

PREDICTION OF DAMAGE RATIO OF REINFORCED CONCRETE BUILDINGS
DUE TO EARTHQUAKE AND COMPARISON WITH ACTUAL DAMAGE RATIO

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SUMMARY

This paper presents a method for predicting damage ratio of reinforced concrete buildings by applying reliability theory. Predicted values related to damage level are obtainable from three parameters; the expected maximum ground acceleration, the distribution ratio of buildings classified by the number of floors and the ductility factor. Load and resistance variables are the acceleration magnification factor and the ultimate shear strength coefficient, respectively. Each probability distribution is modelled as log-normal through statistical research. Good agreements are obtained with the actual examples of three districts in Sendai-city damaged by the 1978 Miyagi-ken-oki Earthquake.

INTRODUCTION

With the distribution function F_r of resistance variable of building R and the F_s of load variable S , the damage ratio is expressed as $P_f = P(R - S \leq 0)$. To represent the effect of plastic deformation and damage level, it is suitable to introduce the index of failure criterion α or α' . Then, $P_f = P(R - \alpha S \leq 0)$ or $P_f = P(\alpha' R - S \leq 0)$. The fundamental concepts of reliability theory are so simple as mentioned above, but there are many tedious tasks to apply for practical use, i.e. the selection of random variables for load and resistance, the statistical research on these variables, the determination of their probability distribution, the estimation of index α and α' , the interpretation of its mechanical meanings and so on.

For these purpose, Shiga et al. made clear that the ultimate shear strength coefficient C_{ys} defined by column and wall area is adequate for resistance variable to low-rised reinforced concrete buildings, and they also indicated the C_{ys} distribution of actual buildings (Ref. 1). Shibata regarded the C_{ys} distribution as log-normal, and proposed a consistent prediction method including the estimation of regional seismic risk (Ref. 2).

The purpose of this study is to predict the damage ratio connected with the damage level, for instance, the damage ratio of slightly damaged, the damage ratio of heavily damaged and so on, by comparison with the actual damage ratio classified by the damage level. Together with this, it is performed that the verification of the whole procedure of the probability method and is proposed a simplified predicting method of damage ratio for reinforced concrete buildings.

STATISTICAL RESEARCH AND MODELLING OF RESISTANCE VARIABLE

The ultimate shear strength coefficient C_{ys} defined by Shiga is adopted as resistance variable. Coefficient 12 and 33 describe the ultimate shear

$$C_{ys} = \frac{12Ac + 33Aw}{1300\Sigma Af} \quad (1)$$

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where, A_c = the column area of the first story (cm^2)
 A_w = the wall area of the first story (cm^2), smaller value in span and longitudinal directions, non-structural elements involved.
 ΣA_f = total floor area of the second floor and above (m^2)

strength (kg/cm^2) of column and wall, respectively. Coefficient 1300 is unit floor weight (kg/m^2).

The research was performed to 405 actual buildings (38% office or commercial, 28% public, 22% school buildings and 12% others). In Japan, the buildings over seven-story in height should usually have the steel framed reinforced concrete construction, these buildings are not involved. Also, special type reinforced concrete buildings such as wall or paneled structure are not included. Most of buildings were constructed between 1965 to 1970.

The frequency distribution of Cys classified on the number of floors are shown in Fig. 1 with modelled log-normal curve. The mean and the variation coefficient at the number of floors are shown in Fig. 2. It is observed two distinct features. The one is that there are skewness in Cys distributions. The other is that variation coefficients are almost same as 0.45. Therefore, it could be considered that Cys is such a random variable that the mean changes with the number of floors, and the variation is equivalent. It is reasonable that the probability density function of Cys is modelled as log-normal by inspection.

$$f_R(r) = \frac{1}{\sqrt{2\pi} \zeta_r} e^{-\frac{1}{2} \left(\frac{\ln r - \lambda r}{\zeta_r} \right)^2} \quad (2)$$

where, r = Cys, $\lambda r = \ln m_r - \frac{1}{2} \zeta_r$, $\zeta_r = \ln(1 + v_r^2)$
 m_r (mean) = $4.9899 \left(\frac{1}{N^2} \right) - 0.9645 \left(\frac{1}{N} \right) + 0.9768$
 N = the number of floors (2~6)
 v_r (variation coefficient) = 0.45 (constant)

STATISTICAL RESEARCH AND MODELLING OF LOAD VARIABLE

The linear acceleration response spectrum could be interpreted as the simplest expression of seismic force, and have been obtained from a large number of strong-motion-accelerometer records. The acceleration magnification factor is adopted as a load variable by this reason. Then, the variation of seismic force is described as that of the acceleration magnification factor at a certain natural period. The research is performed to 206 earthquake records which are almost all obtained in Japan, and have the 30 gal or more maximum ground acceleration.

