NONSTATIONARY RESPONSE OF CONCRETE GRAVITY-DAM-RESERVOIR TO RAMDOM VERTICAL EARTHQUAKE EXCITATION

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SUMMARY

The problem of nonstationary response of concrete gravity dam-reservoir systems to nonstationary random vertical ground acceleration is solved first by a frequency domain approach based on the general method of Priestly and Shinozuka and the non-random solution of Rosenblueth. In addition, nonstationary mean square responses are also obtained by integration of the spectra in the frequency domain. Parallel to the frequency domain approach, a time domain solution for nonstationary mean square responses are obtained by a simple analytic method. An excellent agreement between solutions by these two approaches is found.

INTRODUCTION

The problem of earthquake response of gravity dam-reservoir systems has been investigated extensively by Chopra and his colleagues since 1967 (Ref. 1). Their most recent paper (Ref. 3) gives a comprehensive investigation on the dynamic behavior of gravity dams subjected to either a horizontal or a vertical harmonic ground excitation, taking into account the interaction among the elastic dam, the compressible water and the deformable base of the reservoir. However, most of the past research are based on the assumption that the ground acceleration is a deterministic function of time. The only exceptions known to the author are one paper published by Chopra in 1967 (Ref. 2) and an approximate solution by Rosenblueth in 1968 (Ref. 10) with a brief presentation in the book by Newmark and Rosenblueth (Ref. 8). In Chopra's work, the ground acceleration is considered as a stationary white noise and the mean square stationary responses are obtained in integral form. The integrals, unfortunately, diverge because the system with a rigid reservoir base is undamped and the stationary excitation persists indefinitely in time, as pointed out by Chopra. Rosenblueth, on the other hand, centered his attention on the important design problem of estimating the absolute maximum response of the dam subjected to a segment of stationary white noise by an approximate method.

In view of the recognized significance of stochastic approach to problems of earthquake structural dynamics in general because of the inherent random nature of the earthquake excitations (Refs. 4, 6, 8), and

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the lack of research in the random aspect of the earthquake-dam-reservoir problem relative to the extensive research in the non-random aspect, it is therefore important to seek for fundamental solutions of the problem as a first step towards a comprehensive solution in the future. With this perspective in mind an investigation has been carried out and published by Yang and Charito (Ref. 12) for the two dimensional gravity dam-reservoir problem subjected to an idealized random earthquake excitation in the horizontal direction. The difficult problem of diverging mean square response as experienced by Chopra (Ref. 2) has been removed by considering a nonstationary excitation and response.

As the first stochastic investigation of the problem, namely nonstationary random vibration of the gravity dam-reservoir system subjected to the vertical component of earthquake ground acceleration, a number of simplifying assumptions are made. These include a one-dimensional analysis, a rigid dam, nonviscous water, small motion of the system and negligible surface water wave. Among these assumptions, the effect of dam deformation has been thoroughly studied and can be included in the stochastic analysis with no theoretical difficulty. It will, of course, complicate the numerical analysis considerably. Therefore this effect is left out for simplicity at this time. The effect of deformable reservoir base, on the other hand, is taken into account in this investigation. This effect is important because it serves as a damping mechanism for the system which leads to bounded system response to both stationary and nonstationary excitation. Thus the fundamental difficulties which Chopra encountered in his 1967 stochastic investigation of the problem (Ref. 2) are treated directly by setting up a damping mechanism and by considering time-dependent excitation and response.

