CONTROL OF COUPLED LATERAL-TORSIONAL MOTION OF BUILDINGS UNDER EARTHQUAKE EXCITATION

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SUMMARY

This paper presents a possible application of an active control system to tall buildings under strong earthquakes, taking into account coupled lateral-torsional motions of the building. The earthquake ground motion is modeled as a nonstationary random process. The time dependent statistics of building responses and required active control forces are analyzed. Monte Carlo simulations of response parameters with or without the active control systems are compared. It is shown that a significant reduction for the building response can be achieved by use of an active control system.

INTRODUCTION

The application of active control systems to structures subjected to dynamic loadings has received increasing attention recently (e.g., Refs. 1-5). An important source of information in structural control is given in Ref. 1. Most of the investigations to date, however, are based on greatly simplified structural and excitation models. For instance, in the analysis of actively controlled buildings under strong earthquakes, only the translational motions are considered and the lateral-torsional motions are neglected. The importance of coupled lateral-torsional motions of tall buildings under earthquake excitations was illustrated in Ref. 6.

The purpose of this paper is to investigate the effectiveness of an active mass-damper control system for controlling coupled lateral-torsional motions of tall buildings subjected to strong earthquakes. In this study two active controllers in the x and y directions, respectively, are connected to the mass damper as shown in Fig. 1. The controller considered herein is the electrohydraulic servomechanisms identical to that considered in Refs. 2-5. Sensors are installed only on the top floor and the active control forces are regulated by the measured motions of the top floor. In this arrangement, a direct control for the torsional motions is not provided, since the mass damper exhibits only lateral motions. However, due to the coupling effect between lateral and torsional motions, a reduction in the lateral motions will result in a reduction in torsional motions.

The earthquake ground acceleration is modeled as a nonstationary random

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process, and the random vibration analysis is carried out to determine the stochastic response of tall buildings implemented with an active mass damper system. The time dependent statistics of building responses and required active control forces are obtained. In addition, Monte Carlo simulations are conducted to illustrate the significant reduction of building responses under the active control system. A numerical example is given to demonstrate the effectiveness of such a control system.

FORMULATION

For simplicity, it will be assumed that (1) individual stories are identically constructed, (2) the mass of each story is concentrated at the floor level, (3) the floors are rigid, (4) linear elasticity and damping are provided by massless walls/columns between neighboring floors, and (5) the mass damper controller is regulated by the motion of the top floor.

The motion of the building model may be described by the translations, u(t) and v(t), of the elastic center E of each floor in the x and y directions, respectively, and the rotation $\theta(t)$ about E. Thus, the N-story building model is a 3N degrees of freedom dynamic system. The structural response may be described by these deflection variables and corresponding generalized forces, U(t), V(t), and Q(t), which are, respectively, the shear force in the x direction, the shear force in the y direction, and the torsional moment about E. It is convenient to group the three deflection variables at a given floor and the three generalized forces immediately above this floor, and treat them as components of a state vector. Denote the Fourier transform of such a vector by

$$\{Z\} = \{\bar{u}, \bar{v}, \bar{\theta}, \bar{Q}, \bar{V}, \bar{U}\}' \tag{1}$$

in which an upper-bar signifies the Fourier transform and the prime denotes the transpose of a vector or matrix. Each component in the vector {Z} is