Fig. 3 shows the frequency distribution of the acceleration magnification factor from 0.1 to 1.0 sec natural period in 0.1 sec increment at damping ratio 5%. Fig. 4 shows the mean and the variation coefficient at each natural period. It is presumed that the natural period of low-rised reinforced concrete buildings is less than 0.4 sec or so. In this range, the acceleration magnification factor distribution is similar to the Cys distribution in respect to skewness, constancy of variation coefficient and compatibility with log-normal distribution. The mean changes with natural period in the range

from 1.8 to 2.3, and it is acceptable to regard as constant 2.0 for simplification. Then the probability density function of S is modelled as follows.

$$f_S(s) = \frac{1}{\sqrt{2\pi} \zeta_S} e^{-\frac{1}{2} \left(\frac{\ln s - \lambda_S}{\zeta_S} \right)^2} \quad (3)$$

where, s = acceleration response, $\lambda_S = \ln m_S - \frac{1}{2} v_S^2$, $\zeta_S = \ln(1 + v_S^2)$
 m_S = the mean of acceleration response. (acceleration magnification factor = 2.0, $m_S = 2.0 \times$ maximum ground acceleration)
 v_S (variation coefficient) = 0.40 (constant)

EXPRESSION OF PLASTIC DEFORMATION AND DAMAGE LEVEL

It is necessary for the failure criterion to correspond with definite image of damage level, i.e. slightly damaged, heavily damaged and so on. It is also necessary to express the effect of plastic deformation of the ductile behavior of the building.

For this purpose, it would be reasonable to connect the concept of ductility factor to the failure criterion, because the grade of plastic deformation would be comparable with damage level. The index of failure criterion is defined as shown in Fig. 5 through energy conservation rule.

$$\begin{aligned} \mu > 1 & \quad \alpha = \sqrt{2\mu - 1} & \quad \alpha' = 1/\sqrt{2\mu - 1} \\ \mu < 1 & \quad \alpha = 1/\mu & \quad \alpha' = \mu \end{aligned} \quad (4)$$

It is advantageous in this definition that the energy absorption effect in plastic range is expressed in terms of force.

STANDARD DAMAGE RATIO AND SIMPLIFIED PREDICTION METHOD

The probability of failure P_f is expressed as follows under the condition that R and S are statistically independent.

$$P_f = \int_0^\infty f_S(s) \int_0^{\alpha s} f_R(r) dr ds = \int_0^\infty f_R(r) \int_{\alpha' r}^\infty f_S(s) ds dr \quad (5)$$

The contents of Eq. 5 are shown in Fig. 6. In this study, the probability density function $f_S(s)$ and $f_R(r)$ are both modelled as log-normal, Eq. 5 is replaced by standard normal (Gaussian) probability.

$$\begin{aligned} P_f &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\omega} e^{-\frac{1}{2} t^2} dt, \quad \omega = \frac{\ln \alpha - \lambda_Z}{\zeta_Z}, \quad \alpha = \frac{1}{\sqrt{2\mu - 1}} \\ \lambda_Z &= \ln \left(\frac{m_R}{m_S} \right) - \frac{1}{2} \ln \frac{1 + v_R^2}{1 + v_S^2}, \quad \zeta_Z = \sqrt{\ln(1 + v_R^2)(1 + v_S^2)} \end{aligned} \quad (6)$$

By letting $v_r = 0.45$ and $v_s = 0.40$, and substituting m_r and m_s , which are given by Eqs. 2 and 3, to Eq. 6, it is obtainable the damage ratio classified by the number of floors and the maximum ground acceleration. Tab. 1 and Fig. 7 show the calculated damage ratio in respect to ductility factor from 0.75 to 2.0 in 0.25 increment and ground acceleration from 100 to 500 gal in 50 gal increment.

