A MIXED FINITE ELEMENT METHOD FOR DYNAMIC ANALYSIS OF SEISMICALLY-LOADED STRUCTURES - APPLICATION TO COUPLED SHEAR WALLS

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SUMMARY

A mixed finite element method is presented for the dynamic analysis of seismically-loaded structures. For time-history dynamic analysis of coupled shear walls, the principal internal forces are introduced in the problem formulation and they are computed in the process of backsubstitution. Thus, the computation of a large number of secondary time-histories and element stresses is avoided. Since explicit relations can easily be established between various internal forces, the method lends itself to the normal force-moment and shear-torsion strength interaction relations, plastic design, and collapse analysis.

INTRODUCTION

Design assessment of large structures which are subjected to earthquake excitations usually seeks relatively few principal internal forces and kinematic measures. (By "principal internal forces" we mean forces that are critical to the design process.) On the other hand, finite element analysis of these structures typically involves a large number of nodal variables. Regardless of how small the number of principal internal forces is, computation of the entire system of response variables is generally inevitable; this is a definite shortcoming of the usual finite element analysis. As an alternative, principal internal forces can be introduced in the analysis process via a mixed finite element technique. This technique is particularly advantageous for the time-history analysis of seismically-loaded structures, where the computation of a large number of secondary time-histories is thereby avoided. The technique is applied here to the analysis of coupled shear walls and selected results are obtained utilizing the "ANSYS" computer program (Ref. 2).

Finally, several applications are briefly discussed: imposing interaction strength relations between internal forces, plastic design, and collapse analysis.

DESIGN AND ANALYSIS

The process of structural design requires a series of analyses of alternative trial structures to ensure acceptable performance. Principal internal forces play a key role in member and joint design and in the assessment of structural safety. Design criteria, a basic knowledge of structural behavior, and practical experience are essential elements in

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choosing appropriate principal internal forces. However, natural locations of these forces are found at the structural joints and points of concentrated loads.

To illustrate the approach, design of coupled shear walls in a seismic zone is considered, Fig. 1(a). Under strong horizontal ground motion, the structure is normally allowed to undergo plastic deformation. Since wall collapse is to be avoided, failure should be confined, wherever possible, to the beams. Thus, natural candidates for the principal internal forces are the beam coupling moments. Moreover, the determination of internal forces at several wall sections, especially the base, is needed to insure an adequate design. To incorporate these internal forces in the analysis, a mixed finite element is used. Such elements are introduced to directly bring into the formulation the coupling moments and other internal forces, as explained in the following section.

COUPLED SHEAR WALLS BY MIXED FINITE ELEMENTS

The theoretical background for the method employed here is in Ref. 1; the walls are divided into conventional stiffness elements, and mixed finite elements are used for the coupling beams and at principal joints of wall elements. A typical finite element layout is shown in Fig. 1(b).

For the coupling beam element,* Fig. 1(c), $\underset{\sim}{x}_b$ is the vector $[v_1, \theta_1, v_2, \theta_2, m]^T$; $\underline{A}_b$ is a mixed matrix given below; $\underset{\sim}{b}$ is the vector $[0,0,0,0,\bar{\theta}_b]$ comprising the beam fixed end loads (zero) and the total plastic rotational deformation of the beam ($\bar{\theta}_b$); m is the coupling moment

$$\underline{A}_b = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 2/S \\ & 0 & 0 & 0 & 1 + 2\ell_1/S \\ & & 0 & 0 & -2/S \\ & & & 0 & 1 + 2\ell_2/S \\ \multicolumn{4}{l|}{\text{Symmetrical}} & -f \end{array}\right] \tag{1}$$

where
$$f = \frac{S}{3EI} + \frac{4}{SGA'} \tag{2}$$

A moment-originating element is introduced to bring into the formulation the internal moment at the principal joints of the walls. For this element,* Fig. 1(d) : $\underset{\sim}{Y} = [\theta_i, \theta_j, M_w]^T$; $\underline{B}$ is the matrix given below; and $\underset{\sim}{c} = [0, 0, \bar{\theta}_w]^T$

*See nomenclature.

$$\underline{B} = \begin{bmatrix} 0 & 0 & 1 \\ & 0 & -1 \\ \text{Symmetrical} & & 0 \end{bmatrix} \qquad (3)$$

M_w is the internal moment at the wall joint and $\bar{\theta}_w$ is the corresponding plastic rotational deformation.

