LATERAL-TORSIONAL RESPONSE OF LOW-RISE TIMBER BUILDINGS

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SUMMARY

Knowing the combined lateral and torsional response of a light-frame timber structure is important for seismic design. A three-dimensional model is developed to determine the coupled lateral and torsional natural frequencies and mode shapes for low-rise timber buildings with shear wall and diaphragm construction. Results are presented for a two-story building common to residential construction in the United States.

INTRODUCTION

Structural engineering research for seismic building response and design has centered on lateral motion. Torsional response is just beginning to be investigated. Although a range of values of lateral natural frequency data is known for low-rise timber buildings (Refs. 4,6,11), no corresponding data are available for torsional or combined lateral-torsional motion. Yet, torsional racking was observed (Ref. 2) as the greatest deficiency of light-frame buildings during the 1971 San Fernando earthquake. Obviously, analysis procedures that account for torsional behavior are needed to properly design light-frame buildings.

This study presents a three-dimensional analytical model to determine the coupled lateral and torsional frequencies and mode shapes of low-rise timber buildings. The model, based on linear relationships, is suggested as a middle course between very sophisticated structural models and oversimplified lumped mass models. It corresponds to the accuracy and reliability of available experimental data on timber shear walls while at the same time providing reliable and realistic results.

STRUCTURAL MODEL

Floor and roof diaphragms and vertical shear (racking) walls constitute the lateral load resisting elements of much of light-frame timber structures. The floor and roof diaphragms are assumed rigid in their own plane. This assumption is commonly made in the analysis of high-rise buildings; Shepherd and Donald (Ref. 7) indicated the assumption is also valid for low-rise structures.

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The vertical shear walls generally consist of a sheathing material nailed to a framing system. Itani et al. (Ref. 3) presented a procedure to model each sheet of sheathing by a pair of diagonal springs. The spring stiffness depends on the stiffness of the nailed connection of the sheathing to the framing. The stiffness of the nailed connection is nearly linear at small deformation and becomes nonlinear at larger deformation. Lateral stiffness and strength of the wall is more dependent on number of nails and nail spacing than on wall height (Refs. 9,10). For split-level buildings having half-height walls, the height reduction increases stiffness while the use of fewer nails offsets this increase.

The structural model of a two-story building is given in Fig. 1. The stiffness per unit length of walls on the same floor level is assumed identical, but on different levels it can have different values. The effective length of a wall, used to determine wall stiffness, is assumed to be the total length minus the length of openings.

The foundation is assumed rigid and motionless. The effects of the joints between the foundation and first floor diaphragm are simulated by dummy shear walls. The floors are rigid diaphragms interconnected by pin-ended vertical rigid bars. Walls are connected at nodes at each floor level only.

The general arrangement and nomenclature for the ith floor are given in Fig. 2 for the multi-story building.

Superscripts M or C define the mass center, M, or the stiffness center, C. The mass center, M_i, is the center of gravity of the masses lumped at the floor, i. The stiffness center, C_i, is the point about which the floor, i, rotates under the action of a horizontal couple. The wall lengths in the x and y directions are denoted by a and b, respectively.

Stiffness Matrix

To construct the stiffness matrix of the structure, unit horizontal relative displacements between the floors "i" and "i+1" and "i" and "i-1" are applied. The forces and moments corresponding to these horizontal displacements give the elements of the lateral stiffness matrix.

The expressions for displacements in the x-direction are:

$$(u_i - u_{i+1}) = 1 \quad \begin{cases} K_{xx}^{i+1} = k_{i+1} \Sigma a_{i+1,m} \\ K_{xy}^{i+1} = 0 \\ K_{x\theta}^{i+1} = -k_{i+1} \Sigma a_{i+1,m} y_{i+1,m} \end{cases} \tag{1}$$

Similar expressions were developed (Ref. 5) for displacement u between the i and (i-1) floors and displacements v in the y-direction, and for rotation, θ, about the z-axis. The stiffness coefficients, K, are expressed in terms of the stiffness per unit length of wall, k; the wall lengths, a, b, parallel to the X and Y axes; and the wall coordinates, x, y.

