SEISMIC ANALYSIS OF ROTATING MECHANICAL SYSTEMS

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ABSTRACT

This paper presents the comparison of two models simulating the seismic performance of a rotor-bearing system commonly encountered in a nuclear power-plant. The first model treats the rotor as a rigid body subjected to gyroscopic and coriolis effects. The second model treats the rotor as flexible. A finite-element-beam-model, is developed to study the influence of spin, and base rotation. The results of the above study show that the gyroscopic effects amplify the response of the rotor-bearing systems. The base rotations of the rotor - bearing system under seismic excitation contribute significantly to the responses of the rotor.

INTRODUCTION

In recent years, there is considerable interests in studying the seismic analysis of active mechanical equipments, such as power generator, pumps, motors, etc. housed in a building. The present paper develops two models and compares their relative performance for a rotor-bearing system subjected to seismic activities. The seismic analysis of rotating systems differs from the seismic analysis of structural sytems in two major aspects: (1) rotor-bearing interaction effects (2) gyroscopic effects. In addition, coriolis effects become significant when the base of rotating systems is subjected to rotational motions. Such seismic analysis will permit one to (1) examine the required minimum fluid film thickness (2) withstand the bearing reaction forces and (3) to maintain maximum allowable dynamic stresses induced in the rotor.

MATHEMATICAL DEVELOPMENTS FOR RIGID-BODY MODEL

Figure 1 shows a schematic representation of a rotor-bearing system. Note the three coordinate systems XYZ: associated with the rotor at G. x_b, y_b, z_b, associated with the base, and x_ε, y_ε, z_ε, associated with the rotor but non-spinning. The mass-center G of the rotor is located at height h. The rotor is rotating with angular velocity ω. The newtons laws of motion for the rigid body can be applied as $\overline{F} = m\overline{a}_G$ and $\overline{M}_G = \dot{\overline{H}}_G$ where $\overline{F}$, m, $\overline{a_G}$, $\overline{M}_G$, and $\dot{\overline{H}}_G$ denote respectively the resultant force as rotor, mass of rotor, acceleration of mass center, moment due to external forces, and rate of change of angular momentum. Let ψ, θ, and ϕ define the precession, nutation and spin angles of the rotor. Then, because of the specific orientation of the rotor-reference

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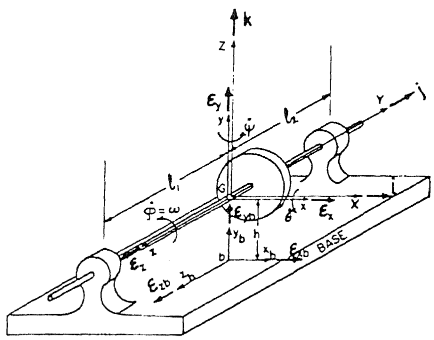

Figure 1. Rotor and Base Reference Axes

system, and small rotational displacements, we are able to linearize the relationships: $\theta = \pi/2 + \theta_{xb} + \theta_x$, and $\psi_x = \psi_{yb} + \theta_y$. These relationships will yield the following A_G^{xb}: Mass center acceleration, F: External forces on the rotor, M: External moments on rotor and F_i: Forces acting on the rotor at the ith station.

$$\begin{aligned}
\underline{a}_G = \{ & \ddot{x}_G - 2\dot{\theta}_{zb}\dot{y}_G + 2\dot{\theta}_{yb}\dot{z}_G - (\dot{\theta}_{yb}^2 + \dot{\theta}_{zb}^2)x_G + (\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})\, y_G - \\
& (\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})z_G + \ddot{x}_b + h(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})\}\, \underline{\varepsilon}_{xb} + \\
\{ & \ddot{y}_G + 2\dot{\theta}_{zb}\dot{x}_G - 2\dot{\theta}_{xb}\dot{z}_G + (\dot{\theta}_{xb}\dot{\theta}_{yb} + \ddot{\theta}_{zb})x_G - \\
& (\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2)\, y_G + (\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})z_G + \ddot{y}_b - h(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2)\}\, \underline{\varepsilon}_{yb} + \\
\{ & \ddot{z}_G - 2\dot{\theta}_{yb}\dot{x}_G + 2\dot{\theta}_{xb}\dot{y}_G + (\dot{\theta}_{zb}\dot{\theta}_{xb} - \ddot{\theta}_{yb})\, x_G + (\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})\, y_G - \\
& (\dot{\theta}_{xb}^2 + \dot{\theta}_{yb}^2)\, z_G + \ddot{z}_b + h(\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})\}\, \underline{\varepsilon}_{zb}
\end{aligned} \qquad (1)$$

