

STRUCTURAL BEHAVIOR OF CONCRETE  
WALLS WITH SHEAR FAILURE

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SUMMARY

The purpose of this paper is to describe briefly the hysteretical behavior of concrete structural walls observed during an experimental program. It discusses the methods for flexural and shear strength prediction, proposes a mathematical model which describes structural behavior of concrete walls with shear failure, and finally, presents some results of the mathematical model subjected to actual earthquake accelerograms in firm soil.

Results show the good agreement of the proposed method for flexural and shear strength predictions. These results were correlated with tests carried out principally in Mexico and in the United States, more emphasis is given to the results of one degree freedom systems whose behavior is that observed during an experimental program and subjected to earthquakes in a step by step analysis.

INTRODUCTION

In order to develop a rational criterium for the design of structural walls when shear predominates over flexure, several experimental and theoretical research programs have been carried out. At present three goals have been achieved: a) a method for flexural and shear strength prediction; b) a mathematical model which describes hysteretical behavior, and c) to initiate with this model step by step analysis with actual earthquakes accelerograms. The final objective of these studies is to propose realistic seismic coefficients or ductility factors for structural walls with shear failure.

FLEXURAL STRENGTH

In ref 1 a method for flexural strength prediction was presented. The method takes into account variables such as: shear span ratio ( $M/Vt$ , flexural moment to shear force and length of wall), concrete and steel strength, amount and position of steel reinforcement, axial load level and the presence of boundary elements (columns or transverse walls).

For flexural strength equations are for

only flexural moment:

$$M_{uo} = (A_{se} + \frac{A}{b\tau} A_{sl}) (\frac{d}{t} - \frac{1}{2}) f_y t \quad (1)$$

and for flexocompression

$$M = M_{uo} \left( 1 + \frac{P a}{f_y (A_{se} + A_{sl})} \right) \quad (2)$$

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where  $A_{se}$  is the total vertical reinforcement placed in both wall ends ( $\text{cm}^2$ );  $A_{s1}$ , total vertical reinforcement in wall web ( $\text{cm}^2$ );  $A$  is the total area of wall including boundary element with a width less than two times wall web ( $b$ );  $t$  is the wall length;  $d$ , effective depth for  $A_{se}$ ;  $f_y$  is the steel yield strength and  $P_a$  the total axial load on the wall.

With these equations experimental results of ref 2 were correlated (Table 1). The result was a mean value of 0.98 and a coefficient of variation of 5%.

#### SHEAR STRENGTH

In a experimental research program on microconcrete wall models, the behavior under high shear level was studied observing the effect of the aforementioned variables. It was also possible to idealize the behavior of walls when shear predominates over flexure and to obtain an equation which predicts shear strength (ref 3).

Equations were deduced with the usual criterium of concrete and steel strength superposition

$$V_u = V_c + V_s \quad (3)$$

The concrete contribution, disregarding favorable effect of axial load, is

$$V_c = V_o = (1.6 - 0.3 (M/Vt)^2) \sqrt{f'_c} A \geq 0.5 \sqrt{f'_c} A \quad (4)$$

where  $M$  is the flexural moment;  $V$ , the ultimate shear on the wall;  $A$  is the wall area (see flexural strength);  $f'_c$ , concrete strength. And including axial load,

$$V_c = V_o \sqrt{1 + P_a/V_o} ; \text{ but } \frac{P_a}{V_o} \leq 5 \quad (5)$$

As usual, the contribution of steel for shear strength is

$$V_s = p f_y A \quad (6)$$

however, it is necessary to define which of the horizontal or vertical steel needs to be used. A better approximation is obtained with

$$V_s = p_v f_{yv} A \quad \text{for } \frac{M}{Vt} < 0.25 \quad (7a)$$

$$V_s = (p_v f_{yv} (\frac{5}{4} - \frac{M}{Vt}) + p_h f_{yh} (\frac{M}{Vt} - \frac{1}{4})) A$$

$$\text{for } 0.25 \leq \frac{M}{Vt} \leq 1.25 \quad (7b)$$

$$V_s = p_h f_{yh} A \quad \text{for } \frac{M}{Vt} > 1.25 \quad (7c)$$

where subscript refer to vertical (v) or horizontal (h) reinforcement;  $p = A_s/S_b$ ;  $A_s$  reinforcement steel in wall web (horizontal or vertical);  $S$ , horizontal or vertical steel separation;  $b$ , web thickness. If  $p_v = p_h = p$  eqs 7 reduce to eq 6.