The individual value means the damage ratio of such an idealized district that all reinforced concrete buildings have the same number of floors. These values could be said as the standard damage ratio Pf_0 . Therefore, the damage ratio of an actual district is easily determinable with the knowledge of the distribution ratio of buildings classified by the number of floors h .

$$Pf = \sum Pf_0 \cdot h \quad (7)$$

When a district is large and divided into several small districts with the difference in ground acceleration, its damage ratio is determined from the damage ratio of individual district given by Eq. 7 and the another distribution ratio defined between individual districts and whole district H .

$$Pf = \sum (\sum Pf_0 \cdot h) H \quad (8)$$

RELATION BETWEEN DUCTILITY FACTOR AND DAMAGE LEVEL THROUGH COMPARISON WITH ACTUAL DAMAGE RATIO

Tab. 2 shows the results of damage survey of reinforced concrete buildings in three districts, Oroshi-machi, Naga-machi and Kamisugi of Sendai-city in the 1978 Miyagi-ken-oki Earthquake. The damage level is graded to five ranks, i.e. 0-nodamaged, I-slightly damaged, II-damaged, III-heavily damaged and IV-collapsed. Values shown in Tab. 2 are the accumulated real number of buildings and damage ratios according to the damage rank and the number of floors.

It is recognized a close connection between the number sets of Tab. 1 and Tab. 2. For example, the number sets in Tab. 2 (I and above-47.73%, II and above-22.73%, III and above-11.36%, IV-6.67%) in the case of three-story buildings in Oroshi-machi lies between those of Tab. 1 ($\mu=0.75$, 33.77~42.57%, $\mu=1.00$, 17.94~24.64%, $\mu=1.25$, 17.94~24.64%, $\mu=1.75$, 4.35~6.94%), which are found in the case of ground acceleration of 350 and 400 gal. Similarly, number sets of two-story building in Oroshi-machi, three- and four-story buildings in Naga-machi correspond to those of 300-350 gal, 300 gal and 300-350 gal, respectively. The relation between ductility factor and the damage level is deduced as follows.

	I and above	II and above	III and above	IV
ductility factor μ	0.75	1.00	1.25	1.75

COMPARISON WITH PREDICTED VALUE TO ACTUAL DAMAGE RATIO

Tab. 3 shows the predicted damage ratio of previous three districts based on Eq. 7 and the above table. The actual damage ratio is also given the number sets in the row titled "real".

It is presumed that the maximum ground acceleration in Oroshi-machi was 300-350 gal, and Naga-machi 300 gal. The predicted value of Kamisugi is not well fitted to the actual value, but taking into a consideration of "II and above" value, it is assumed that the maximum ground acceleration was less than 200 gal. These results coincide well with the presumed values by various investigations, i.e. 320 gal for Oroshi-machi and Naga-machi and 180 gal Kamisugi (Ref. 3).

CONCLUSION

It is shown that the probabilistic method could be considerably effective in predicting the damage ratio of a group of buildings. But many problems should be discussed. For example the sufficient comparison has not been performed for five- and six-story buildings, because these middle high-rised buildings are few in Oroshi-machi and Naga-machi and many in Kamisugi, but they had not been damaged. Further investigations are required.

