A PROBABILISTIC STUDY OF THE DYNAMIC RESPONSE OF A STIFFNESS-DEGRADING SINGLE DEGREE OF FREEDOM STRUCTURE EXCITED BY A STRONG GROUND MOTION WITH SPECIFIC PROBABILISTIC CHARACTERISTICS

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SUMMARY

The question of "how stiffness degradation is influenced by the probability distribution of the input earthquake accelerogram peaks" is investigated in this paper. The earthquake response of a Single Degree Of Freedom (SDOF) model of a reinforced concrete structure is studied with the aid of NLSPEC, a nonlinear dynamic analysis computer program (Ref. 1) in which a modified Clough's model has been implemented. From two classes of accelerogram peak distributions, six records have been selected as the ground motion inputs. To interpret the stiffness-degrading behavior of the model a nonhomogeneous Markov model has been defined and probability-transition matrices are estimated using the response data. Finally, the results corresponding to the two earthquake classes are compared and conclusions are made.

INTRODUCTION

In this paper, the response of a stiffness-degrading SDOF reinforced concrete structure excited by two different types of earthquake time histories is studied. The accelerograms are classified primarily based on their fitted Exponential Half Tail (EHT) models (Ref. 2). To investigate the significance of this classification, other influential parameters, namely Richter magnitude, epicentral distance, and local site conditions, are held constant. The amplitudes of the accelerograms have also been scaled to make the results comparable within each class and between the classes.

The structural model is a stiffness-degrading SDOF oscillator with fixed yielding and dynamic characteristics. Taking the initial stiffness of the model as K_o and the reloading stiffness throughout a response time history as K_t, the ratio $r(t) = K_t/K_o$ has been used to quantify the level of stiffness degradation. Initial observations of the dynamic responses have indicated that for both classes of the earthquake inputs, r(t) is time-dependent. This phenomenon is basically due to the time-variant characteristics of the input ground motion and is modeled as a nonhomogeneous Markov chain.

The method presented in this study can be applied in development of a damage assessment procedure based on the stiffness degradation models and probabilistic ground motion characteristics.

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PROBABILISTIC CHARACTERISTICS OF GROUND MOTION

De Herrera and Zsutty (Ref. 2), in their investigation of defining a ground motion characteristic which specifies load cycles in terms of number and value distribution of peak ground accelerations, proposed the EHT model which is briefly described below.

The peaks, defined as the absolute value of the maximum between two zero crossings in a time history, are ordered in decreasing order $X_1, X_2, \ldots, X_r$. The X_i are random variables with unknown probability density function (pdf). Assuming statistical regularity, the model predicts what would be the mean and standard deviation of the i^{th} largest peak if several similar events in terms of magnitude and epicentral distance occurred.

An exponential pdf on the X_i or the transformed variable $Y_i = X_i^n$ [n = 1/2, Weibull; n = 2, Rayleigh] is shown to be appropriate for the 400 strong ground motions studied.

We recall that an exponential pdf is defined as

$$P_x = \lambda\, e^{-\lambda x}$$

where λ is the parameter of the pdf.

In order to identify the effects of the type of the accelerogram peaks pdf on the nonlinear response of a SDOF system, the two extremes of the probabilistic description of peaks are considered (Rayleigh and Weibull). The acceleration records used in this study are shown in Table 1 and classified as Rayleigh (RY) or Weibull (WE). The acceleration record No. 1 has been taken as the reference record and the other five records have been selected and scaled such that the scaled PGA of all the records are almost the same and also that large variations among the scaled pdf parameters of each class are prevented.

STIFFNESS DEGRADATION MODEL

The main purpose of this paper is to study several earthquake responses of a reinforced concrete SDOF model exhibiting a degrading stiffness property. This property is assumed to be associated with the crack-closing phenomenon (Ref. 3). The dynamic characteristics as well as the load-deflection curve of the selected structural model are shown in Fig. 1. The implemented stiffness degradation model is a modified version of Clough's model as discussed in Ref. 4.