It can be shown (ref 3 and Table 2) that a good correlations between calculated and experimental values exists. The ratio is about 0.99 with 6% coefficient of variation for  $p < 0.7\%$  and small differences between horizontal and vertical reinforcement (1:2 unimportant to important reinforce-

ment).

Eqs. 3 to 7 lead to those obtained in ref 4 for the particular case of a concrete square specimen under shear and tension on the reinforcement, simulating stress behavior of a containment under inner pressure. Equations of ref 4 for concrete strength are:

$$V_u = (2.25 - 1.06 f_s/f_y) \sqrt{f'_c} \quad ; \text{ for maximum strength (kg/cm}^2\text{)} \quad (8)$$

$$V_u = (1.95 - 0.98 f_s/f_y) \sqrt{f'_c} \quad ; \text{ for alternating load strength (kg/cm}^2\text{)}$$

if in eqs 4 and 9,  $M/Vt = 1$  (square specimen) and in eq 8  $f_s = f_y$ ; results

$$V_u = 1.2 \sqrt{f'_c} \quad ; \quad \frac{V_o}{A} = v_o = 1.3 \sqrt{f'_c} \quad \text{eq 4}$$

$$\text{eqs 8} \quad V_u = 0.97 \sqrt{f'_c} \quad ; \quad \frac{V_o}{A} = v_o = 0.97 \sqrt{f'_c} \quad \text{eq 9}$$

#### MATHEMATICAL MODEL

In fig 1 the idealized hysteresis loops, obtained from the experimental research program on concrete walls with shear failure under alternating loads, are shown. For low loads behavior is practically linear elastic. After cracking, wall losses strength and rigidity; from laboratory test on microconcrete models it was observed that for first cycle loading path, similar to that shows by a continuous line in fig 1, are obtained. Under repetitions of lateral load for the same deformation level, wall strength deteriorates and after several cycles hysteresis loops stabilize (discontinuous path in fig 1). This stabilized stress (or force) is named sustained stress and the largest sustained strength, after maximum load shear walls lose strength abruptly. Two stress (or force) envelopes can be defined, one for the stresses of the first cycle of each level of load and another for the sustained stresses. Expressions for these envelopes were deduced after drawing all experimental values on dimensionless axes; equations obtained by least squares fitting are shown on fig 1, where  $v_u$  and  $\gamma_u$  are the stress (or force) and distortion (ratio of displacement at the top, to height of the wall) at failure. In the same way expressions for the hysteresis cycles were proposed in ref 5 and are also shown on fig 1, in these equations  $v_c$  and  $\gamma_c$  represent the limit stress (or force) and distortion of a given cycle. In ref 3 additional expressions for internal paths in hysteresis loops are given.

This model can be easily formed since all parameters are known i.e. equations for envelopes, hysteretical cycles and internal paths; also the maximum strength can be predicted using eqs 3 to 7. Sustained strength is approximately 85% of maximum, or can be calculated with good precision with eq 9 instead eq 3

$$V_{us} = V_o = (1.2 - 0.23 (M/Vt)^2) \sqrt{f'_c} A \geq 0.3 \sqrt{f'_c} A \quad (9)$$

Distortion at failure ( $\gamma_u$ ) is known from eq 9 which is an empirical expression deduced from the experimental program for  $0.5 < M/Vt < 2$

$$\gamma_u = 0.00025 \left( 20.5 + \frac{V_s}{A} + \left( \frac{M}{Vt} \right)^2 \left( 0.7 \frac{V_s}{A} - 2.1 \right) \right)$$

Constants which define hysteretical cycles (A to D, fig 1) were found using least squares for all the walls tested in ref 3 and their values, those shown in same fig 1. Fig 2 depicts calculated points and experimental curves; a good correlation as shown despite general values were used instead of particular values for the wall.

#### INELASTIC ANALYSIS

During a strong earthquake structures show inelastic behavior which produce reduction in strength, rigidity and energy dissipation in members when shear predominates over flexure. There are few studies on the subject; as a contribution, results of step by step analyses under actual accelerograms are presented.

Single degree freedom systems, whose behavior were alike to that previously described, were subjected to four actual earthquakes on firm soils. The criterium for analysis was that, the excitation induced maximum strength on wall, then, comparing it with the response of an elastic system under the same earthquake. As the elastic system one with a rigidity equal to  $K_f = V_u/\gamma_u$  (at failure of inelastic system) was chosen. Due to the shortness of this paper it was not possible to discuss further this subject however, ref 3 presents this discussion.

Response as a function of maximum strength of inelastic system ( $V_u$ ). If the response the inelastic system is  $V_u$  and  $V_e$  the elastic response for the same earthquake, figs 3 and 5 show their spectra for periods between 0.1 and 3 sec. Actual structures with this behavior have periods less than 0.6 - 0.8 sec.