The shear-originating element is found, from the preceding element, by replacing θ_i, θ_j, M_w, and $\bar{\theta}_w$ by u_i, u_j, V_w, and $\bar{u}$, respectively. V_w is the shearing force at the joint and $\bar{u}$ is the corresponding plastic displacement. Similarly, a normal force-originating element is obtained from the shear element by changing the horizontal displacement and shearing force to vertical displacement and normal force, respectively.

The major steps in the time-history dynamic analysis are shown in Fig. 2:

(i) Formulation of element

(ii) Assembly of elements

(iii) Coordinate condensation (A) into dynamic degrees of freedom and principal internal forces

(iv) Coordinate condensation (B) into dynamic degrees of freedom

(v) Solution

(vi) Utilizing the process of back-substitution, determining the time-histories of the principal internal forces, which were condensed in step (iv)

(vii) For nonlinear analysis, steps (iv) and (vi) are eliminated.

Numerical Application

To examine the method presented in the linear range, an 18-story coupled shear wall structure (taken with minor modification from Ref. 3 was analyzed. For this structure;* $h = 105$, $S = 38$, $\ell_1 = 63.2$, $\ell_2 = 67.5$ in; $A_1 = 5550$, $A_2 = 5650$, $A = 672$ sq. in.; $I_1 = 66 \times 10^5$, $I_2 = 90 \times 10^5$, $I = 1.52 \times 10^4$ in^4; $E = 5000$ k/in^2; and lumped mass/story = 0.5 k - sec^2 - in^{-1}.

The principal internal forces included coupling moments and forces at the wall bases. This structure was subjected to the first four seconds of the El-Centro Earthquake record, N-S component, May 18, 1940, scaled by 1/3. Considering Rayleigh damping, a damping ratio of 5% is assumed for the first and second modes. Numerical integration was carried out using a time step of .01 second.

*See nomenclature.

After performing the solution, using the "ANSYS" computer program (Ref. 2), the backsubstitution process resulted in the time-histories of the principal internal forces, shown in Fig. 3. The maximum values of coupling moment at the 2nd, 10th, and 18th stories are 3972, 33831 and 957 kip-in, respectively. The maximum forces at wall bases are: $M_1 = .789 \times 10^5$ and $M_2 = 1.027 \times 10^5$ kip-in; $v_1 = 269$ and $v_2 = 304$ kip; and $N_1 = 2836$ kips.

To evaluate the computational saving of the mixed method, a block diagram is provided in Fig. 4 for the stiffness formulation of the coupled shear walls. The dynamic relations are first established in terms of the entire coordinate system (114 coordinate). By condensation, these relations are reduced to 19 dynamic degrees of freedom. Solving these relations yield their time-histories. By backsubstitution, the time-histories of the remaining 95 degrees of freedom are computed. Finally, the time-histories of the element internal forces are found using the time-histories of the element nodal variables and element relations. The first two steps are referred to as the "Displacement Pass," while the last two steps are termed the "Stress Pass." However, by using the mixed finite element technique, the "stress pass" is reduced to the backsubstitution in 24 relations: 18 coupling moments and 6 forces acting at the bases. Although the exact saving in the computational time is not available, taking the above discussion into consideration, it is estimated that the computational time is reduced by over 50%. This saving generally increases for taller shear walls and other large structures.

DISCUSSION

In the mixed finite element analysis, unlike the typical stiffness formulation, internal forces are included in the model coordinate system. Naturally, explicit relations can easily be established between various internal forces. Consequently, for an indeterminate structure, the method lends itself to imposing the normal force-moment and shear-torsion strength interaction relations. Moreover, if the principal internal forces are chosen among design variables, the method will be a useful tool for plastic design and collapse analysis.

CONCLUSION

A mixed finite element method is presented for the dynamic analysis of seismically-loaded structures. The method utilizes a mixed finite element model whose coordinate system includes principal internal forces. The method is applied to time-history dynamic analysis of coupled shear walls, where the computation of secondary time-histories is avoided. Numerical results are obtained for linear analysis. Since explicit relations can easily be established between various internal forces, the method lends itself to the normal force-moment and shear-torsion strength interaction linearized relations, plastic design and collapse analysis.