$$\begin{aligned}
\underline{F} = -\{ & (k_{xx1} + k_{xx2})\, x_G + (k_{xy1} + k_{xy2})\, y_G + (-\ell_1 k_{xy1} + \ell_2 k_{xy2})\theta_x \\
& + (\ell_1 k_{xx1} - \ell_2 k_{xx2})\theta_y + (c_{xx1} + c_{xx2})\dot{x}_G + (c_{xy1} + c_{xy2})\dot{y}_G + \\
& (-\ell_1 c_{xy1} + \ell_2 c_{xy2})\dot{\theta}_x + (\ell_1 c_{xx1} - \ell_2 c_{xx2})\dot{\theta}_y\}\, \underline{\varepsilon}_{xb} \\
-\{ & (k_{yx1} + k_{yx2})x_G + (k_{yy1} + k_{yy2})y_G + (-\ell_1 k_{yy1} + \ell_2 k_{yy2})\theta_x \\
& + (\ell_1 k_{yx1} - \ell_2 k_{yx2})\theta_y + (c_{yx1} + c_{yx2})\dot{x}_G + (c_{yy1} + c_{yy2})\dot{y}_G + \\
& (-\ell_1 c_{yy1} + \ell_2 c_{yy2})\dot{\theta}_x + (\ell_1 c_{yx1} - \ell_2 c_{yx2})\dot{\theta}_y\}\, \underline{\varepsilon}_{yb} \\
-\{ & (k_{zz1} + k_{zz2})z_G + (c_{zz1} + c_{zz2})\dot{z}_G\}\, \underline{\varepsilon}_{zb}
\end{aligned} \qquad (2)$$

$$
\begin{aligned}
\mathbf{M}_G = & -\{(-\ell_1 k_{yx1} + \ell_2 k_{yx2})x_G + (-\ell_1 k_{yy1} - \ell_2 k_{yy2})y_G \\
& + (\ell_1^2 k_{yy1} + \ell_2^2 k_{yy2})\theta_x + (-\ell_1^2 k_{yx1} - \ell_2^2 k_{yx2})\theta_y \\
& + (-\ell_1 c_{yx1} + \ell_2 c_{yx2})\dot{x}_G + (-\ell_1 c_{yy1} - \ell_2 c_{yy2})\dot{y}_G \\
& + (\ell_1^2 c_{yy1} + \ell_2^2 c_{yy2})\dot{\theta}_x + (-\ell_1^2 c_{yx1} - \ell_2^2 c_{yx2})\dot{\theta}_y\} \, \boldsymbol{\varepsilon}_{xb} \\
& -\{(\ell_1 k_{xx1} - \ell_2 k_{xx2})x_G + (\ell_1 k_{xy1} - \ell_2 k_{xy2})y_G \\
& + (-\ell_1^2 k_{xy1} - \ell_2^2 k_{xy2})\theta_x + (\ell_1^2 k_{xx1} + \ell_2^2 k_{xx2})\theta_y \\
& + (\ell_1 c_{xx1} - \ell_2 c_{xx2})\dot{x}_G + (\ell_1 c_{xy1} - \ell_2 c_{xy2})\dot{y}_G \\
& + (-\ell_1^2 c_{xy1} - \ell_2^2 c_{xy2})\dot{\theta}_x + (\ell_1^2 c_{xx1} - \ell_2^2 c_{xx2})\dot{\theta}_y\} \, \boldsymbol{\varepsilon}_{yb}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\mathbf{F}_i = & -(k_{xxi} x_i + k_{xyi} y_i + c_{xxi}\dot{x}_i + c_{xyi}\dot{y}_i) \, \boldsymbol{\varepsilon}_{xb} \\
& -(k_{yxi} x_i + k_{yyi} y_i + c_{yxi}\dot{x}_i + c_{yyi}\dot{y}_i) \, \boldsymbol{\varepsilon}_{yb} \\
& -(k_{zzi} z_i + c_{zzi}\dot{z}_i) \, \boldsymbol{\varepsilon}_{zb}
\end{aligned}
\tag{4}
$$

where $C_{xy_1} = C_{yx_1}$ denote damping coefficients and k_{xxi}, k_{yyi}, etc. denote the stiffness coefficient due to lubricant film. These relationships when rearranged will yield the equation of motion

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \tag{5}$$

where [M], [C], [K] and [F] denote respectively the mass, damping stiffness matrices and external forces due to seismic excitation which includes base rotation and translation. This can be solved for [X]-rotor displacement using the Newmark numerical technique.