Response as a function of excitation ( $V_{re} = ma\ddot{x}$ ). In ref 6 the ratio  $V_u/ma\ddot{x}$  is proposed as a measurement of inelastic behavior:  $m$  is the mass of the system and  $a\ddot{x}$  is a representative acceleration of the earthquake. Even though maximum acceleration is not the best parameter, it was chosen for  $a\ddot{x}$ . In figs 4 and 5 spectra for  $V_u/V_{re}$  are shown.

Response as a function of system "ductility" ( $\delta_u/\delta_e$ ). This is the same representation than  $V_u/V_e$  because  $K_f = V_e/\delta_e = V_u/\delta_u$ . If some other definition of  $K_f$  would be chosen  $\delta_u/\delta_e$  would not be a simple valued ratio (ref 6).

It is almost impossible to compare results of the proposed model with others; however, the model shows general tendencies alike to classic models.

#### PRELIMINARY ANALYSIS OF RESULTS

From figs 2 to 5 the following behavior tendencies can be standed out for earthquakes on firm soil:

- a) Although earthquakes characteristics are so different, the responses are over a narrow band; preliminary results show the some behavior for systems with degradation of strength and rigidity on soft soils.
- b) Spectra show a radical change in response for  $T > 0.8$  sec.
- c) For  $0.1 < T < 0.8$  sec, response is practically the same; the maximum -

envelop corresponds to  $V_u/V_e = \delta_u/\delta_e = 1.1$ , and  $V_u/V_{re} = 2.0$ . Mean values are 0.9 and 1.7 respectively.

d) Managua and Acapulco earthquakes increased response when  $0.8 < T < 1.2$ ; the maximum value is almost 2.0. Both earthquakes have short duration.

e)  $V_u/V_e$  is practically constant and tends to 1.0 when T increases (except for the range discussed in point d).

f)  $V_u/V_{re}$  decreases abruptly when  $T > 0.8$  sec and is less than 1 for  $T > 1$  sec.

#### FINAL COMMENTS

The proposed methods for flexural and shear strength prediction are more general than those specified in some codes because these take into account parameters which have significant influence on the behavior of concrete walls, and can predict strength under any system of load, shape of specimen and test method.

Step by step analyses of the proposed model under actual accelerograms on firm soil show no reduction in elastic response ( $V_u/V_e = \delta_u/\delta_e \approx 1.0$ ); studies on model for earthquakes in soft soil are in progress. The goal is to provide realistic seismic coefficients, or ductility factors, for structures whose behavior can be associated to the model.

#### REFERENCES

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TABLE 1 FLEXURAL STRENGTH

SPECIMEN	SECTION	REINFORCEMENT PERCENT (%)			$f_{y,}$ kg/cm <sup>2</sup>		$f'_c$ kg/cm <sup>2</sup>	$q_e$	$q_1$	$P_n$ kg.s.	-d/t	A/bt	$M_{UD}$ T-m	$\frac{P_n}{bt'c}$	$M_u$ T-m	$M'_u$ T-m	$M_{med}$ T-m	$M_{L}$ T-m	$M'_{L}$ T-m
		$P_n/z$	$P_h$	$P_v$	$f_{yv}$	$f_{yh}$													
R1	1	0.147	0.31	0.25	5320	5215	455	0.040	0.034	-----	0.95	1.0	47.9	0	47.9	53.1	46.2	1.04	1.15
R2	1	0.40	0.31	0.25	5320*	4590	470	0.092	0.033	-----	0.95	1.0	83.6	0	83.6	86.4	88.6	0.94	0.97
B1	2	0.5328	0.31	0.29	5310	4580	540	0.106	0.034	-----	0.925	1.32	109.5	0	109.5	109.8	108.1	1.01	1.02
B3	2	0.5328	0.31	0.29	4560	4460	480	0.116	0.035	-----	0.925	1.32	104.7	0	104.7	104.6	109.1	0.98	0.96
B4(1)	2	0.5328	0.31	0.29	5310*	4590	450	0.125	0.039	-----	0.925	1.32	109.1	0	109.1	107.8	115.7	0.94	0.93
B2	2	1.762	0.63	0.29	5425	4180	545	0.318	0.034	-----	0.925	1.32	265.9	0	265.9	261.1	271.2	0.98	0.96
B5	2	1.762	0.63	0.29	5310*	4525	460	0.408	0.039	-----	0.925	1.32	284.1	0	284.1	267.7	292.4	0.97	0.92
B6	2	1.762	0.63	0.29	5310*	4490	220	0.846	0.082	94400	0.925	1.32	282.2	0.26	361.3	344.4	363.5	0.98	0.93
E7	2	1.762	0.63	0.29	5310*	4665	500	0.387	0.036	121625	0.925	1.32	292.0	0.15	395.5	379.6	397.3	1.00	0.96
B3	2	1.762	1.38	0.29	5310*	4560	425	0.445	0.043	121625	0.925	1.32	286.7	0.17	386.6	369.7	400.5	0.97	0.92
F1	3	1.867	0.71	0.30	5355	4530	390	0.510	0.048	-----	0.92	1.11	292.0	0	292.0	302.3	319.1	0.92	0.95
F2	3	2.088	0.63	0.31	5355*	4385	465	0.463	0.042	121560	0.92	1.11	314.8	0.16	414.5	411.8	382.0	1.05	1.07
Prom.																	0.98	0.98	
C.V.(%)																	5	7	