MARKOV MODEL

State-space vectors as well as the probability-transition matrices are estimated using the actual response time histories data. Ten states have been defined based on equal intervals of r(t) between 0 and 1. For example, state "1" is defined as r(t) being between 0.9 and 1.0 inclusive, and for state "10" between 0.0 and 0.1 inclusive.

A step is defined as transition from one state to another, including the possibility of returning to the initial state. In this study, a "step" can be interpreted as transition between any two consecutive zero force positions on the response hysteresis diagram (Fig. 1). In other words, on this diagram each cycle corresponds to two successive steps.

From each response time history, the total number of state transitions (N) as well as the highest level of states visited (S_m) have been observed. Then each response time history is subdivided into six stages (not necessarily equivalent in time) to account for the nonhomogeneity of the model. An S_m by S_m probability-transition matrix has been estimated at the end of each and every stage. The number of stages is taken as six since for all the cases N/6 is much larger than S_m. The estimates of the elements of the k^{th} stage probability-transition matrix P_k, $(P_{ij})_k$, are calculated as

$$(P_{ij})_k = \frac{(n_{ij})_k + 1}{S_m + (n_{i.})_k} \qquad (1)$$

where $(n_{ij})_k$ = number of observed transitions from state i to state j at the stage k,

$(n_{i.})_k$ = number of observed transitions initiated from state i.

The state-space vectors at the end of each stage can be found as follows. Given the initial state-space vector, q_o, an S_m dimensional vector {1,0,0,...,0}, the state-space vector at the end of the first stage, q_1, is the transpose of the first row of P_1. Similarly, q_2 is the transpose of the first row of the product $P_1 \times P_2$. q_3 up to q_6 are the transpose of the first row of the products $P_1 \times P_2 \times P_3$, etc. A numerical example of this process being applied to one of the six response time histories is presented in the next section.

A CASE STUDY

Table 2 contains the six probability-transition matrices of the six-stage response time history calculated using earthquake record No. 1 as the input. The numbers written in parentheses are $(n_{ij})_k$ and their sum for each row is $(n_{i.})_k$. Products of the matrices are not shown; however, the six-stage state-space vectors are given in Table 3. For this case the observed values of N and S_m are 218 and 6, respectively.

DISCUSSION

Referring to Table 4, the final results indicate that the highest level of stiffness degradation in Rayleigh-type cases is on the average 20% higher than that of the cases with Weibull-type inputs. This difference may be due to the fact that the accelerograms of these two types of pdf have identical PGA but different levels of "average peaks." For the Weibull distribution used in Ref. 3, the expected value of the upper half tail peaks (greater than the median) is related to the parameter λ by $(A_{avg})_W = 2/\lambda$ and a similar expression for the Rayleigh distribution is $(A_{avg})_R = \sqrt{\pi/2}\,(1/\lambda) \simeq 1.253/\lambda$. Using the values of λ given in Table 1, we find the following average peak values:

$$(A_{avg})_W \simeq 6.1 \qquad (A_{avg})_R \simeq 67$$

As can be seen, the average peak value of the Rayleigh-type accelerograms are set to a level more than ten times larger than the average peak value of the Weibull-type.

Another noticeable difference between the two classes of the results can be seen by comparing the final state-space vectors and treating them as pdf with states being taken as the random variable X. The basic parameters of these discrete pdf, such as the mean ($\bar{x}$), the standard deviation (S), and the coefficient of skewness (g), are shown in Fig. 2. It can be seen that on the average the final state-space vectors with the Rayleigh-type accelerograms as inputs have higher mean and standard deviation values and are more skewed towards the higher states compared with those corresponding to the Weibull-type inputs. This comparison indicates that on the average the level of stiffness degradation has been higher for the Rayleigh than for the Weibull cases, which is consistent with the previous observation. Also, it can be stated that at the end of the response time histories, the likelihood of the structural model being at a high-level state is higher for the cases with Rayleigh-type inputs compared with Weibull ones.