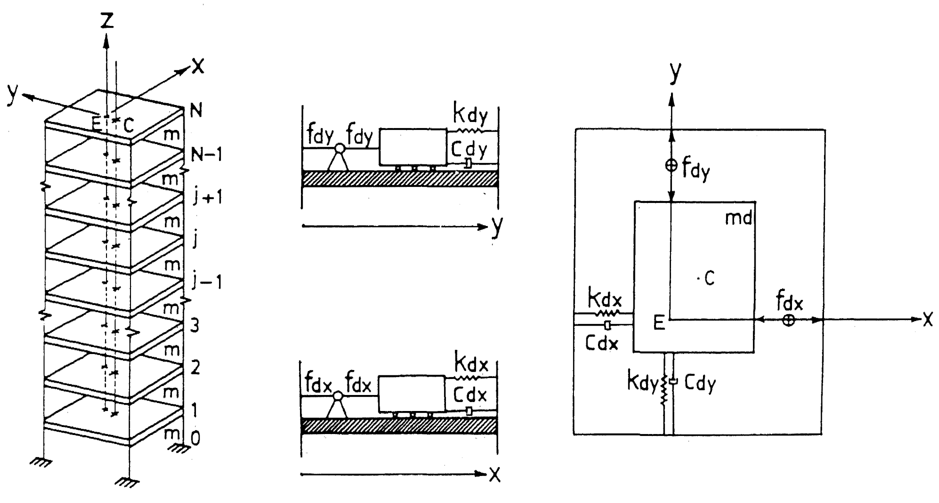

Fig. 1: Building Model and Arrangement of Active Mass Damper at Top Floor.

frequency dependent. The frequency variable ω, however, has been dropped because of simplicity of notation.

The state vector of the jth floor is related to that of the j-1th floor through (Ref. 6).

$$\{Z\}_j = [T]\{Z\}_{j-1} \tag{2}$$

in which the subscripts j and j-1 signify the respective floors, and [T] is a transfer matrix. Physically, [T] represents the propagation mechanism by which the state vector is transferred from one story unit to another. With the coordinate origin at E and the x and y directions parallel to the two sides of the building, as shown in Fig. 1, the [T] matrix is given in Ref. 6. The elements of [T] are functions of the frequency ω and the following structural properties of the story unit: m=floor mass; β=external damping; I_0=rotational inertia of a floor about E; k_x, k_y and k_θ = stiffnesses; c_x, c_y and c_θ = internal damping coefficients; x_c, y_c = eccentricities of the elastic center.

The state vector at the top floor $\{Z\}_N$ is related to the state vector at the ground $\{Z\}_0$ through

$$\{Z\}_N = [T]^N\{Z\}_0 + \{F\} \tag{3}$$

in which

$$\begin{aligned} \{Z\}_N &= \{\bar{u}_N, \bar{v}_N, \bar{\theta}_N, 0, \bar{V}_N, \bar{U}_N\}' \\ \{Z\}_0 &= \{\bar{u}_0, \bar{v}_0, \bar{\theta}_0, \bar{Q}_0, \bar{V}_0, \bar{U}_0\}' \\ \{F\} &= \{0, 0, 0, 0, \bar{f}_y(\omega), \bar{f}_x(\omega)\}' \end{aligned} \tag{4}$$

where $\bar{f}_x(\omega)$ and $\bar{f}_y(\omega)$ are the Fourier transforms of the control forces exerted on the mass damper, respectively, in the x and y directions. They are regulated by the motions of the top floor

$$\bar{f}_x(\omega) = g_{xr}(\omega;\, \varepsilon_x, \tau_x)\, \bar{u}_N \quad ; \quad \bar{f}_y(\omega) = g_{yr}(\omega;\, \varepsilon_y, \tau_y)\, \bar{v}_N \tag{5}$$

in which g_{xr} and g_{yr} are the gains of the controllers, which are functions of the loop gains, ε_x and ε_y, and the feedback gains, τ_x and τ_y (see Refs. 2-3). The subscript r in Eq. 5 indicates the type of sensor used to regulate the controller, with r=0 for displacement sensor, r=1 for velocity sensor, and r=2 for acceleration sensor. Note that for high-rise buildings, the computation of $[T]^N$ can be carried out easily using a previously developed procedure (Ref. 6).

The mass damper is connected to the top floor through springs and dashpots in the x and y directions such that the elastic center of the mass damper coincides with that of the top floor. There is no torsional stiffness between the mass damper and the top floor such that $\bar{Q}_N$=0 as shown in Eq. 4. Hence, the mass damper does not undergo rotational motions. The state vector of the mass damper is defined by

$$\{Z\}_{N+1} = \{\bar{u}_{N+1}, \bar{v}_{N+1}, 0, 0\}' \tag{6}$$

in which $u_{N+1}(t)$ and $v_{N+1}(t)$ are, respectively, the displacements of the mass damper in the x and y directions.