BEAM MODEL

The beam model of a rotor bearing system subjected to seismic response permits us to include the following effects: rotary inertia, shear deformation, gyroscopic effects, rotor-bearing interaction, intermediate disks and flywheels, axial thrust, axial torque, base translation, and base rotation. The beam-model for the rotor is developed considering the shaft as made up of series of circular discs. An elemental disc is shown in Figure 2. Because of small displacements in a seismic activity, we are able to

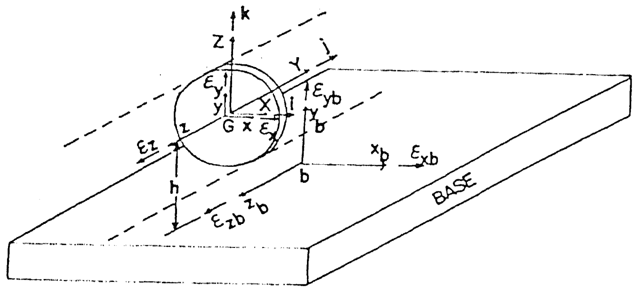

Figure 2. Rotor and Base Reference Frames

follow linearize development of equations of motion. The governing equations of motions for an elemental disc may be written as:

$$
\begin{aligned}
\frac{\partial Q_x}{\partial s} + P\frac{\partial \theta_y}{\partial s} + f_x &= \rho A a_x \\
\frac{\partial Q_y}{\partial s} - P\frac{\partial \theta_x}{\partial s} + f_y &= \rho A a_y \\
\frac{\partial M_x}{\partial s} - Q_y + P\left(\frac{\partial u_y}{\partial s} + \theta_x\right) + T\frac{\partial \theta_y}{\partial s} &= \rho\left\{ I_T(\ddot{\theta}_{xb} + \ddot{\theta}_x) + I_P\,\omega(\dot{\theta}_{yb} + \dot{\theta}_y)\right\} \\
\frac{\partial M_y}{\partial s} + Q_x - P\left(\frac{\partial u_x}{\partial s} - \theta_y\right) - T\frac{\partial \theta_x}{\partial s} &= \rho\left\{ I_T(\ddot{\theta}_{yb} + \ddot{\theta}_y) - I_P\omega(\dot{\theta}_{xb} + \dot{\theta}_x)\right\}
\end{aligned}
\tag{6}
$$

Where Q_x, Q_y, denote shear forces, M_x, M_y denote moment, T -denotes external torque, P -represent axial force, ω denotes rotor angular velocity, A -denotes cross-sectional area of the disc, ρ -denotes specific density, a_x, a_y describe the mass-center acceleration components of the disk, U_x, U_y, θ_x, θ_y describe the displacements of the disc along and about x, y axes, and I_T and I_P the moment of inertia about X (or Y) and Z axes - respectively. The external forces f_x and f_y per unit length distributed along the rotor axis in x_b, y_b directions can be expressed as

$$
\begin{aligned}
f_x &= -\sum_{i=1}^{n} \left\{(k_{xx})_i(u_x)_i + (k_{xy})_i(u_y)_i + (c_{xx})_i(\dot{u}_x)_i + (c_{xy})_i(\dot{u}_y)_i\right\} \delta(s - s_i) \\
f_y &= -\sum_{i=1}^{n} \left\{(k_{yx})_i(u_x)_i + (k_{yy})_i(u_y)_i + (c_{yx})_i(\dot{u}_x)_i + (c_{yy})_i(\dot{u}_y)_i\right\} \delta(s - s_i)
\end{aligned}
\tag{7}
$$

where in k_{xx}, k_{yy}, etc. describe the stiffness properties and c_{xx}, c_{yy}, etc. describe the damping of the bearing field.