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References

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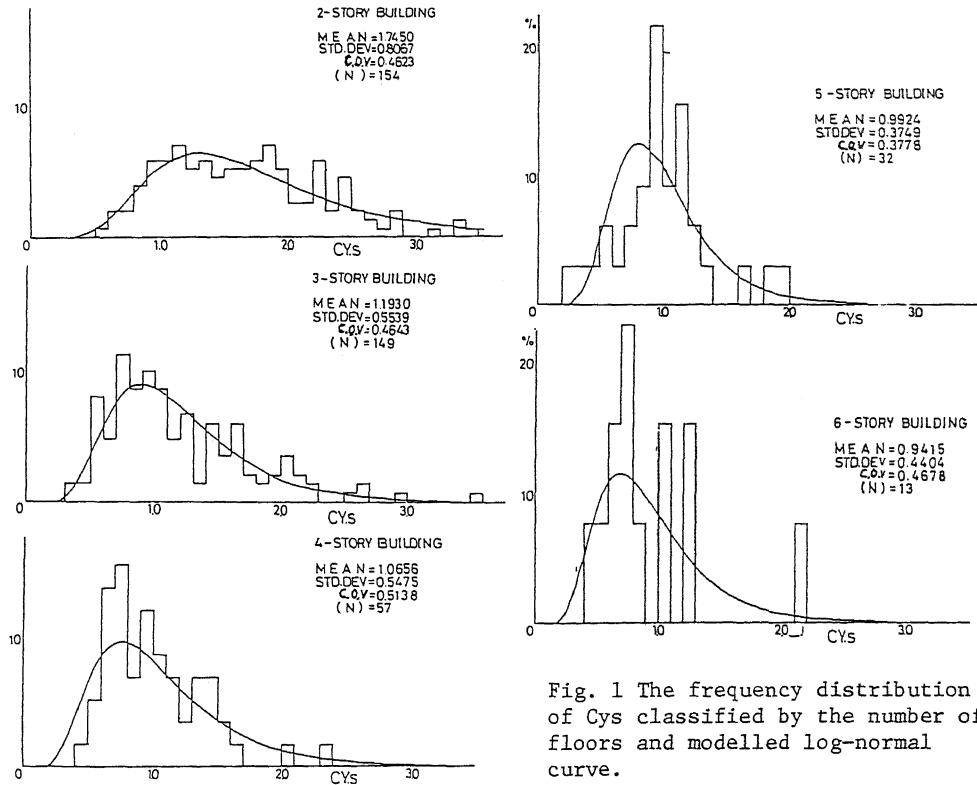


Fig. 1 The frequency distribution of Cys classified by the number of floors and modelled log-normal curve.