From the equations of motion and the force-displacement relation of the mass damper, it can be shown that

$$\{Z\}_{N+1} = [T_d]\{\tilde{Z}\}_N - \{\tilde{F}\} \tag{7}$$

in which $\{\tilde{Z}\}_N$ and $\{\tilde{F}\}$ are the reduced vectors of $\{Z\}_N$ and $\{F\}$, respectively,

$$\{\tilde{Z}\}_N = \{\bar{u}_N, \bar{v}_N, \bar{V}_N, \bar{U}_N\}' \quad ; \quad \{\tilde{F}\} = \{0, 0, \bar{f}_y(\omega), \bar{f}_x(\omega)\}' \tag{8}$$

and $[T_d]$ is the transfer matrix for the mass damper and the top floor,

$$[T_d] = \begin{bmatrix} 1 & 0 & 0 & (k_{dx} + i\omega c_{dx})^{-1} \\ 0 & 1 & (k_{dy} + i\omega c_{dy})^{-1} & 0 \\ 0 & -m_d\omega^2 & 1-m_d\omega^2(k_{dy} + i\omega c_{dy})^{-1} & 0 \\ -m_d\omega^2 & 0 & 0 & 1-m_d\omega^2(k_{dx} + i\omega c_{dx})^{-1} \end{bmatrix} \tag{9}$$

where m_d=mass of the mass damper, k_{dx} and k_{dy}=spring constants of the mass damper, and c_{dx} and c_{dy}=dampings of the mass damper.

Let the component of the ground displacement in the x direction be a dirac delta function and other components be zero, so that the Fourier transform $\bar{u}_0$ is unity, i.e.,

$$\bar{u}_0 = 1 \quad , \quad \bar{v}_0 = \bar{\theta}_0 = 0 \tag{10}$$

Then, all the state vectors $\{Z\}_n$ for n=1,2,.. are the frequency response vectors due to the ground displacement in the x direction.

With the application of Eqs. 5 and 10, matrix equations given by Eqs. 3 and 7 involve 10 equations with 10 unknowns, i.e., $\bar{u}_N$, $\bar{v}_N$, $\bar{\theta}_N$, $\bar{V}_N$, $\bar{U}_N$, $\bar{Q}_0$, $\bar{V}_0$, $\bar{U}_0$, u_{N+1}, and v_{N+1}. These ten unknowns, including the state vector $\{Z\}_0$, can be solved from ten equations given by Eqs. 3 and 7.

Knowing the state vector $\{Z\}_0$ at the ground level, any other state vector $\{Z\}_m$ at an arbitrary floor m can be obtained using an appropriate transfer matrix. Thus, the frequency response function of any quantity, including the required active control force, Eq. 5, can be obtained.

To simulate the ground motions, a model which allows specifications of variable intensity and frequency contents is a uniformly modulated random process

$$\ddot{u}_0(t) = \psi(t)\ddot{x}(t) \tag{11}$$

in which $\psi(t)$ is a deterministic envelope function and $\ddot{x}(t)$ is a stationary random process with zero mean. A number of envelope functions have been suggested in the literature. The particular one used herein is: $\psi(t)=0$ for $t<0$; $\psi(t)=(t/t_1)^2$ for $0\le t\le t_1$; $\psi(t) = 1$ for $t_1\le t\le t_2$; $\psi(t) = \exp[-c(t-t_2)]$ for $t>t_2$; where t_1, t_2 and c are parameters which should be chosen appropriately to reflect the shape and duration of the earthquake ground acceleration.

The frequency contents in earthquake ground excitations are described by the spectral density of the stationary random process $\ddot{x}(t)$ in Eq. 11. The following frequently used spectrum is employed,

$$\Phi_{\ddot{x}\ddot{x}}(\omega) = \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} S^2 \tag{12}$$

in which ω_g, ζ_g and S are parameters depending on geological conditions.