NON-RANDOM SOLUTION

The fundamental solution in the form of a unit impulse response function for the hydrodynamic pressure due to a Dirac delta function acceleration in the vertical direction at the reservoir base as given by Rosenblueth (Ref. 5, 6) is

$$h_p(y,t) = \frac{1+\alpha}{2}\ \frac{wc}{g} \sum_{n=0}^{\infty} (-\alpha)^n \left[U\left(t-\frac{2nH+y}{c}\right) - U\left(t-\frac{2nH+2H-y}{c}\right)\right], \text{ for } 0\leq y\leq H \qquad (1)$$

and $t\geq 0$ where $\alpha = (c_0w_0-cw)/(c_0w_0+cw)$ is the refraction coefficient representing the reduction of response during the refraction and reflection process of the propagating waves. Also U is the Heaviside unit step function w_0 the unit weight of reservoir base, w that of water, c_0 the sound speed in reservoir base, c that in water and g the gravitation acceleration. The function $h_p(y,t)$ is zero, or course, for $t<0$ as required by the initial condition. For the idealized case of rigid reservoir base with $\alpha=1.0$, $h_p(t)$ is undamped and periodic with period of $4H/c$. For a deformable reservoir base with $\alpha<1.0$, $h_p(t)$ is damped periodic. Once the hydrodynamic pressure solutions are obtained, the shear force and overturning moment solutions can be readily determined by simple integrations.

RANDOM EXCITATION AND RESPONSE

A well known nonstationary random process model, which is in the form of a modulated stationary process, was first proposed by Pristley (Ref. 9). Since the model is capable of characterizing the main features of earthquake accelerations (random magnitude, frequency content as well as temporal variation) and is suitable for analytic work, it has been gradually accepted for earthquake modeling, for example, by Shinozuka (Ref 11), Liu (Ref. 7), Corotis and Venmarke (Ref. 5), Clough and Penzien (Ref. 4), and Yang and Charito (Ref. 12). Priestley's model will be used here for the vertical earthquake ground acceleration a(t) in the following form:

$$a(t) = \int_{-\infty}^{\infty} B(t,\omega)e^{i\omega t}dA(\omega) \tag{2}$$

in which $B(t,\omega)$ is a deterministic modulation function and $dA(\omega)$ is defined as a zero mean random orthogonal process in that the expected value $E[dA(\omega)^{*}dA(\omega')] = 0$, when $\omega \neq \omega'$. When $B(t,\omega)$ takes the special value of unity the random process a(t) reduced to the associated stationary process $a_s(t)$.

When the reservoir-dam system is excited by such a nonstationary random earthquake acceleration a(t), the response hydrodynamic force F(t) on the dam is also a nonstationary random process with an integral representation similar to that for a(t). The input-output transfer relationship for linear systems with these nonstationary random excitation and response processes has been obtained by Shinozuka (Ref. 11). For the hydrodynamic response force on the dam, the nonstationary power spectral density is given by

$$S_f(t,\omega) = \left| \int_{\xi=0}^{t} B(t-\xi,\omega)e^{-i\omega\xi}h_f(\xi)d\xi \right|^2 S_a(\omega) \tag{3}$$

in which $S_a(\omega)$ is the power spectral density of the associated stationary process of the vertical earthquake ground acceleration $a_s(t)$, $h_f(t)$ the impulse response function for the hydrodynamic force on the dam and $B(t,\omega)$ is the modulation function in Eq. 2. On the basis of this transfer relation in the frequency domain, one can determine the transcient frequency distribution of the mean square hydrodynamic force on the dam from the specified vertical ground acceleration in terms of $a_s(t)$ and $B(t,\omega)$ and the non-ramdom response solution $h_f(t)$ by Rosenblueth. The total mean square hydrodynamic force $E[F^2(t)]$ can be obtained by integrating $S_f(t,\omega)$ over the frequency domain.

NUMERICAL AND ANALYTICAL SOLUTIONS

For the first problem, the nonstationary random vertical ground acceleration process a(t) is assumed to be a suddenly applied stationary white noise with a unit step modulation function $B(t,\omega) = U(t)$ and a uniform density S_o for the associated stationary process $a_s(t)$. Although this is the simplest nonstationary model, it nevertheless is transcient in nature and is fundamentally different from the stationary case. The transcient spectral densities $S_f(t,\omega)$ of the hydrodynamic force on the dam are calculated at various multiples of the natural period $T_1 = 4H/c$ including the long time limiting case where $t\to\infty$, corresponding to the stationary response. For each time, the response spectral density $S_f(t,\omega)$ covers a frequency range of up to five and half natural frequencies ($\omega_1 = \pi c/2H$). These results are obtained by a simple numerical integration technique with very fine time and frequency increments and are presented in non-dimensional variables, Fig. 2. The total mean square hydrodynamic forces $E[F^2(t)]$ are calculated as a function of time for values of the reflection coefficient $0.815 \leq \alpha < 1.0$ and are shown in Fig. 3.