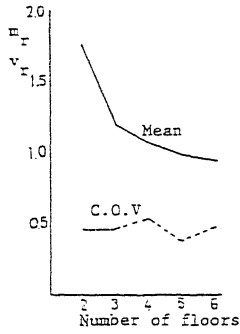


Fig. 2 Mean and variation coefficient of Cys vs. the number of floors

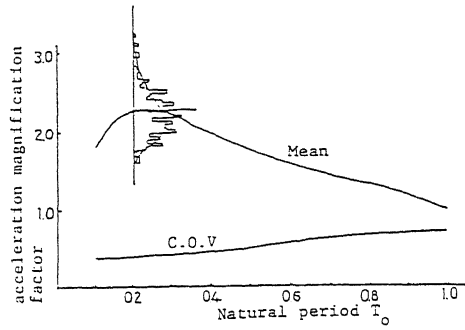


Fig. 4 Mean and variation coefficient of acc. mag. factor

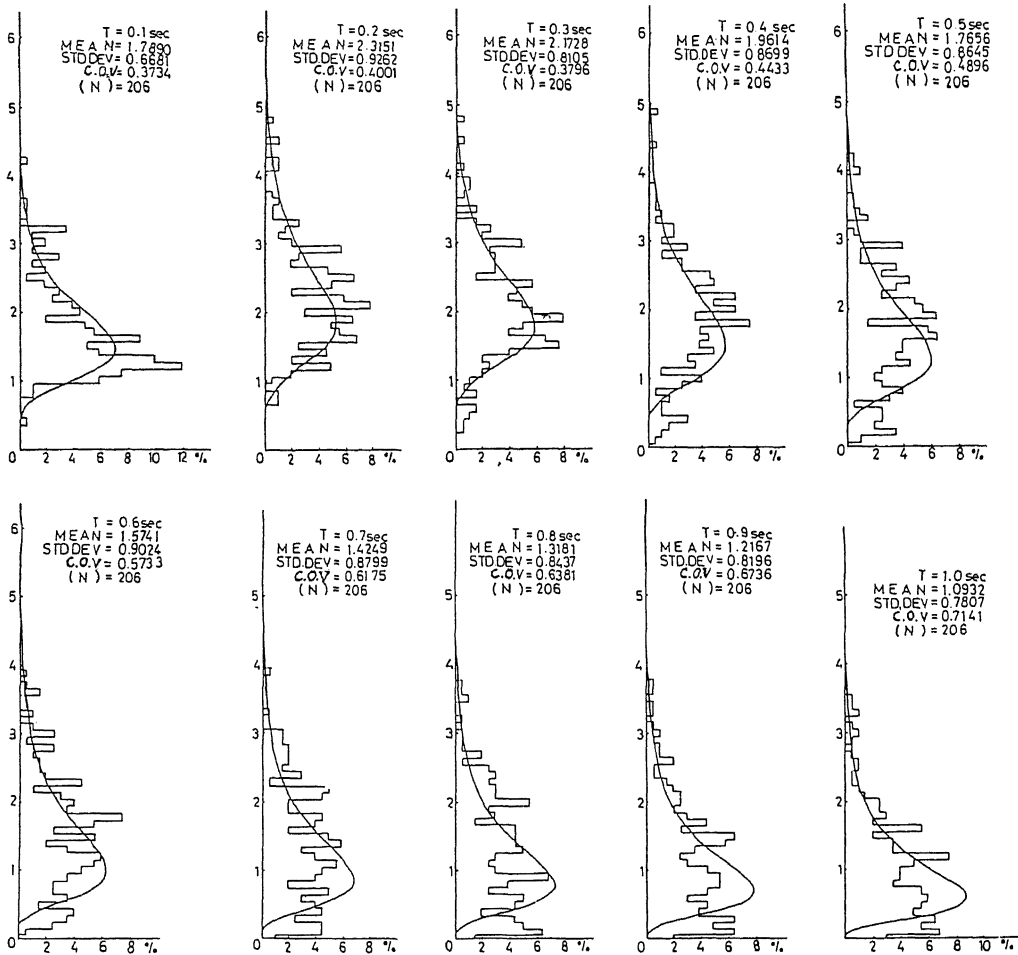


Fig. 3 Frequency distribution of acceleration magnification factor at each natural period and modelled log-normal curve

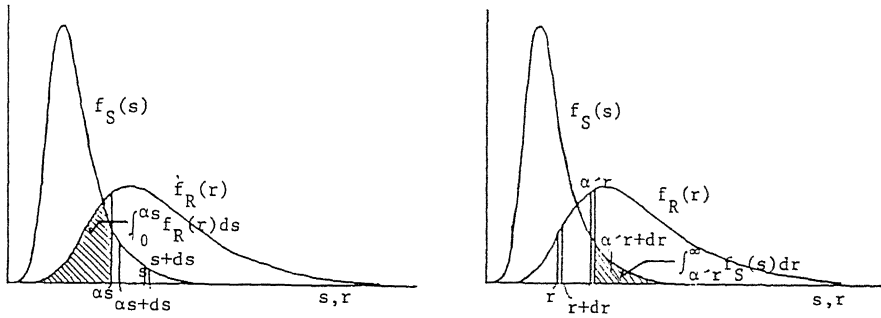


Fig. 5 Schematic representation of failure probability

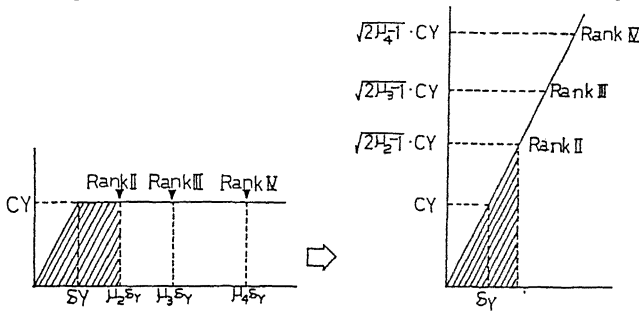


Fig. 6 Schematic illustration of the index of failure criterion by energy conservation rule