The above equation of motion is in the form of partial differential equations involving spatial variations S and tamporal variant t. A numerical

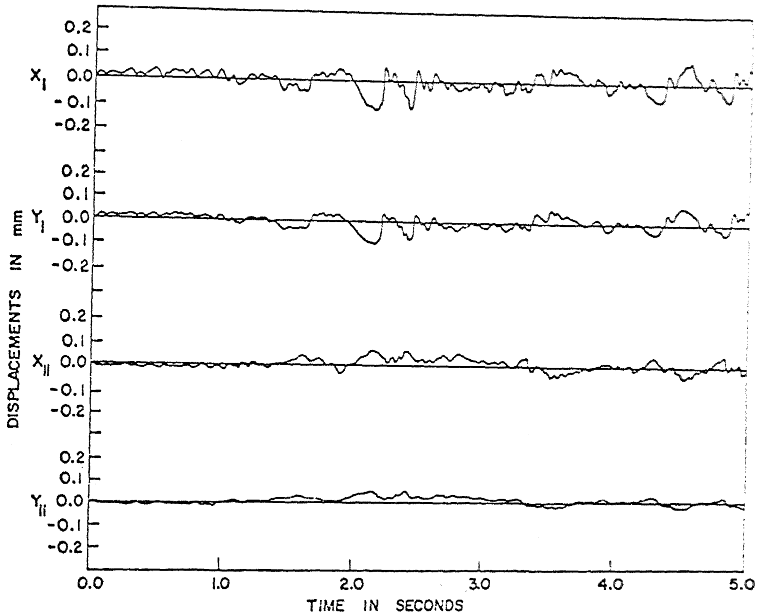

Figure 3. Displacements of Rotor in the Bearings
(Rigid Body Model)

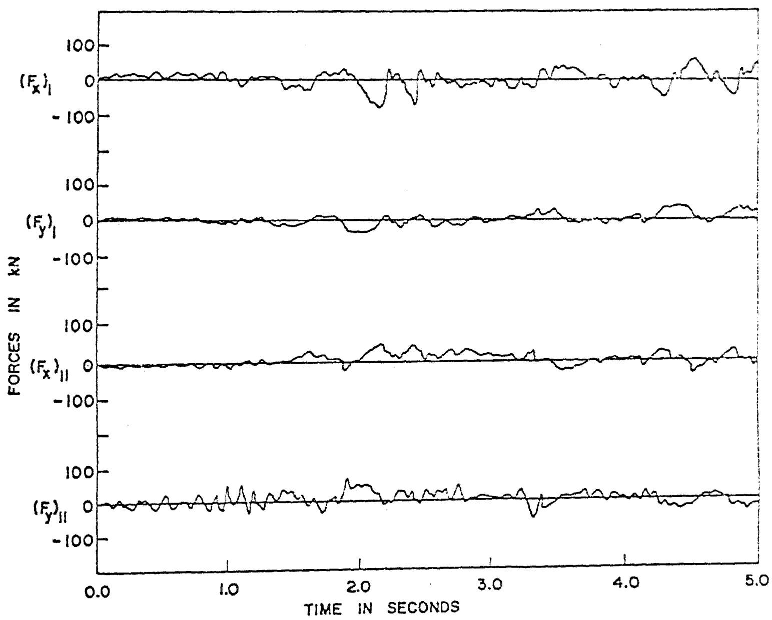

Figure 4. Dynamic Reaction Forces in the Bearings
(Rigid Body Model)

solution to the problem is obtained using finite elements in the spatial domain and finite differences in the time domain. However before proceeding for solution of the above equations, these must be presented in an integral form. This is achieved by applying Galerkin's technique.

Using simple linear interpolation for displacements and rotations for a typical rotor element with two nodes at S_1 and S_2 (e.g. $U_x = (U_x)_1 N_1(S) + (U_x)_2 N_2(S)$, $\theta_x = (\theta_x)_1 N_1(S) + (\theta_x)_2 N_2(S)$, etc. where $N_1 = (S_2 - S)/(S_2 - S_1)$, the equation of motion can be written as

$$\delta\{q\}_e^T \; [[\bar{M}]_e\{\ddot{q}\}_e + [\bar{C}]_e\{\dot{q}\}_e + [K]_e\{q\}_e] = \delta\{q\}_e^T \{Q\}_e \quad (8)$$

where $\{q\}^T = \{U_x)_1, (U_y)_1, (\theta_x)_1, (\theta_y)_1, (U_x)_2, (U_y)_2(\theta_x)_2(\theta_y)_2]$, $[\bar{M}]_e$ denotes elemental inertia matrix, $[C]_e$ describes a sum total of gyroscopic matrix, coriolis matrix due to base rotation, and damping matrix due to bearings located at nodes, $[\bar{K}]_e$ describes a sum total of conventional stiffness matrix for the beam element, geometric stiffness matrix due to axial force, geometric stiffness matrix due initial axial torque, supplemental stiffness matrix due to base rotation and stiffness matrix due to bearings located at the nodes; $[\bar{Q}]_e$ denotes a vector of nodal forces and moments due to base translations and rotation and $\{q\}_e$ is a column vector of rotational and linear displacements at two nodes. The above equations can be modified to include the effect of intermediate disks and flywheels.