The frequency response vector $\{Z\}$ due to $u_0(t) = \exp(i\omega t)$ as obtained above can be converted into the impulse response vector due to $\ddot{u}_0(t) = \delta(t)$ through the Fourier transform

$$\{h_Z(t)\} = \frac{-1}{2\pi} \int_{-\infty}^{\infty} \omega^2\{Z\}d\omega \tag{13}$$

Since the ground acceleration has a zero mean, the mean value of the response vector is zero. The variance is given by (Ref. 7).

$$\{\sigma_Z{}^2(t)\} = \int_{-\infty}^{\infty} |\{M_Z(t;\omega)\}|^2 \; \Phi_{\ddot{x}\ddot{x}}(\omega)d\omega \tag{14}$$

in which

$$\{M_Z(t;\omega)\} = \int_0^t \{h_Z(\tau)\}\psi(t-\tau)e^{-i\omega t}d\tau \tag{15}$$

can be computed using the Fast Fourier Transform (FFT) technique.

In addition to evaluating nonstationary standard deviations of building responses using Eq. 14, the random response vectors have also been simulated using the frequency response vectors obtained above and the FFT technique. The simulation approach using the FFT technique has been described for instance in Ref. 7.

NUMERICAL EXAMPLE

An eight-story building with identical story units (15m by 24m) is considered for illustrative purpose. The properties of each story unit are: m=345.6 tons; I_0=2.607 X 10^4 KN/m; k_x=3.404 X 10^5 KN/m; k_y=4.502 X 10^5 KN/m; k_t=3.84 X 10^7 KN/rad; β=100 KN/m/sec; and c_x=c_y=c_t=0.0. The parameters of the earthquake model are: ω_g=18.85 rad/sec; ζ_g=0.65; S^2=4.65 X 10^{-4} $m^2/sec^3/rad$; t_1=3 sec; t_2=13 sec; and c=0.26. The properties of mass damper are: m_d=34.56 tons (10% of the floor mass); c_{dx}=56.4 tons/sec; c_{dy}=67.18 tons/sec; k_{dx}=1534.1 KN/m; k_{dy}=2029.0 KN/m. The loop gains and feedback gains for the controller are: ε_x=2.0; ε_y=1.0; τ_x=2.0; and τ_y=5.0. Acceleration sensors are used on the top floor.

Without the active mass damper, the nonstationary standard deviation of the top floor relative displacement in the x direction with respect to the ground is presented in Fig. 2 as Curve 1. The corresponding results are computed from Eq. 14 and presented as Curve 2 in Fig. 2 for the building with the active control system. A comparison between Curves 1 and 2 indicates a significant reduction of the building response in the x direction due to the active mass damper. Furthermore, the nonstationary standard deviation of the top

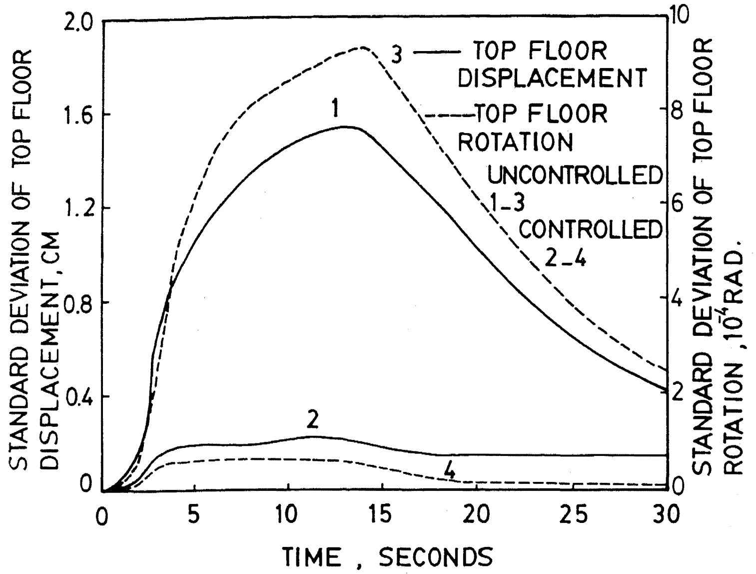

Fig. 2: Nonstationary Standard Deviation of Top Floor Relative Displacement and Rotation.