CONCLUSION

It has been observed that for the specific structural model being used in this study, the highest level of stiffness degradation as well as the likelihood of the model being at different degradation states is influenced by the type of the pdf of the input accelerogram peaks. "Average peak" value of the input accelerograms as opposed to the PGA seems to be better related to the maximum level of stiffness degradation in the earthquake response of a typical SDOF model. The method presented in this paper can be directly applied in development of a damage assessment model by assigning appropriate loss factors to the different levels of stiffness degradation.

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Table 1. Earthquake Data

No.	Description	M	R (km)	S	PGA cm^2/sec^2 Orig.	PGA cm^2/sec^2 Scaled	C	$1/\lambda$ Orig.	$1/\lambda$ Scaled	Earthquake File
1	IIA019 COMP S00W El Centro E.Q. 1968 El Centro Site Imperial Valley Irrigation District	6.4	45	SS	127.8	127.8	WE	3.1	3.1	CIT 035
2	IIF098 COMP S53E San Fernando E.Q. 1971 846 South Olive Ave., Basement, L.A.	6.4	42	SS	236.4	132.4	WE	5.6	3.1	CIT 122
3	IIU300 COMP N45W N. Cal. E.Q. 1941 City Hall, Ferndale	6.4	50	SS	118.6	127.8	WE	2.9	3.1	CIT 294
4	IIB024 COMP S00W Lower California E.Q. 1934 El Centro Imperial Valley	6.5	64	SS	156.8	127.8	RY	66.1	53.9	CIT 043
5	IIH124 COMP S90W San Fernando E.Q. 1971 2600 Nutwood Ave., Basement Fullerton	6.4	58	SS	34.5	127.8	RY	15.3	56.7	CIT 153
6	IIT286 COMP EAST Borrego Valley E.Q. 1942 El Centro, Imperial Valley Irrigation District	6.5	48	SS	46.5	127.8	RY	18.1	49.8	CIT 273

Abbreviations: M, magnitude; R, epicentral distance; S, local site conditions; C, distribution class; SS, soft soil.

Table 2. Six-Stage Probability-Transition Matrices with Record No. 1 as the Input

	0.829 (28)	0.057 (1)	0.029 (0)	0.029 (0)	0.029 (0)	0.029 (0)
	0.143 (0)	0.143 (0)	0.143 (0)	0.143 (0)	0.285 (1)	0.143 (0)
P_1 =	0.143 (0)	0.143 (0)	0.143 (0)	0.285 (1)	0.143 (0)	0.143 (0)
k = 1	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)
	0.111 (0)	0.111 (0)	0.111 (0)	0.333 (2)	0.223 (1)	0.111 (0)
	0.125 (0)	0.125 (0)	0.25 (1)	0.125 (0)	0.25 (1)	0.125 (0)
	0.334 (2)	0.111 (0)	0.111 (0)	0.111 (0)	0.222 (1)	0.111 (0)
	0.125 (0)	0.125 (0)	0.250 (1)	0.125 (0)	0.250 (1)	0.125 (0)
P_2 =	0.083 (0)	0.167 (1)	0.083 (0)	0.251 (2)	0.333 (3)	0.083 (0)
k = 2	0.105 (1)	0.105 (1)	0.263 (4)	0.316 (5)	0.158 (2)	0.053 (0)
	0.056 (0)	0.056 (0)	0.111 (1)	0.389 (6)	0.332 (5)	0.056 (0)
	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)
	0.125 (0)	0.250 (1)	0.125 (0)	0.125 (0)	0.250 (1)	0.125 (0)
	0.083 (0)	0.083 (0)	0.167 (1)	0.083 (0)	0.502 (5)	0.083 (0)
P_3 =	0.077 (0)	0.231 (2)	0.231 (2)	0.231 (2)	0.154 (1)	0.077 (0)
k = 3	0.083 (0)	0.167 (1)	0.167 (1)	0.250 (2)	0.250 (2)	0.083 (0)
	0.136 (2)	0.136 (2)	0.182 (3)	0.136 (2)	0.318 (6)	0.045 (0)
	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)
	0.143 (0)	0.143 (0)	0.143 (0)	0.143 (0)	0.286 (1)	0.143 (0)
	0.059 (0)	0.235 (3)	0.176 (2)	0.235 (3)	0.235 (3)	0.059 (0)
P_4 =	0.083 (0)	0.167 (1)	0.167 (1)	0.333 (3)	0.167 (1)	0.083 (0)
k = 4	0.125 (1)	0.313 (4)	0.188 (2)	0.063 (0)	0.250 (3)	0.063 (0)
	0.071 (0)	0.286 (3)	0.143 (1)	0.357 (4)	0.071 (0)	0.071 (0)
	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)
	0.182 (1)	0.182 (1)	0.091 (0)	0.182 (1)	0.273 (2)	0.091 (0)
	0.067 (0)	0.133 (1)	0.267 (3)	0.267 (3)	0.200 (2)	0.067 (0)
P_5 =	0.091 (0)	0.455 (4)	0.091 (0)	0.182 (1)	0.091 (0)	0.091 (0)
k = 5	0.250 (3)	0.250 (3)	0.125 (1)	0.188 (2)	0.125 (1)	0.062 (0)
	0.154 (1)	0.077 (0)	0.231 (2)	0.308 (3)	0.154 (1)	0.077 (0)
	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)
	0.077 (0)	0.154 (1)	0.154 (1)	0.308 (3)	0.231 (2)	0.077 (1)
	0.091 (0)	0.091 (0)	0.182 (1)	0.455 (4)	0.091 (0)	0.091 (0)
P_6 =	0.300 (2)	0.100 (0)	0.100 (0)	0.200 (1)	0.200 (1)	0.100 (0)
k = 6	0.190 (3)	0.143 (2)	0.095 (1)	0.333 (6)	0.190 (3)	0.048 (0)
	0.250 (2)	0.250 (2)	0.083 (0)	0.250 (2)	0.083 (0)	0.083 (0)
	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)	0.167 (0)

