RANDOM SEISMIC RESPONSE ANALYSIS OF HIGH-ELEVATED MULTI-SPAN CONTINUOUS BRIDGE

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SUMMARY

This paper presents a random seismic response analysis of a high-elevated multi-span continuous bridge structure in the perpendicular to the bridge axis. This emphasis is placed upon the dynamic soil-structure interaction and travelling effect of seismic motions. The dynamic soil-structure interaction effect is investigated with the available substructure method. The phase difference effects of input motions on structural responses are also examined by means of a simplified cross spectral matrix which denotes the travelling effect of seismic motions.

INTRODUCTION

The substructure method is effectively applicable to the dynamic soil-structure interaction analysis of the high-elevated multi-span continuous bridge structure. The substructure method is consisted of an independent formulation for the superstructure and the soil-foundation system, and combinating them at their interface continuity (Ref.1,Ref.2). Therefore, taking the reduction of the degree-of-freedom due to the first few vibration modes, the substructure method is very available to the seismic response analysis of dynamic soi-structure interaction system(Ref.3).

Moreover, for the high-elevated multi-span continuous bridge structure, seismic motions are assumed to arrive at the respective pier foot with a phase time lag due to an apparent propagation along the bridge axis(Ref.2). The significant frequency range, in view of a seismic response analysis of the present structure, is generally limited to the first few vibration modes. A seismic incident wave, which is spatially variable, may be evaluated with a simplified cross spectral matrix which denotes the correlation of input intensity, the attenuation and the input phase difference due to the travelling seismic motions(Ref.4). If the travelling effect of seismic motions is given by the cross spectral matrix, random seismic response analyses of the structure can be easily taken in terms of the complex mode analysis.

FORMULATION

Substructure Method The substructure method, which deals with the soil-foundation system and superstructure separately and then combines them with use of continuity requirements at their interface, is very available to the seismic response analysis of the dynamic soil-structure interaction system. The efficient analysis, which is formulated with the accurate evaluation of the dynamic properties of the structure and the substantial

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reduction of degree-of-freedom, is then given by this method. For the high-elevated multi-span continuous bridge structure as shown in Fig.1, the system can be separated by the soil-caisson foundation system and the superstructure. The caisson foundation is modelled as a rigid body which has frequency independent soil impedance functions, denoting two degree-of-freedom of horizontal translation and rocking about its gravity center. This impedance function corresponds to the soil conditions that an uniform layer overlines a firm half-space substratum(Ref.5).

Common finite element formulation gives the governing equation for the superstructure. The mass matrix is expressed with the consistent mass matrix. The damping ratio of each vibration mode gives the constant value for the superstructure system. The modal matrix for the rigidly supported superstructure can be determined by the eigenvalue analysis of the undamped vibration system. The substantial reduction of degree-of-freedom is taken in terms of the modal matrix. The significant vibration modes are now denoted by $[\Phi]$. The relative displacement of the superstructure for the rigidly supported system denotes $\{x_a\}$ and the displacement of the soil-foundation system $\{x_b\}$. These displacement can be transformed by the significant vibration modes $[\Phi]$ and the diagonal element of the mass matrix with respect to the soil-foundation system $[\tilde{m}]$ as follows:

$$\begin{Bmatrix}\{x_a\}\\ \{x_b\}\end{Bmatrix} = \begin{bmatrix}[\Phi] & [0]\\ [0] & [\tilde{m}]\end{bmatrix}\begin{Bmatrix}\{q\}\\ \{p\}\end{Bmatrix} = [\tilde{\Phi}]\{s\} \qquad (1)$$

Using the substructure method, the governing equation of the total system can be expressed by

$$[M_s]\{\ddot{s}\} + [C_s]\{\dot{s}\} + [K_s]\{s\} = [F_s]\{\ddot{x}_g\} \qquad (2)$$

in which

$$[M_s] = \begin{bmatrix}[I] & [M_1]\\ [M_1]^T & [M_2]\end{bmatrix},\quad [C_s] = \begin{bmatrix}\lceil 2\beta_j\omega_j \rfloor & [0]\\ [0] & [C_2]\end{bmatrix},\quad [K_s] = \begin{bmatrix}\lceil \omega_j^2 \rfloor & [0]\\ [0] & [K_2]\end{bmatrix}$$

and $[M_1]$, $[M_2]$, $[C_2]$ and $[K_2]$ denote the transformed matrix, which can be evaluated with the application of Eq.(1) on the interaction force and the equation of motion of the soil-foundation system. β_j and ω_j also denote the damping ratio and natural frequency of jth mode for the superstructure, respectively. $\{\ddot{x}_g\}$ denotes the input acceleration at each pier foundation.

