The mathematical model of the bridge which has been previously described is presented in Fig. 2a.

The main characteristics of the mathematical model - The total number of the finite elements used for the discretization of the structure is 88. 28 rectangular elements with four nodes have been used for the discretization of the bridge deck, while the beams and the towers have been discretized by using 40 beam elements. The discretization of the stays has been accomplished by 16 flexible elements.

Input data have been acquired by using the program for automatic generation and plotting of the mesh of the mathematical model.

Dynamic characteristics - The natural periods and modes of vibration of the system have been computed and some of the computed natural periods have been presented in Table 1. Some characteristic mode shapes are shown in Fig. 3.

For better understanding of the dynamic behaviour of these systems, some modifications of the basic model have been used. Cross-beams have been fixed, Fig.2b, at each end of the towers in order to obtain portal frames. The dynamic characteristics have been acquired for such a modified basic model.

The influence of the stays upon the dynamic characteristics of the cable-stayed bridges has been analysed by comparison of the results obtained by using the basic model with those acquired by the modified one i.e., those which have been obtained by adding stays. The modified model is shown in Fig.2b, and the vertical vibrations periods for the basic and the modified model are shown in Table 2.

Response analysis - Response analyses for the basic model and the two modified ones have been accomplished by using the mode superposition method.

For an earthquake effect in longitudinal and transversal direction, time histories of displacements for the most characteristic points of the bridge model have been obtained. The record of April 15, 1979 Ulcinj2, Montenegro earthquake, W-E component, has been used with adopted maximum peak acceleration of 0.24 g.

Damping of 2.5%, 5% and 7% has been used in order to analyse the influence of damping upon this type of structures. The maximum displacements for the characteristic points of the model for different damping degree are given in Table 3. For damping value of 5%, the variation of the displacements at the characteristic points of the model which occur due to the Ulcinj2, Montenegro earthquake are presented in Fig. 4.

CONCLUSIONS

From the performed analyses, the following conclusions can be drawn:

- The discretization of these structures should be done by using three-dimensional models in order to understand the complex mode shapes as well as to find out the number of necessary modes for the response analysis by mode superposition.

- During the response analysis by mode superposition, the exact selection of the number of mode shapes by means of three-dimensional models gives reliable results for practical purposes.

- The great differences between the maximum displacement values which have been obtained for different damping degree point to the importance of exact evaluation of the damping parameter.

- The influence of the number of stays upon the dynamic characteristics of the structure is more expressed during lower mode shapes while slight differences are observed with respect to the circular frequencies in case of higher modes. This can be explained by the fact that in case of higher modes of vibration, the bridge stays do not participate during vibration.

- This type of structures requires portal frames or A-frames which decrease the torsional effect upon the edge beams as well as the influence of the bending moment upon the bridge deck during an earthquake effect in transversal direction.

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2. Podolky, W. and J.F. Fleming, "Historical Development of Cable-Stayed Buildings", Journal of the Struct.Division, ASCE,No.ST8, September 1972, pp. 2079-2095.
3. "All-Concrete Cable-Stayed Railway Bridge", "Concrete International Design & Construction",December 1979, Vol. I, No. 12.

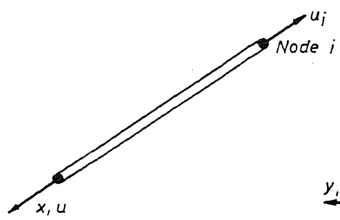


Fig. 1a. Cable element

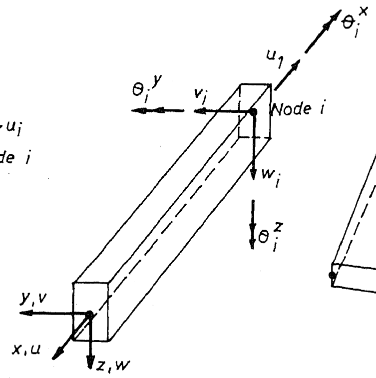


Fig. 1b. Beam element

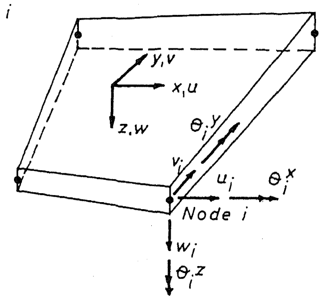
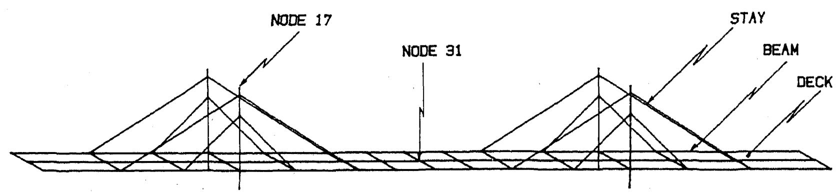
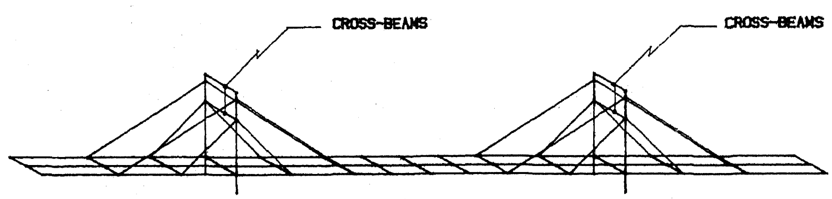