The results obtained so far are based on the Priestley-Shinozuka solution in the frequency domain. As an alternate approach it is instructive and useful to consider the time-domain solution for the problem, particularly to insure the accuracy of the numerical results. For the simple nonstationary problem, the mean square response is given by

$$E[F^2(t)] = 2\pi S_o \int_o^t h_f^2(t-\tau)\,d\tau \qquad (4)$$

The simplicity of this solution is a direct result of the assumed simple model for the ground acceleration. Moreover, because of the periodicity of the impulse response function $h_f(t)$ as mentioned previously, this integral can be readily carried out analytically to give

$$E[F^2(t)] = \frac{\pi S_o (1+\alpha)^2 \rho^2 c H^3}{3} \sum_{n=1}^{N} \alpha^{2(n-1)} \qquad (5)$$

where N is the number of half natural periods in time and in the limiting stationary response $E[F^2]\to\pi S_o \rho^2 c H^3 (1+\alpha)/3(1-\alpha)$. The mean square response forces from Eq. 5 are plotted in Fig. 3 in the same nondimensional format as previously used for frequency domain numerical solutions, for the case of rigid foundation with $\alpha = 1.0$ and of elastic foundation with $\alpha = 0.815$. The time domain solution for $\alpha = 0.815$ is almost identical to the previous frequency domain solution.

DISCUSSION OF RESULTS

Fig. 2 shows that the nonstationary spectral densities of the response hydrodynamic force approach to the stationary case in about ten natural periods (τ=10). The initial spectra are relatively broad in frequency band than those at later times. All spectra are centered around the first natural frequency (Ω=1.0) with negligible density distribution beyond two natural frequencies. Fig. 3 shows that the total mean square hydrodynamic force for the case of elastic reservoir base (α=0.815) increases with time and approaches to the stationary case in about ten natural periods. As the rigidity of the reservoir base increases, the refraction coefficient α increases accordingly, resulting in higher and higher mean square responses.

The limiting case of a perfectly rigid reservoir base with $\alpha = 1.0$ is shown by a straight line and is unbounded as $t \to \infty$. This unbounded response is analogous to the undamped vibration under a steady state excitation. The deformability of the reservoir base serves as a damping mechanism for the system and that the system response converges as time increases. Note that the dimensional mean square hydrodynamic force on the dam depends on the magnitude of the spectral density S_o of the vertical ground acceleration, the hydrostatic shear force ($F_o = wH^2/2$) and the first natural frequency ($\omega_1 = \pi c/2H$) in addition to the refraction coefficient α and the non-dimensional time ($\tau = t/T_1$; $T_1 = 4H/c$).

To gain some engineering insight of the problem, a crude quantitative analysis can be worked out as follows. First, it is observed that the response spectra as shown in Fig. 2 have negligible magnitude beyond the range of two natural frequencies ($\Omega = 2.0$). Consequently, the solution is applicable for ground excitations with a white spectrum cut-off beyond $\Omega = 2.0$ or $\omega = 2\omega_1$, the so called band-limited spectrum. On this basis, consider the uniform magnitude of the two-sided spectrum S_o to be $g^2/4\omega_1$, meaning that the mean square acceleration excitation equals g^2 with root mean square (RMS) value of one gravitational acceleration. With this assumed random acceleration, $S_o = g^2/4\omega_1$, the stationary root mean square (RMS) hydrodynamic force response can be estimated from the curve with $\alpha = 0.815$ in Fig. 2 to be $\sqrt{20(32/\pi^4)}\, F_o$ or about 2.6 times the hydrostatic force. Similarly when $S_o = g^2/400\omega_1$ then the RMS acceleration equals 0.1g and the estimated stationary RMS force becomes about 0.26 times the hydrostatic force.