Random Vibration Analysis   The equation of motion of the dynamic interaction system expressing in Eq.(2) generally takes the nonproportional damping matrix.Therefore, a random vibration analysis based on the eigenvalue analysis of the undamped vibration system can not be directly applied to Eq.(2). Then, in order to make use of the complex mode analysis, Eq.(2) can be transformed to the first order differential equation:

$$\{\dot{u}\} + [D]\{u\} = [Q]\{\ddot{x}_g\} \qquad (3)$$

in which

$$[D] = \begin{bmatrix}[M_s]^{-1}[C_s] & [M_s]^{-1}[K_s]\\ -[I] & [0]\end{bmatrix},\quad [Q] = \begin{bmatrix}[M_s]^{-1}[F_s]\\ [0]\end{bmatrix},\quad \{u\} = \begin{Bmatrix}\{\dot{s}\}\\ \{s\}\end{Bmatrix}$$

Generally, the random vibration analysis of Eq.(3) can be taken by the application of the complex eigenvalue analysis, which may be easily carried out using the substantial reduction of degree-of-freedom for the superstructure system. Using the complex eigenvalue λ_j and the complex mode matrix $[V]$, Eq.(3) can be rewritten by

$$\{\dot{r}\} + [\lambda_j]\{r\} = [W]\{\ddot{x}_g\} \tag{4}$$

in which

$$[V]^{-1}[D][V] = [\lambda_j] \quad , \quad [V]^{-1}[Q] = [W] \quad , \quad \{u\} = [V]\{r\}$$

Now assume that the input acceleration at each pier foundation is expressed by

$$\{\ddot{x}_g\} = \{f\}\,\ddot{z}_g \tag{5}$$

in which an acceleration $\ddot{z}_g$ denotes a white noise excitation with zero mean and of intensity level S_0. After directly postmultiplying Eq.(4) $\{r\}^T$, and premultiplying the transpose of the same equation by $\{r\}$, then taking the expectations for the respective terms leads to the differential equation of covariance matrix $[R_{rr}]$.

$$\frac{d}{dt}[R_{rr}] + [\lambda_j][R_{rr}] + [R_{rr}][\lambda_j] = 2\pi S_0[F_W] \tag{6}$$

in which

$$[R_{rr}] = E[\{r\}\{r\}^T] \quad , \quad [F_W] = [W]\{r\}\{r\}^T[W]^T$$

and $E[.]$ denotes the mathematical expectation. In the stationary response, the covariance matrix can be easily evaluated by neglecting the the first term of Eq.(6):

$$[R_{rr}]_{jk} = 2\pi S_0[F_W]_{jk}/(\lambda_j + \lambda_k) \tag{7}$$

Using the covariance matrix, the rms response of the structure can be evaluated in terms of the transformation matrices which are determined by means of the eigenvalue analyses:

$$[R_{uu}] = E[\{u\}\{u\}^T] = [V][R_{rr}][V]^T \qquad [R_{xx}] = E\begin{bmatrix}\{x_a\}\{x_a\}^T \\ \{x_b\}\{x_b\}\end{bmatrix} = [\tilde{\Phi}]^T[R_{uu}][\tilde{\Phi}] \tag{8}$$

Generally, the input acceleration depends mainly upon the dynamic properties of stratum. Then, the power spectral density of $\ddot{z}_g$ is useful for a filtered white noise rather than a white noise. The power spectral density of a filtered white noise is expressed in terms of the output absolute acceleration of a single degree-of-freedom system, denoting the predominant frequency ω_g and damping ratio h_g.