REFERENCES

1. El-Kamshoshy, F. M., "Mixed Finite Element Method for Optimal Seismic Design of Earthquake Resistant Structures," Ph.D. Dissertation, University of California, Berkeley, June 1981.

2. "ANSYS" Engineering Analysis System Users Manual, Revision 4, Swanson Analysis Systems, Inc., Houston, TX, U.S.A., 1982.

3. Nayar, K. K., and Coull, A., "Elasto-Plastic Analysis of Coupled Shear Walls," J. of Structural Division, ASCE, Proc. Paper, Vol. 102, No. ST9, Sept. 1979, pp. 1845-1860.

NOMENCLATURE

A_i Cross section area of wall (i), (i = 1,2)

A Beam cross section area

A' Beam shear area

E Young's modulus

G Shear modulus

h Story height

I Beam moment of inertia

I_i Moment of inertia of wall (i), (i = 1,2)

ℓ_1 Distance from centroidal axis of wall (1) to connected beam end

ℓ_2 Distance from centroidal axis of wall (2) to connected beam end

m_j Coupling moment at story (j), (j = 1,18)

M_i Moment at base of wall (i), (i = 1,2)

N_i Normal force at base of wall (i), (i = 1,2)

r_j Lateral displacement of story (j), (j = 1,18)

S Beam clear span

u Horizontal displacement

v_i Vertical displacement of wall (i), (i = 1,2)

V_i Shearing force at base of wall (i), (i = 1,2)