CONCLUSION

The problem of nonstationary response of a concrete gravity dam-reservoir system to nonstationary random vertical ground acceleration is solved first by a frequency domain approach based on the general method of Priestly and Shinozuka and the non-random solution of Rosenblueth. Parallel to the frequency domain approach, a time domain solution for nonstationary mean square responses are obtained by a simple analytic method. An excellent agreement between solutions by these two approaches is obtained for the mean square hydrodynamic force.

ACKNOWLEDGMENT

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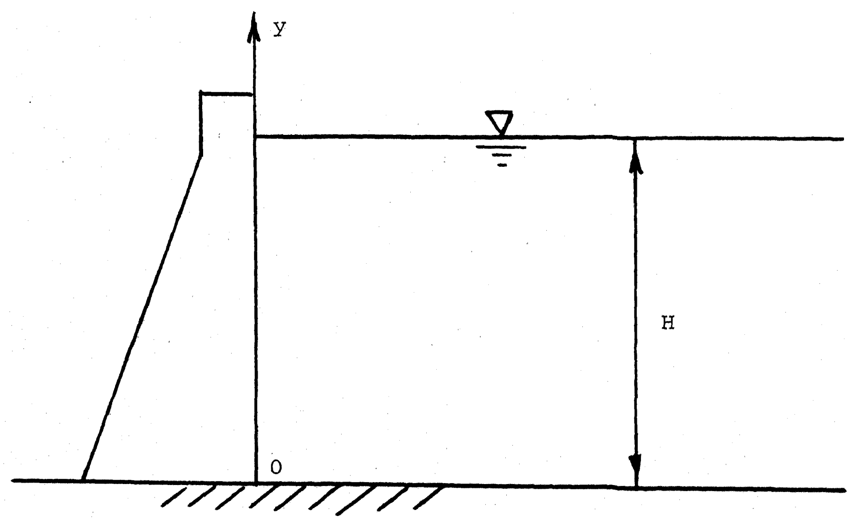