For the high-elevated multi-span continuous bridge structure, a seismic motions are assumed to arrive at the respective pier foot with a phase time lag and variation of dynamic characteristics. Now assume that a seismic motion is evaluated with the simplified cross spectral matrix which represents the correlation of input intensity, attenuation and input phase difference due to the propagation. The cross spectral matrix of input acceleration $\{\ddot{x}_g\}$ in Eq.(4) is expressed by $[S_{\ddot{x}_g}(\omega)]$. Taking the Fourier transformation of Eq.(4) in the stationary response, the cross spectral matrix $[S_{rr}(\omega)]$ of the response $\{r(\omega)\}$ can be expressed by

$$[S_{rr}(\omega)] = [G][W][S_{\ddot{x}_g}(\omega)][W]^T[G]^T \tag{9}$$

in which $[G] = \lceil \lambda_j + i\omega \rfloor^{-1}$

and ω is a frequency and $i = \sqrt{-1}$. Then, assume that the element of the simplified cross spectral matrix may be expressed by

$$S_{\ddot{x}_g}(\omega)_{lm} = A_{lm}\exp(-v_{lm})\, S_0 \tag{10}$$

in which $v_{lm} = a_{lm} + i\omega b_{lm}$

and A_{lm} denotes the correlation of intensity level. Moreover, v_{lm} is expressed in terms of a distance L_{lm} of respective pier foot and a phase velocity C_0 of input excitation. Namely,

$$a_{lm} = \kappa L_{lm} \quad , \quad b_{lm} = L_{lm}/C_0$$

For the propagation of seismic motions, a_{lm} denotes the attenuation effect of intensity level and b_{lm} the phase difference effect due to a phase velocity C_0. Using the expression of Eq.(10) for the cross spectral matrix, the covariance matrix $[R_{rr}]$ can be evaluated with the residue theorm. Then, the element of $[R_{rr}]$ is expressed by

$$[R_{rr}]_{jk} = 2\pi S_0[[W][B][W]^T]_{jk}/(\lambda_j + \lambda_k) \tag{11}$$

in which the element of $[B]$ is

$$[B]_{lm} = \exp[(2-j-k)a_{lm}]\,\{\exp[iIm(\lambda_j)(j-k)b_{lm} + \exp[-iIm(\lambda_k)(j-k)b_{lm}]\}/2 \tag{12}$$

and $I_m(\lambda_j)$ denotes the imaginary part of λ_j.

NUMERICAL RESULTS AND DISCUSSION

Dynamic Soil-Structure Interaction A numerical computation is carried out for the high-elevated three-span continuous bridge structure as shown in Fig.1. A seismic response analysis in the direction perpendicular to the bridge axis is taken by means of the substantial reduction of degree-of-freedom and the complex eigenvalue analysis. The governing equation of motion for the superstructure system is formulated with the predominant vibration modes corresponding to frequency under 30 rad/s. The foundation is made of a caisson structure embedded 20m in the surface layer of stratum on the firm substratum. Dynamic soil-structure interaction effects are significant for variations of V_{s2}/V_{s1}, denoting a ratio of the shear wave velocity of strata. For the situation of $V_{s2}/V_{s1}=0.3$ and $V_{s1}=300$ m/s, a seismic response analysis is carried out for the dynamic soil-structure interaction system as shown in Fig.1. Namely, Fig.2 shows the girder vibration modes, which denote the dynamic characteristics of the soil-structure interaction system together with the rigidly supported system. The vibration mode shapes are normalized with respect to the mass matrix. The corresponding natural frequency also represents in Table. Changes in vibration modes due to the soil-structure interaction are recognized for the rigidly supported system. Fig.3 also shows the rms displacement response σ_x and the rms bending moment response σ_M for respective nodes of girder. These responses are normalized with respect to the rms response associated with the situation of the rigidly supported structure. The solid line denotes the rms responses for a white noise excitation. The dash line also denotes the rms responses for a filtered white noise excitation, expressing with characteristic value of $\omega_g =4\pi$ rad/s and h_g=0.6. Moreover,an abscissa denotes the ratio of the natural frequency for the superstructure and the stratum. Namely,

$$e = \Omega_s / \Omega_b$$

in which Ω_S denotes the natural frequency of the superstructure in the situation of the rigidly supported system, and Ω_b denoted the natural frequency of the strata. Then, it is considered that the parameter e indicates the interaction effect between the superstructure and the soil-foundation system. In this study, since the depth of strata takes 60m, the natural frequency Ω_b can be approximately evaluated with the shear wave velocity. The rms response σ_x also corresponds to the absolute displacement response which do not include the displacement of seismic motion, and σ_x^c to the dynamic relative displacement. An increase of parameter e corresponds to the situation of softer strata. For the increase of the parameter e , the displacement response σ_x indicates a trend of increase, while the relative displacement response σ_x^c and the bending moment response σ_M give a trend of decrease.