NOTATION

$P_n$  relationship between steel area in the boundary elements to web wall area  
 $P_h$  horizontal reinforcement in web wall  
 $P_v$  vertical reinforcement in web wall  
 $A$  cross section of wall, included effective area of transverse walls or columns

SECTION:

1 rectangular  
 2 wall confined by columns  
 3 wall confined by transverse walls

$f_{yv}$  vertical yield stress  
 $f_{yh}$  horizontal yield stress  
 $f_{ye}$  yield stress of reinforcement in boundary elements

$q_e = \frac{A_s \cdot f_{ye}}{bt'c}$  ++ bending moment measured at yield of reinforcement  
 $q_1 = \frac{A_s \cdot f_{yh}}{bt'c}$  (1) monotonic loading

TABLE 2 RECTANGULAR CROSS SECTION WALLS. LAB AND CALCULATED STRENGTH UNDER ALTERNATING LOADS

IDENT.	REF.	H/L	$f'_c$	$P_h$	$f_y$	$P_v$	$f_y$	$\sigma$	$V_o$	$V_c$	$V_s$	$V_{calc}$	$V_m$	$V_{calc}$
		M/LV	--	--	--	--	--	--	--	--	--	--	--	$V_m$
2	1	1.95	306	0.0035	3100	0.0035	3100	22	8.7+	16.4	10.9	27.3	26.2	1.04
5	1	1.95	305	0.0035	3100	0.0035	3100	22	8.7+	16.4	10.9	27.3	29.2	0.93
7	1	1.95	296	0.0035	3100	0.0035	3100	22	8.6+	16.2	10.9	27.1	26.5	1.02
8	1	1.95	292	0.0035	3100	0.0035	3100	22	8.5+	16.2	10.9	27.1	27.0	1.00
10	1	0.67	378	0.0035	3100	0.0035	3100	22	28.5	37.9	10.9	48.8	55.1	0.89
12	1	0.67	280	0.0035	3100	0.0035	3100	22	24.5	33.8	10.9	44.7	44.0	1.02
13	1	2.00	293	0.0035	3350	0.0035	3350	22	8.6+	16.2	11.7	27.9	27.7	1.01
21	1	2.00	250	0.0035	3630	0.0035	3630	22	8.6+	15.4	12.7	28.1	29.0	0.97