Figure 1. Dam-reservoir system

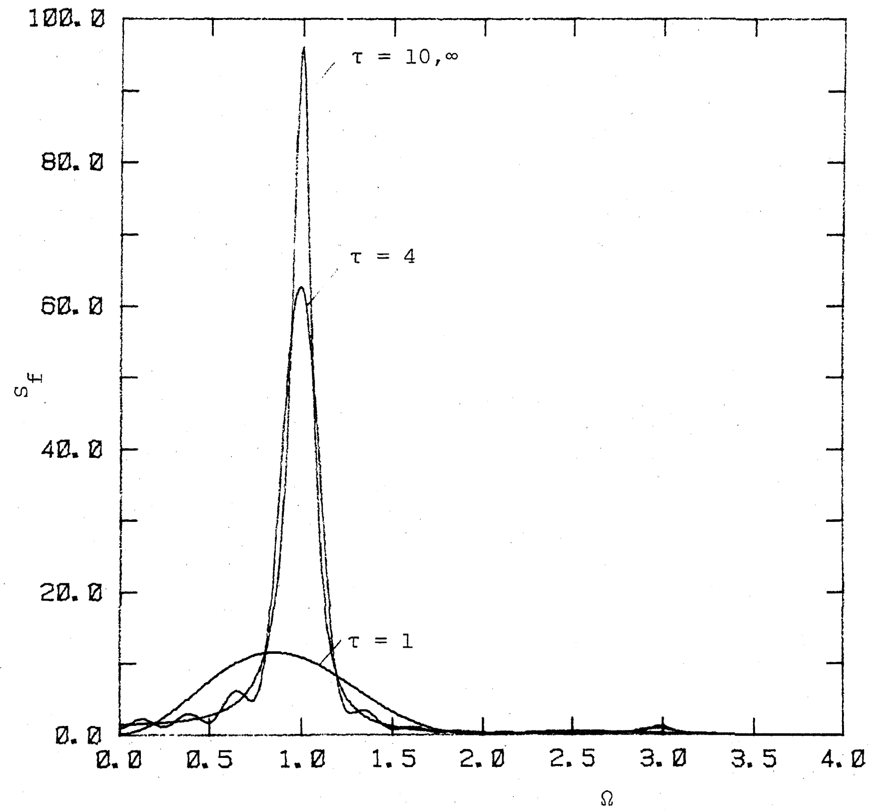

Figure 2. Nonstationary spectral densities for force S_f versus frequency Ω and time τ where $S_f = S_f(\tau,\Omega,0)/S_o\left(\frac{8F_o}{\pi^2 g}\right)^2$, $\Omega = \omega/\omega_1 = \omega/(\pi c/2H)$, $\tau = t/T_1 = t\omega_1/2\pi$; $\alpha = 0.815$, $z = 0$ (reservoir base).

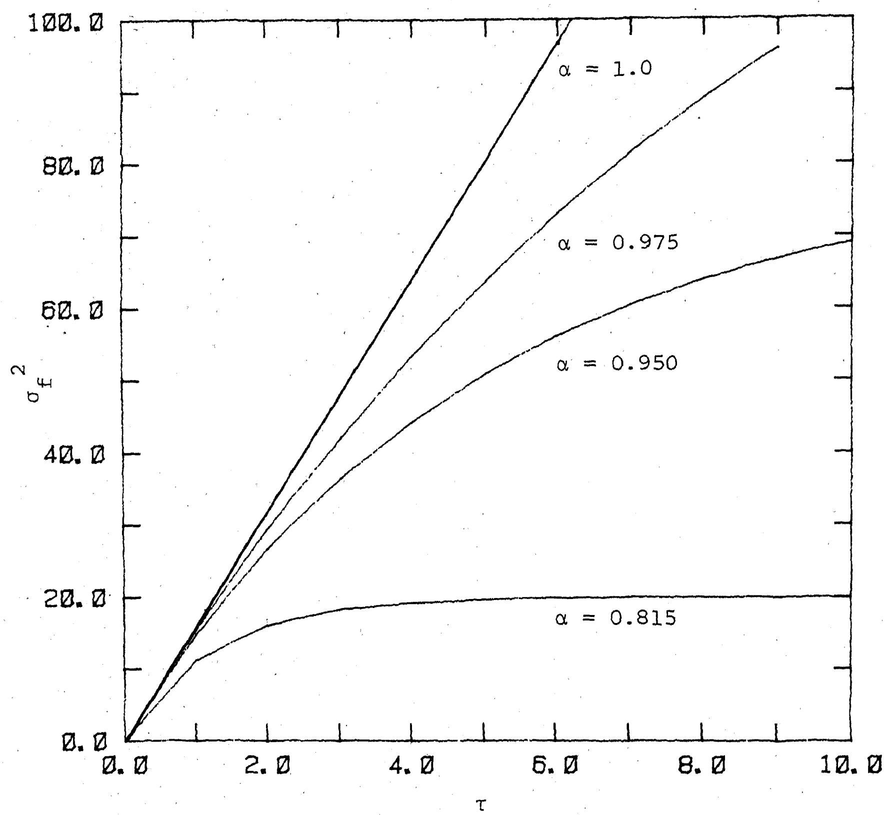

Figure 3. Nonstationary mean square response force σ_f^2 versus time τ and refraction coefficient α, where $\sigma_f^2 = \sigma_f^2(\tau,0)/2S_o\omega_1\left(\frac{8F_o}{\pi^2 g}\right)^2$, $\tau = t/T_1 = t\omega_1/2\pi$.