θ Rotation

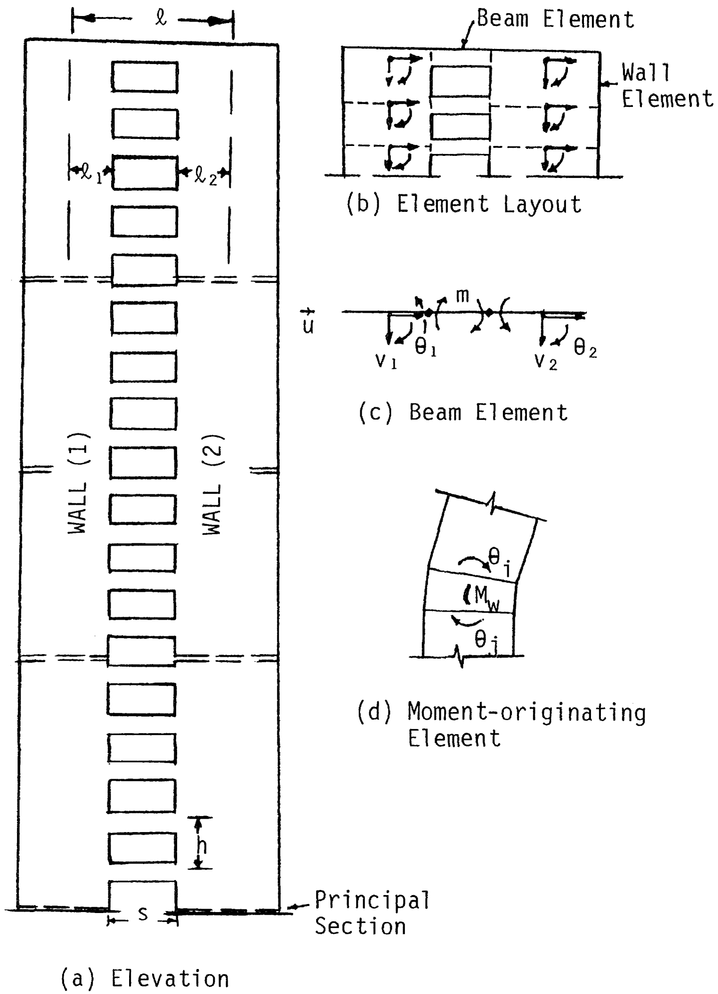

Fig. 1. Coupled Shear Walls

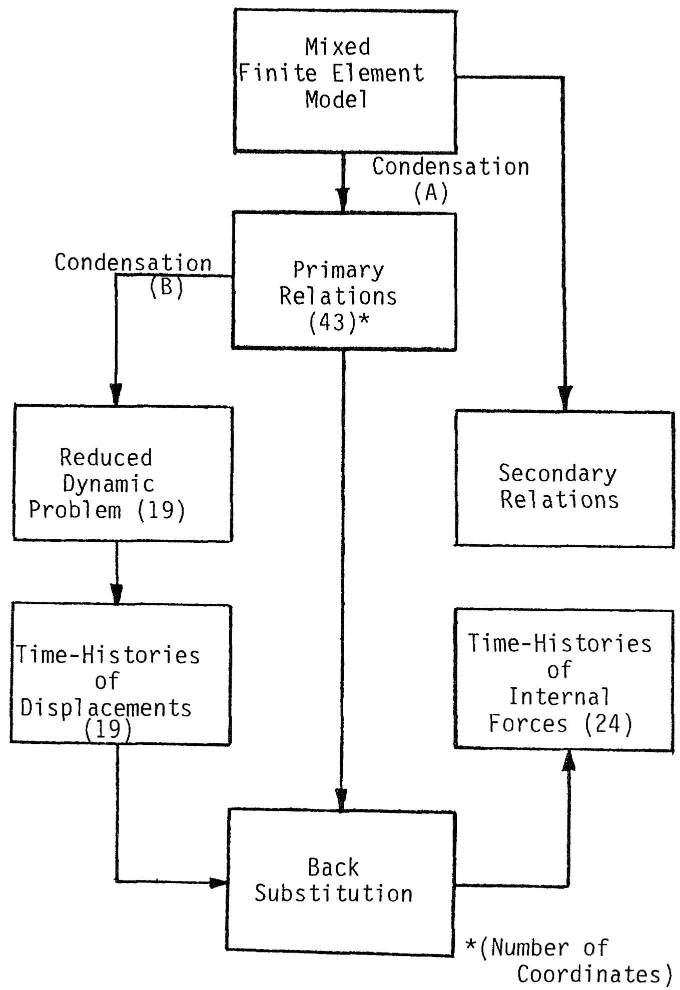

Fig. 2. Time-History of Primary Variables

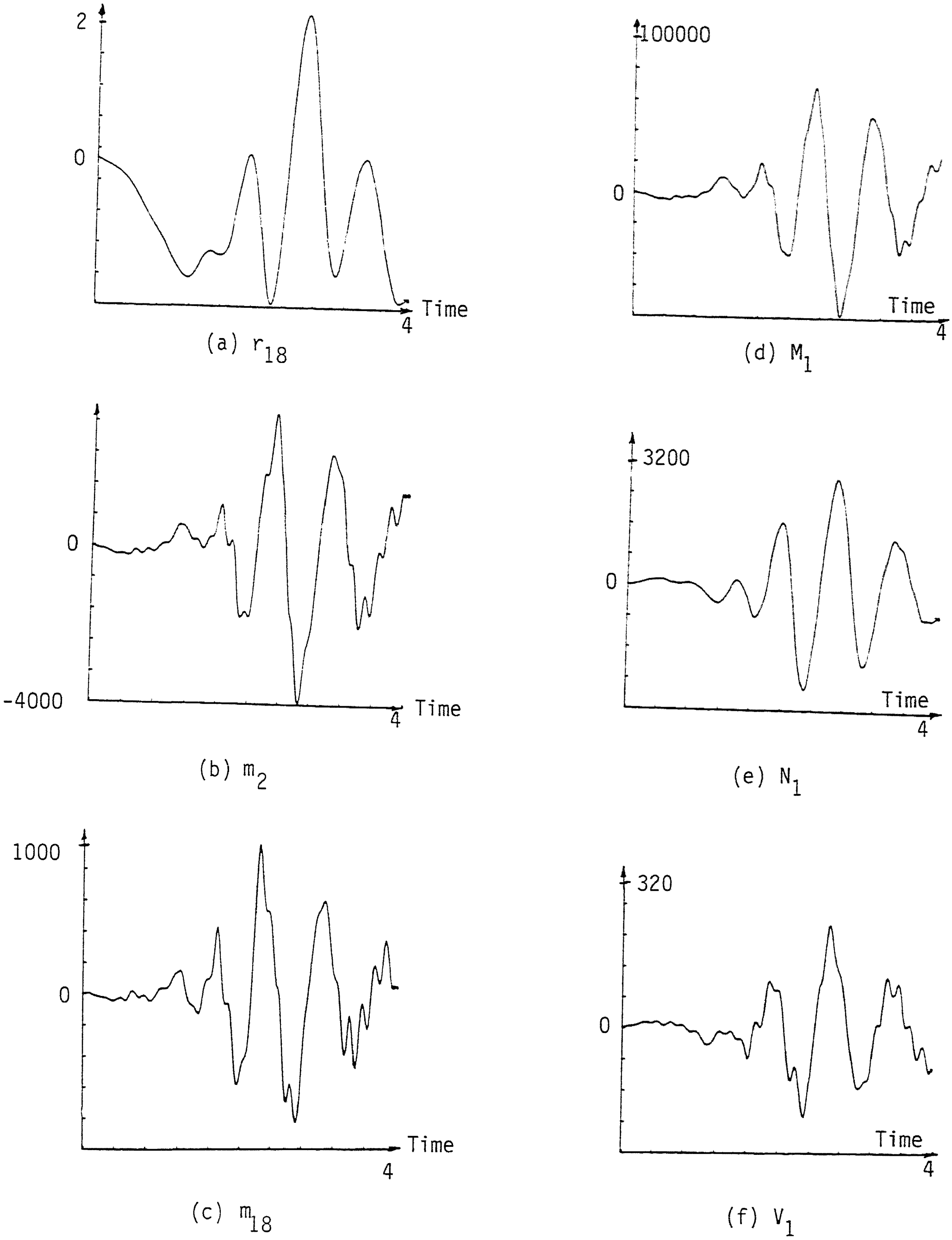