The governing equations for the rotor can be obtained by properly assembling the elemental disk matrcies and vectors. Such governing equations may be expressed as

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \quad (9)$$

which can be solved using Newmark's technique.

COMPARISON STUDY OF THE TWO MODELS

The equations of motions obtained for the rigid body model and for the beam model were solved to obtain the seismic response in terms of rotor displacements at the bearings. For the rotor mass of 24,000 Kg and flywheel mass 5000 Kg rotating at 3000 rpm, and moment of inertia for rotor 4.57 X 10^3 kgm^2 and 2500 kgm^2 for the flywheel, and for the Elcentro-excitation data, figures 3, 4, 5 and 6 show the rotor behavior in seismic response for the two models. The two models show that the effect of base rotation and spin of the shaft contribute significantly in rotor displacements and dynamic reaction forces at the bearings. For the the rigid body model however, the rotor displacement and dynamic reaction forces at the bearings are significantly lower than those obtained from the response of the beam model. Computationally, rigid-body model is relatively simple. For the beam model, a total of 19 elements for the total rotor length of 8.5 m, the time involved was relatively low on IBM 370/165.

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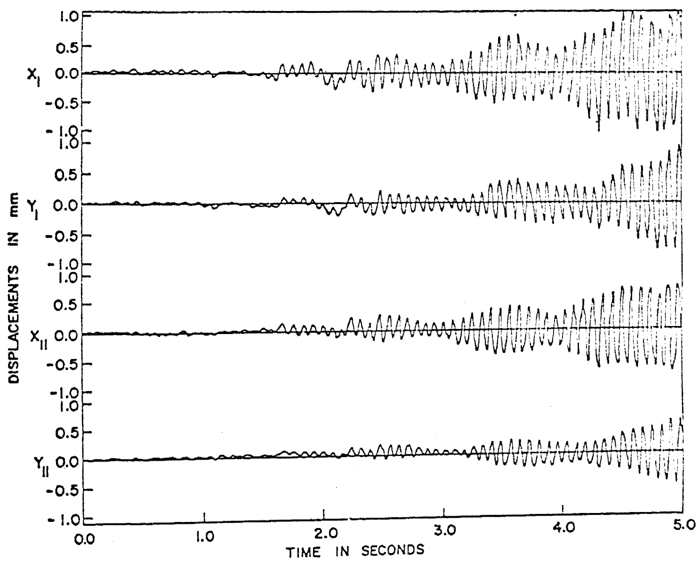

Figure 5. Displacements of Rotor in the Bearings (Base Rotation Included)

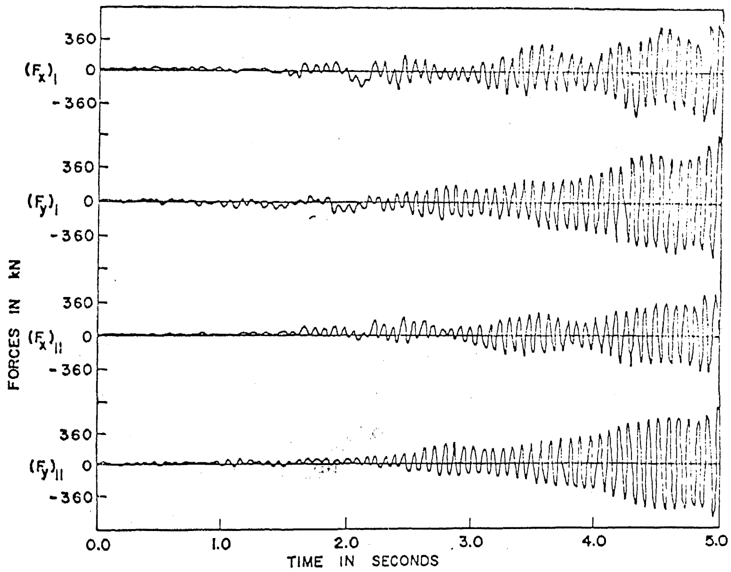

Figure 6. Dynamic Reaction Forces in the Bearings (Base Rotation Included)