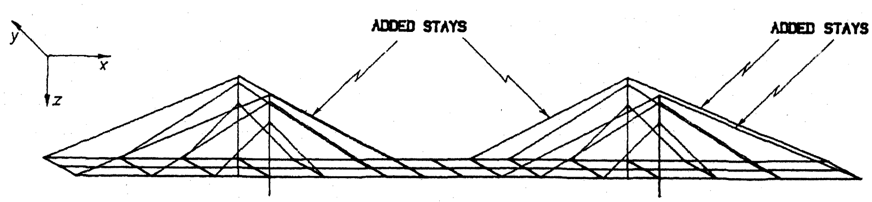
Fig. 1c. Rectangular element (obtained by superimposing plate bending and plane stress)



a) Basic model

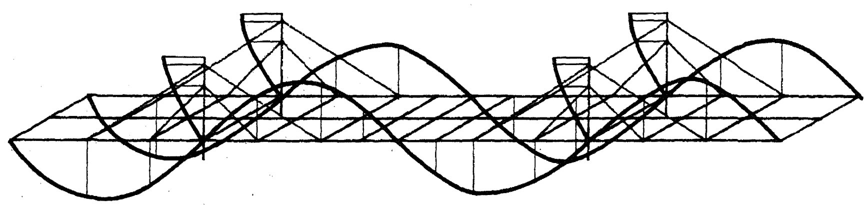


b) Basic model + cross beams

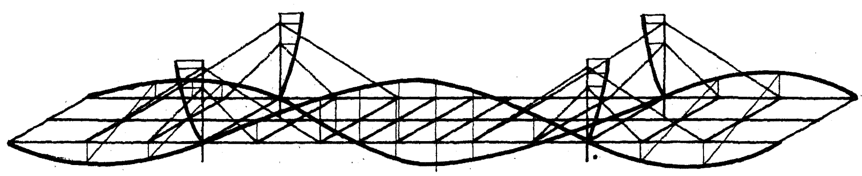


c) Basic model + added stays

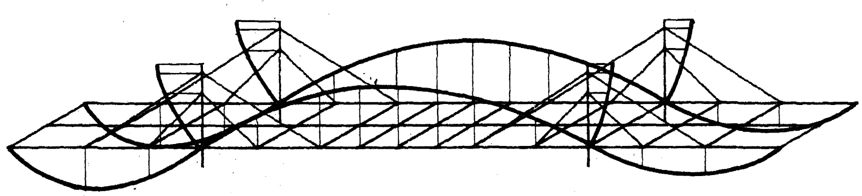
Fig. 2. Mathematical models – input data mesh generation



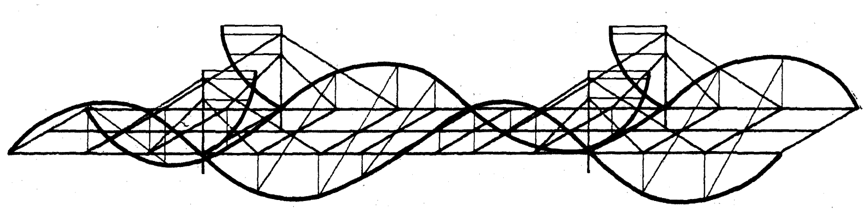
Vertical mode $T_2 = 0.726$ sec.



Torsional mode $T_3 = 0.706$ sec.



Longitudinal mode $T_7 = 0.345$ sec.



Torsional mode $T_9 = 0.234$ sec.

Fig. 3. ***Presentation of selected mode shapes***

Mode order	Period T(sec)	Types of vibrations
1	1.274	vertical
2	0.726	vertical
3	0.706	torsional
4	0.482	vertical
5	0.430	torsional
6	0.401	transversal
7	0.345	longitud.
8	0.343	torsional
9	0.234	torsional
10	0.233	longitud.
11	0.224	vertical
12	0.205	torsional
13	0.199	vertical
14	0.182	transversal
15	0.152	transversal
16	0.120	transversal

Table 1. Some of the computed natural periods for a basic mathematical model

Mode order	Vertical vibrations			
	CSB—B		CSB—AS	
	Tsym. (sec)	Tant.sym. (sec)	Tsym. (sec)	Tant.sym. (sec)
1	1.274		0.847	
2		0.726		0.584
3*	0.706		0.572	
4	0.482		0.428	
5*		0.430		0.383
6*	0.343		0.316	
7		0.224		0.223