Phase Difference Effects For the high-elevated three-span continuous bridge as shown in Fig.1, examine about phase difference effects on responses resulting from the propagation of seismic motion. The input excitation properties of respective pier foot are expressed in terms of the simplified cross spectral matrix expressed in Eq.(10). The seismic response analysis is carried out in the situation of neglecting the attenuation effect. For the representative nodes of girder, Fig.4 shows phase difference effects of input excitation on the rms displacement responses. These responses are normalized with respect to the rms response σ_{x_O} due to no phase input situations. The response variations on the input phase difference depend mainly upon the predominant vibration modes. Similary, Fig.5 shows the input phase difference effects on the rms bending moment response. These responses are normalized with respect σ_{M_O} due to no phase input situations. It is shown that the input phase difference gives significant effects on the bending moment response rather than the displacement response.The input phase difference effects depend also upon the phase velocity of seismic motion. Then, with respect to the input phase difference effects, an accurate evaluation of phase velocity provides the good estimation of structural responses. Fig.6 also shows the input phase difference effects on the rms bending moment response for the representative nodes of pier. It is noticed that the first mode effect indicates a decaying tendency for the input phase difference, while the higher modes give significant effects on the responses. Then, it is important to evaluate the significant effects of higher modes on the response for the travelling seismic motion. From these results, the use of conventional response spectrum may underestimate responses where higher modes contributions are significant, while it gives a possible maximum response for the presominant contributions of the first mode.

CONCLUSION

From these investigations, one may concluse that:

For the present structure, the dynamic soil-structure interaction effects indicate the increase of absolute displacement, and the decrease of relative displacement and bending moment of the superstructure. It is shown that the effects are mainly depended upon the ratio of a natural frequency between the superstructure and stratum.

The phase difference effects of input excitation on structural responses is closely related to the vibration modes in such a way that the first mode contribution is most predominant at no phase input situation, while higher modes contribution increases at specific situations of input phase difference. It is, therefore, important for the extending structures as dealt herein to investigate the travelling effect of input motions on structural responses.

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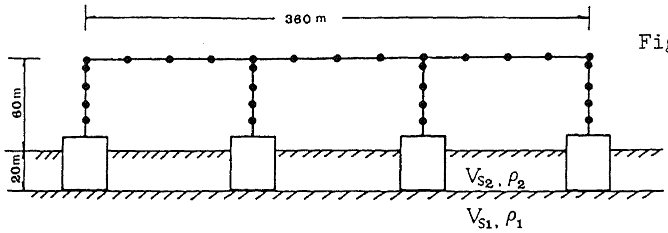
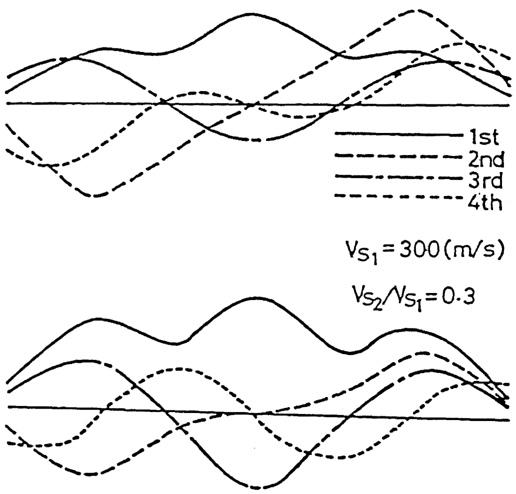