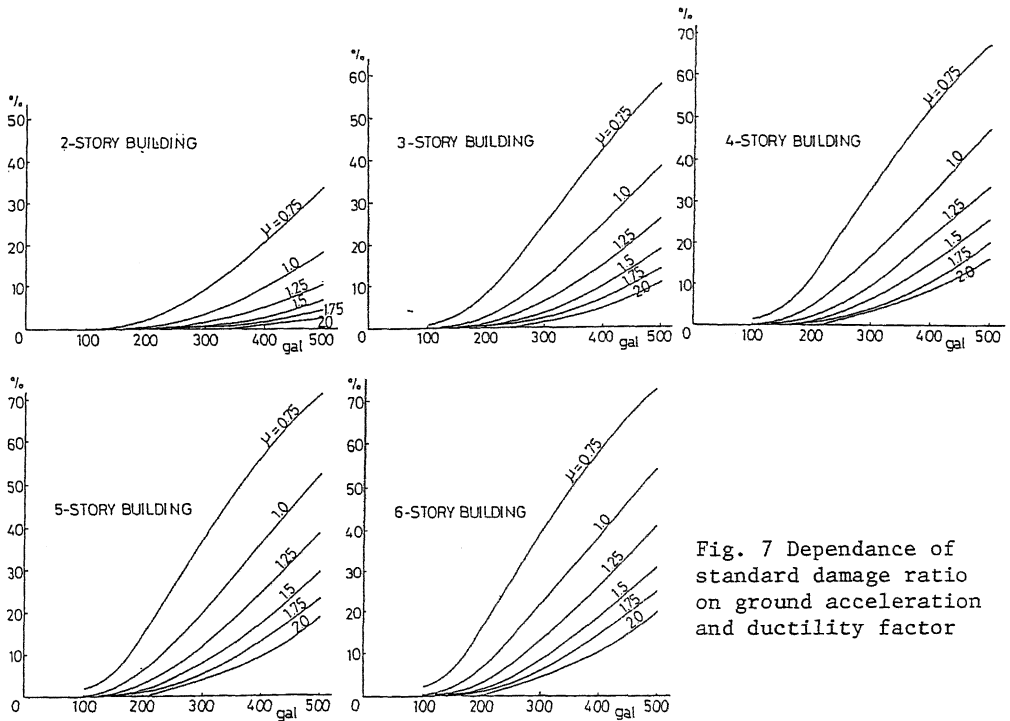


Fig. 7 Dependence of standard damage ratio on ground acceleration and ductility factor

Tab. 1 Table of standard damage ratio Pf_0

No. of floors	2 story						3 story						4 story					
	0.75	1.00	1.25	1.50	1.75	2.00	0.75	1.00	1.25	1.50	1.75	2.00	0.75	1.00	1.25	1.50	1.75	2.00
100	0.06						0.48	0.10	0.03	0.01	0.01		0.96	0.23	0.07	0.03	0.01	0.01
150	0.59	0.13	0.04	0.01	0.01		2.95	0.85	0.31	0.14	0.07	0.04	5.07	1.63	0.64	0.31	0.17	0.10
200	2.17	0.59	0.20	0.09	0.05	0.03	8.24	2.95	1.26	0.64	0.37	0.23	12.72	5.07	2.33	1.26	0.75	0.48
250	5.12	1.65	0.65	0.31	0.17	0.10	15.82	6.67	3.20	1.78	1.09	0.71	22.58	10.54	5.45	3.20	2.04	1.38
300	9.38	3.47	1.51	0.78	0.45	0.28	24.64	11.81	6.23	3.71	2.39	1.63	33.11	17.48	9.91	6.22	4.18	2.95
350	14.68	6.07	2.87	1.58	0.96	0.62	33.77	17.94	10.22	6.45	4.35	3.08	43.26	25.20	15.39	10.22	7.18	5.26
400	20.64	9.38	4.74	2.75	1.74	1.16	42.57	24.64	14.98	9.91	6.94	5.07	52.46	33.11	21.53	14.97	10.92	8.24
450	26.93	13.27	7.15	4.32	2.82	1.94	50.67	31.49	20.23	13.95	10.10	7.58	60.48	40.80	27.96	20.23	15.22	11.80
500	33.28	17.60	9.99	6.28	4.23	2.98	57.90	38.24	25.76	18.41	13.71	10.54	67.31	48.00	34.40	25.76	19.93	15.81