WALLS WITH BOUNDARY ELEMENTS. LAB AND CALCULATED STRENGTH UNDER ALTERNATING LOADS

3	1	1.95	280	0.0035	3100	0.0035	3100	22	8.4+	16.0	10.9	26.9	26.4	1.02
4	1	1.95	290	0.0035	3100	0.0035	3100	22	8.5+	16.1	10.9	27.0	26.0	1.01
6	1	1.95	345	0.0035	3100	0.0035	3100	22	9.3+	17.1	10.9	28.0	26.8	1.04
9	1	0.50	360	0.0035	3100	0.0035	3100	22	28.9	38.4	10.9	49.3	46.1	1.07
11	1	0.50	390	0.0035	3100	0.0035	3100	22	26.4	35.8	10.9	46.7	44.5	1.05
14	1	2.00	247	0.0035	3800	0.0035	3800	22	7.9+	15.4	13.3	28.7	26.9	1.07
15	1	2.00	320	0.0035	3575	0.0035	3575	22	8.9+	16.6	12.5	29.1	29.2	1.00
16	1	2.00	209	0.0070	3100	0.0070	3100	22	7.2+	14.5	21.7	36.2	35.2	0.95
17	1	2.00	175	0.0070	3100	0.0035	3100	22	6.6+	13.7	21.7	35.4	33.0	1.06
18	1	0.50	230	0.0035	3100	0.0070	3100	22	23.1	32.3	21.7	54.0	55.6	0.97
19	1	2.00	137	0.0070	3500	0.0070	3500	22	6.9+	14.2	24.6	38.8	38.2	1.01
20	1	2.00	258	0.0070	2650	0.0070	2650	22	8.0+	15.5	18.6	34.1	33.5	1.02
WB-1	4	0.54	160++	0.0025	3000	0.0025	3000	0	19.1	19.1	7.5	26.6	26.0	1.02
WB-2	4	0.54	160++	0.0025	3000	0.0025	3000	0	19.1	19.1	7.5	26.6	27.6	0.97
WB-3	4	0.54	160++	0.0025	3000	0.0025	3000	0	19.1	19.1	7.5	26.6	31.0	0.86
WB-6	4	0.54	160++	0.0050	3000	0.0050	3000	0	19.1	19.1	15.0	34.1	35.3	0.97
WB-7	4	0.54	160++	0.0050	3000	0.0050	3000	25	19.1	19.1	15.0	44.1	45.6	0.97
2	7	2.05	373	0.0033	5160	0.0033	5160	27	9.6+	19.6	17.1	36.7	37.3	0.98
1	7	2.05	378	0.0033	5160	0.0033	5160	27	9.7+	19.6	17.1	36.7	37.8	0.97
B3-2	8	0.50	276	0.0050	5230	0.0050	5554	0	25.3	25.3	27.8	53.1	52.8	1.00
B6-4	8	0.50	216	0.0050	5062	0.0025	5062	0	22.4	22.4	12.7	35.1	41.7	0.84
B7-5	8	0.25	262	0.0050	5111	0.0050	5413	0	25.6	25.6	27.1	52.7	52.3	1.00
B8-5	8	1.00*	240	0.0050	5050	0.0050	5378	0	20.1	20.1	25.2	45.4	42.2	1.00

\*\* in kg/cm<sup>2</sup>; +,  $0.5 \sqrt{f'_c}$ ; ++ General datum of the reference

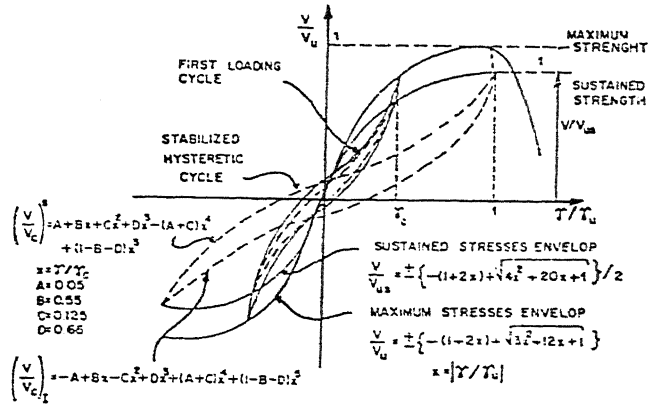


Fig 1 Idealized behavior of walls with shear failure

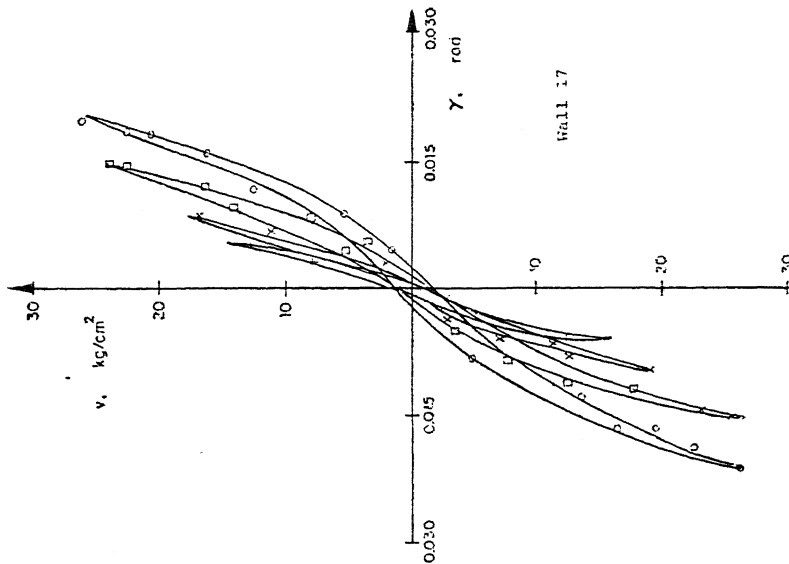


Fig 2 Hysteresis loops and calculated points