Fig. 3. Numerical Results (kip-in-sec.)

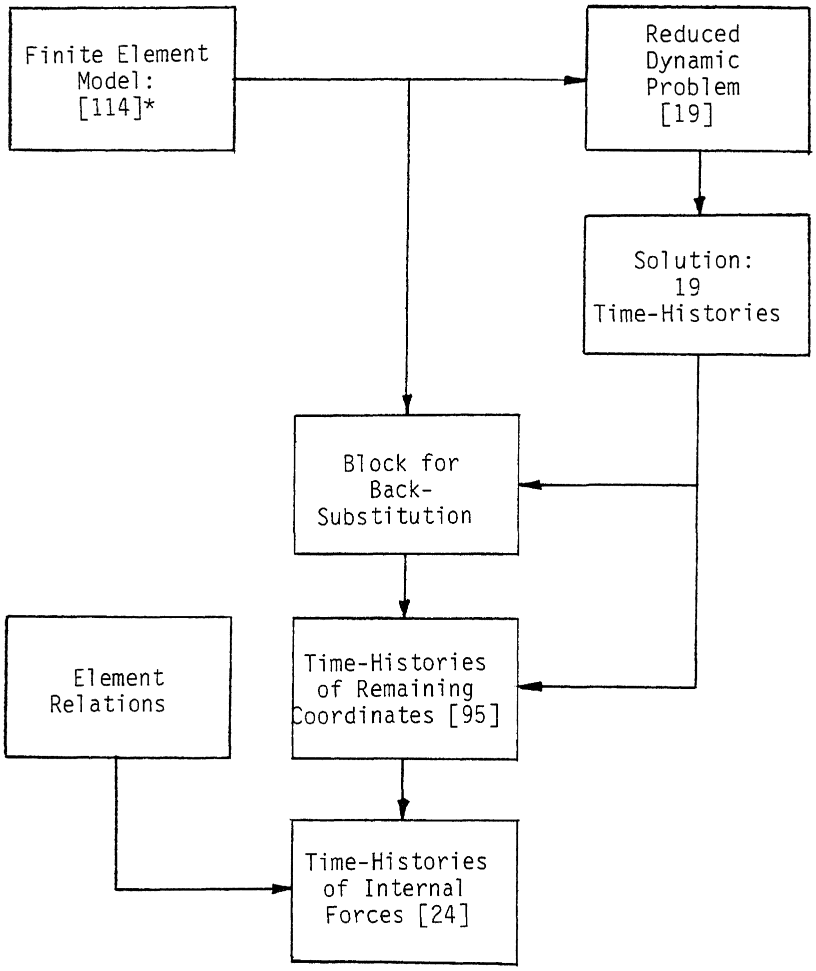

Fig. 4. Typical Finite Element Time-History Dynamic Analysis (Stiffness Formulation)

*[Number of coordinates]