No. of floors	5 story						6 story					
	0.75	1.00	1.25	1.50	1.75	2.00	0.75	1.00	1.25	1.50	1.75	2.00
100	1.29	0.32	0.10	0.04	0.02	0.01	1.46	0.37	0.12	0.05	0.03	0.01
150	6.33	2.14	0.87	0.43	0.24	0.14	6.98	2.41	1.00	0.50	0.28	0.17
200	15.18	6.33	3.01	1.67	1.01	0.66	16.39	6.98	3.37	1.89	1.16	0.76
250	26.05	12.70	6.79	4.08	2.65	1.82	27.70	13.78	7.47	4.53	2.97	2.05
300	37.22	20.48	11.98	7.70	5.28	3.78	39.13	21.93	13.01	8.45	5.84	4.21
350	47.66	28.86	18.17	12.34	8.83	6.56	49.66	30.60	19.53	13.39	9.66	7.22
400	56.86	37.22	24.91	17.71	13.13	10.06	58.82	39.13	26.53	19.04	14.23	10.97
450	64.69	45.15	31.81	23.50	17.99	14.14	66.53	47.14	33.61	25.07	19.33	15.30
500	71.21	52.43	38.57	29.47	23.17	18.64	72.89	54.42	40.50	31.22	24.73	20.01

unit=percentage

Tab. 2 The number of damaged buildings and the damage ratio in three districts of Sendai-city

No. of floors	Distribution ratio	total	Oroshi-machi				Naga-machi				Kamisugi						
			I & above	II & above	III & above	IV	I & above	II & above	III & above	IV	I & above	II & above	III & above	IV			
2	0.71875	138	22	6	3	0	15.94	4.35	2.17	0.00							
3	0.22917	44	21	10	5	3	47.73	22.73	11.36	6.67							
4	0.03125	6	3	0	0	0	50.00	0.00	0.00	0.00							
5	0.00521	1	0	0	0	0	0.00	0.00	0.00	0.00							
6	0.01563	3	1	1	0	0	33.33	33.33	0.00	0.00							
Total		192	47	17	8	3	24.48	8.85	4.17	1.56							

Oroshi-machi

No. of floors	Distribution ratio	total	Oroshi-machi				Naga-machi				Kamisugi						
			I & above	II & above	III & above	IV	I & above	II & above	III & above	IV	I & above	II & above	III & above	IV			
2	0.35897	14	2	0	0	0	14.29	0.00	0.00	0.00							
3	0.41026	16	3	2	1	0	18.75	12.50	6.25	0.00							
4	0.17949	7	3	1	1	0	42.86	14.29	14.29	0.00							
5	0.05128	2	1	1	0	0	50.00	50.00	0.00	0.00							
Total		39	9	4	2	0	23.08	10.26	5.13	0.00							

Naga-machi

No. of floors	Distribution ratio	total	Oroshi-machi				Naga-machi				Kamisugi						
			I & above	II & above	III & above	IV	I & above	II & above	III & above	IV	I & above	II & above	III & above	IV			
2	0.13158	15	5	1	0	0	33.33	6.67	0.00	0.00							
3	0.27193	31	7	0	0	0	22.58	0.00	0.00	0.00							
4	0.32456	37	7	0	0	0	18.92	0.00	0.00	0.00							
5	0.16667	19	6	0	0	0	31.58	0.00	0.00	0.00							
6	0.10526	12	4	2	0	0	33.33	16.67	0.00	0.00							
Total		114	29	3	0	0	25.44	2.63	0.00	0.00							

Kamisugi

Tab. 3 Comparison between predicted damaged value and actual damage ratio

district	ground acc. (gal)	actual damage ratio			
		I & above $\mu=0.75$	II & above $\mu=1.00$	III & above $\mu=1.25$	IV $\mu=1.75$
Oroshi-machi	350	20.67	9.89	5.29	2.10
	real	24.48	8.85	4.17	1.56
	300	14.23	6.20	3.09	1.12
Naga-machi	300	21.33	10.28	5.49	2.16
	real	23.08	10.26	5.13	0.00
	250	13.72	5.87	2.87	1.01
Kamisugi	250	19.56	9.02	4.64	1.74
	200	10.91	4.32	1.98	0.64
	150	4.32	1.39	0.55	0.15
real	25.44	2.63	0.00	0.00	