Table 3

n	State-Space Vectors
0	{1, 0, 0, 0, 0, 0}
1	{0.829, 0.057, 0.029, 0.029, 0.029, 0.029}
2	{0.296, 0.113, 0.124, 0.132, 0.227, 0.109}
3	{0.116, 0.183, 0.166, 0.157, 0.273, 0.095}
4	{0.096, 0.231, 0.161, 0.238, 0.179, 0.086}
5	{0.149, 0.209, 0.170, 0.223, 0.159, 0.082}
6	{0.177, 0.144, 0.126, 0.303, 0.157, 0.085}

Table 4

Record No.	Final State-Space Vector	Class
1	{0.177, 0.144, 0.126, 0.303, 0.157, 0.085}	WE
2	{0.187, 0.193, 0.250, 0.282, 0.087}	WE
3	{0.125, 0.157, 0.195, 0.23, 0.209, 0.084}	WE
4	{0.121, 0.097, 0.141, 0.092, 0.194, 0.279, 0.077}	RY
5	{0.1, 0.126, 0.114, 0.114, 0.113, 0.126, 0.18, 0.125}	RY
6	{0.129, 0.147, 0.126, 0.117, 0.154, 0.165, 0.164}	RY

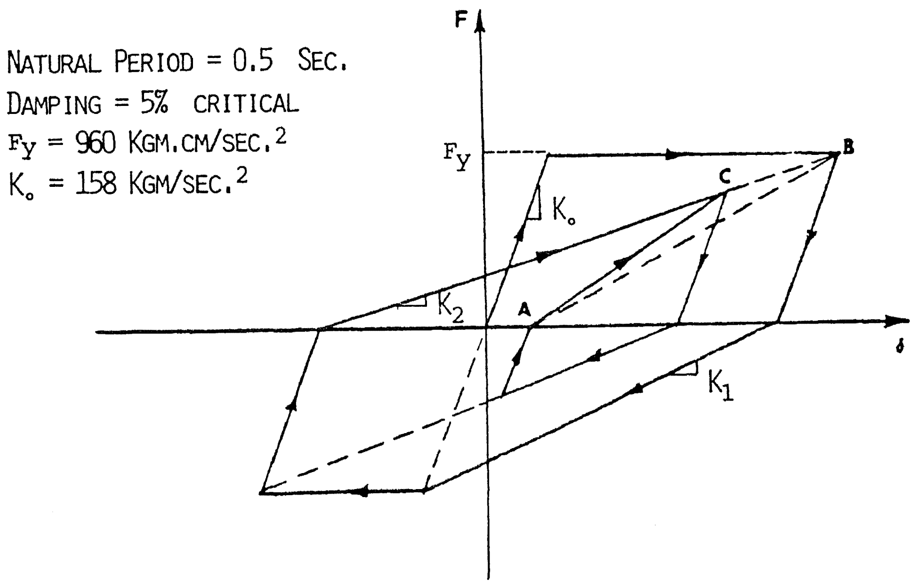
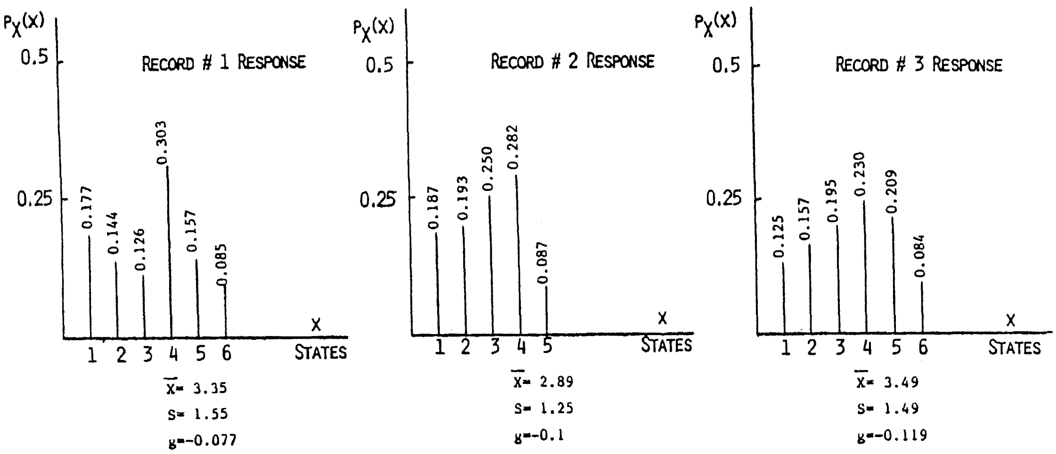
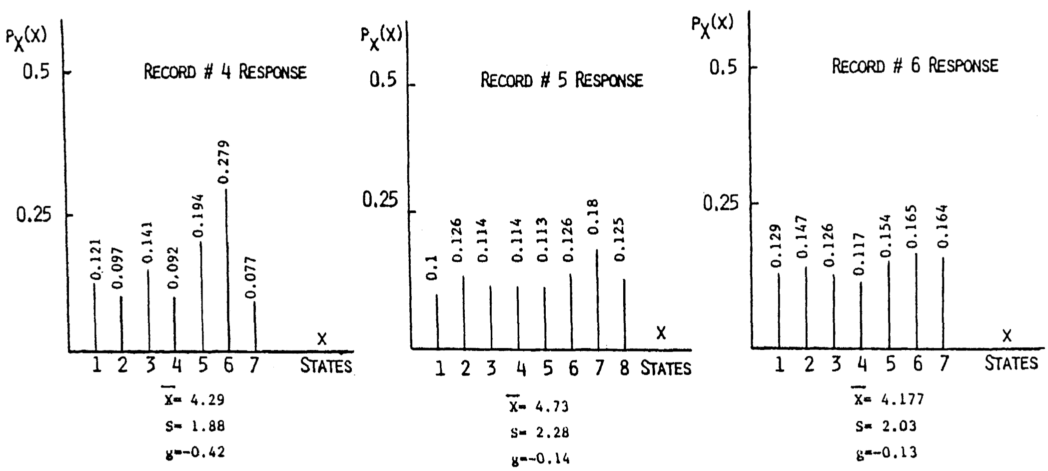


Figure 1. Load-deflection curve (Ref. 4) and parameters of the stiffness degradation model.



Weibull Type Input



Rayleigh Type Input

Figure 2. Final state-space vectors for two classes of earthquake inputs.