CSB—B Cable - stayed bridge - basic model

CSB—AS Cable - stayed bridge - added stays

* — Torsional vibration of cable - stayed bridge

Table 2. Comparison of the cable stays effect upon the dynamic characteristics of the two considered models

Damping %	Max. displacements at the characteristic points of the mathematical model			
	31—LDV (cm)	31—TDT (cm)	17—TDT (cm)	17P—TDT (cm)
2.5	3.64	2.88	20.68	7.58
5	2.67	2.26	16.56	5.37
7.5	1.95	2.00	14.36	4.29

LDV — Vertical displacement at the nodal point under earthquake effect in longitudinal direction

TDT — Transversal displacement at the nodal point under earthquake effect in transversal direction

17P — Nodal point 17, Portal Frame

Table 3. Maximum displacements for the characteristic points of the model for different damping degree

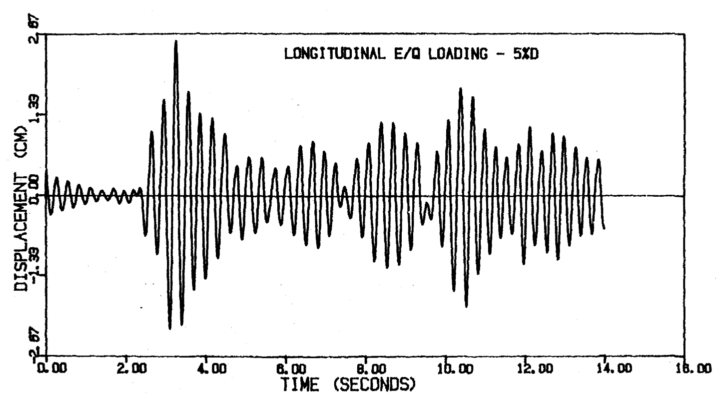


Fig. 4a. Variation of vertical displacement at node 31

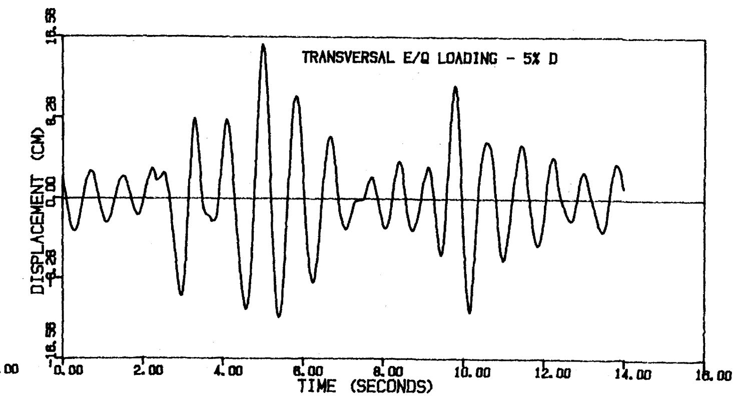


Fig. 4c. Variation of transversal displacement at node 17

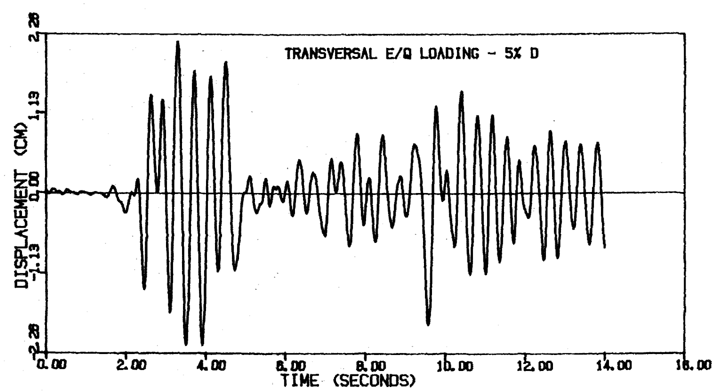


Fig. 4b. Variation of transversal displacement at node 31

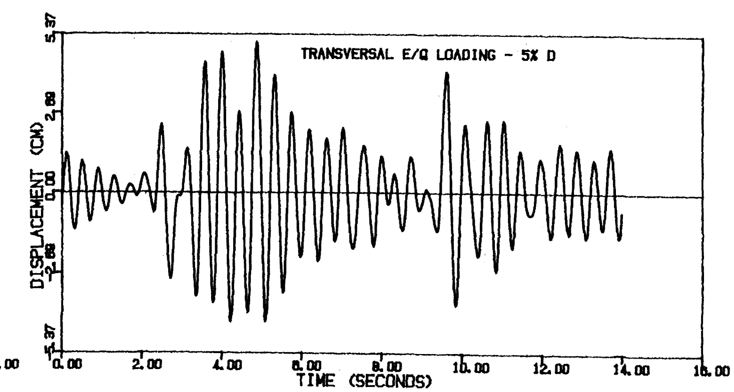


Fig. 4.d. Variation of transversal displacement at node 17 of the Portal frame