floor rotation with and without the active control system are shown in Fig. 2 as Curves 3 and 4, respectively. Although no control is provided by the active mass damper for the torsional motion, the reduction in the top floor rotation is significant because of the coupling effect. The maximum standard deviation of the required active control force from the controllers is computed to be 113 KN, that is within the practical range.

Sample functions of the relative top floor displacement in the x direction have been simulated for the building with and without the active control system. The results are presented in Fig. 3 for comparison. The simulated sample functions of the top floor rotation are displayed in Fig. 4. As expected, the active control system is capable of significantly reducing the building response quantities. Extensive numerical simulation results and nonstationary standard deviations of other response quantities have been obtained but not presented herein, because of the page limitation. All the results exhibit the same behavior as those presented in Fig. 2-4.

CONCLUSION

It has been demonstrated that the active mass damper system can be used effectively to reduce the building response under earthquake excitations. While no control is provided for the torsional motion, a substantial reduction in the rotation and torsional moment can be achieved through the coupling effect. The required active control forces are linearly proportional to the earthquake intensity S. For the example given herein, the earthquake intensity

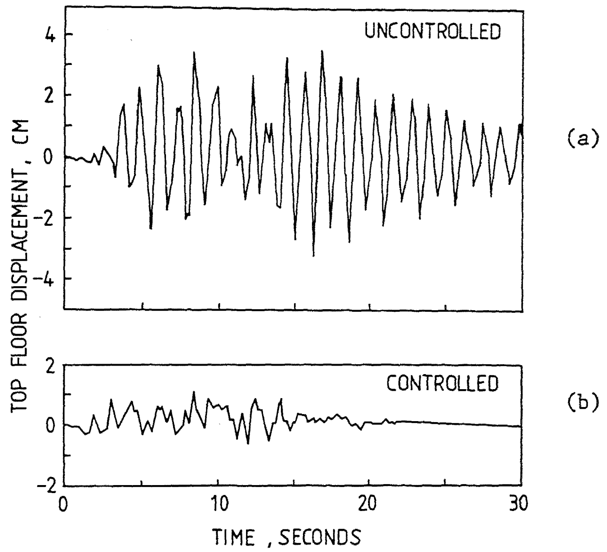

Fig. 3: Sample Function of Top Floor Relative Displacement; (a) Without Control, and (b) With Active Control System.

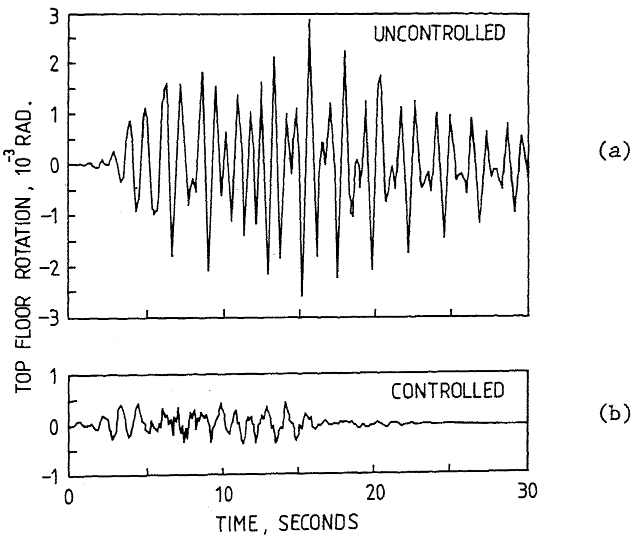

Fig. 4: Sample Function of Top Floor Rotation; (a) Without Control, and (b) With Active Control System.

corresponds to the one associated with the Housner's average response spectra, and the maximum required active control force is within the practical range.

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