Fig.1 Soil-Footing Structure System



(a) Rigidly Supported System

(b) Soil-Structure Interaction System

Fig.2 Girder Vibration Mode Shapes

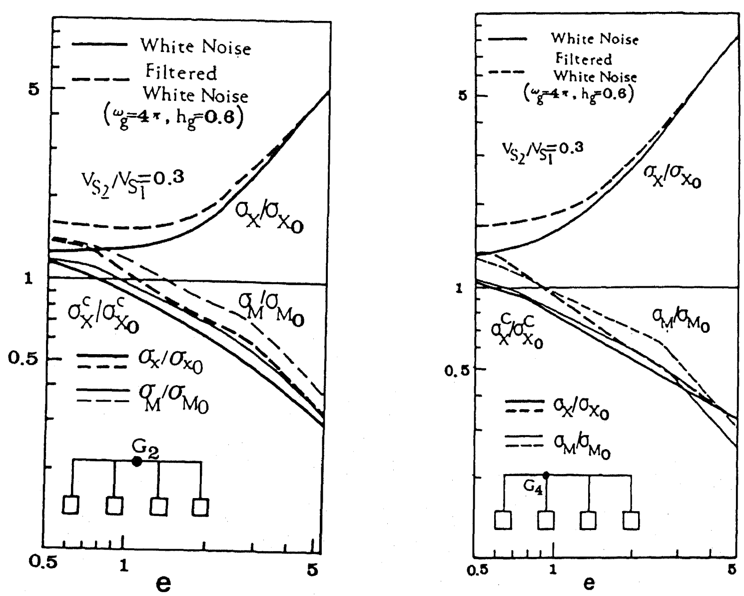


Fig.3 rms Response vs e for Soil-Structure Interaction System

Table Natural Frequencies

Modes	Rigidly Supported System rad/s (s)	Interaction System (rad/s) $V_{s1}/V_{s2} = 0.3$	
		V_{s1} = 300 m/s	V_{s1} = 500 m/s
1	7.83 (0.802)	3.93	5.56
2	8.01 (0.784)	4.30	5.98
3	8.34 (0.754)	4.89	6.84
4	12.0 (0.524)	5.37	7.65
5	13.6 (0.462)	9.34	9.95

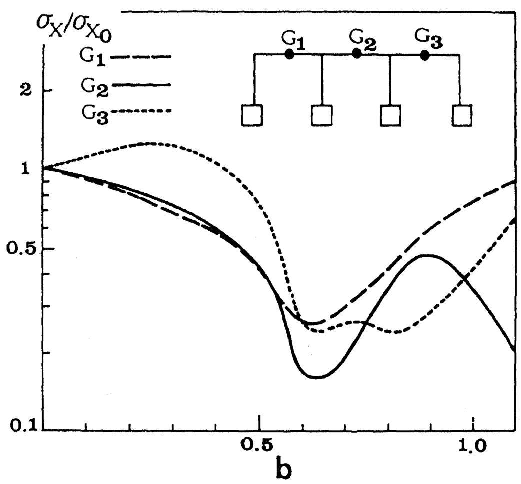


Fig.4 rms Displacement Response vs Phase Lag

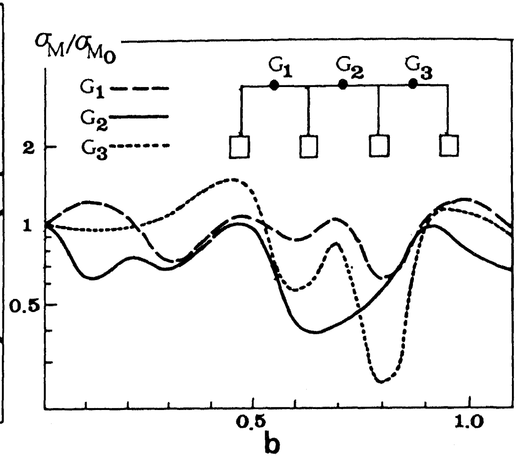


Fig.5 rms Bending Moment Response vs Phase Lag

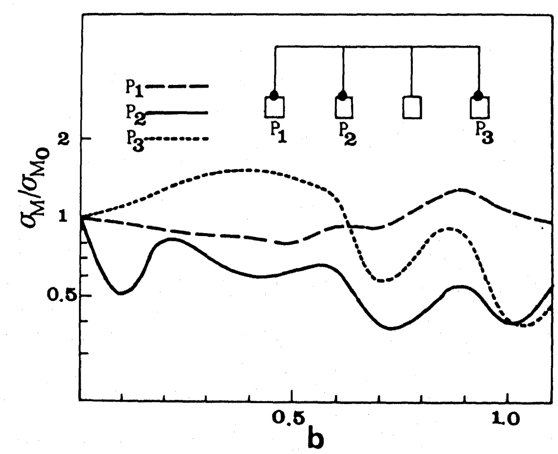


Fig.6 rms Bending Moment Response vs Phase Lag