The stiffness coefficients derived above can be assembled in the submatrices $[K_{i,i-1}]$, $[K_{i,i}]$, and $[K_{i,i+1}]$ for the floor "i". These submatrices correspond to the displacement vectors $[u_{i-1}\ v_{i-1}\ \theta_{i-1}]$, $[u_i\ v_i\ \theta_i]$, and $[u_{i+1}\ v_{i+1}\ \theta_{i+1}]$. The matrix $[K_{i,i-1}]$ is given below. The other matrices are given in Ref. 5.

$$[K_{i-1,i}] = \begin{bmatrix} -K^i_{xx} & 0 & -K^i_{x\theta} \\ 0 & -K^i_{yy} & -K^i_{y\theta} \\ -K^i_{x\theta} & -K^i_{y\theta} & -K^i_{\theta\theta} \end{bmatrix} \tag{2}$$

The submatrices for each floor can then be assembled into the stiffness matrix for the entire structure.

The stiffness matrix must be modified for a split-level building since this type of building consists of two parts each having floor diaphragms at different levels. If the two parts are assumed disconnected, their individual stiffness matrices $[K_A]$ and $[K_B]$ can be determined as discussed for the two-story building. Thus, the stiffness matrix of the entire uncoupled structure, $[K_o]$, has the symbolic form:

$$K_o = \left[\begin{array}{c|c} K_A & 0 \\ \hline 0 & K_B \end{array}\right] \tag{3}$$

The interaction between the two parts is expressed by a connection matrix $[K_c]$. If the lateral stiffness of the interconnecting walls is denoted by $\bar{k}$, and the distance from these walls to the y axis is denoted by X, then the stiffness matrix of the entire coupled structure is:

$$[K] = [K_o] + [K_c] \tag{4}$$

where

$$[K_c] = \left[\begin{array}{ccc|cc} \bar{K} & & & -\bar{K} & \\ & 2\bar{K} & & -\bar{K} & -\bar{K} \\ & & \bar{K} & & -\bar{K} \\ \hline -\bar{K} & -\bar{K} & & 2\bar{K} & \\ & -\bar{K} & -\bar{K} & & 2\bar{K} \end{array}\right] \tag{5}$$

and the submatrix

$$[\bar{K}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{k} & \bar{k}X \\ 0 & \bar{k}X & \bar{k}X^2 \end{bmatrix} \tag{6}$$

Mass Matrix

The masses are lumped at each floor level by assuming they are uniformly distributed over the entire floor plan. A typical floor plan (Fig. 3) is subdivided into rectangular parts with dimensions s x r; thus s_{ik} and r_{ik} indicate sides of the rectangular part k at floor level i. Similarly, the mass center of the rectangular part k is denoted M_{ik}.

Using these notations for the floor "i," the area A_i, the coordinates of the mass center x_i^M and y_i^M, and the moment of inertia about the point 0, J_{oi} can be obtained as below:

$$A_i = \sum_k r_{ik}\, s_{ik} \tag{7}$$

$$x_i^M = \frac{\sum_k r_{ik}\, s_{ik}\, x_{ik}^M}{A_i} \tag{8a}$$

$$y_i^M = \frac{\sum_k r_{ik}\, s_{ik}\, y_{ik}^M}{A_i} \tag{8b}$$

$$J_{oi} = m_i \sum_k r_{ik}\, s_{ik}\left[\frac{1}{12}(r_{ik}^2 + s_{ik}^2) + (x_{ik}^M)^2 + (y_{ik}^M)^2\right] \tag{9}$$

Thus, the mass matrix of floor "i" has the form

$$[M_i] = \begin{bmatrix} A_i m_i & 0 & -y_i^M A_i m_i \\ 0 & A_i m_i & x_i^M A_i m_i \\ -y_i^M A_i m_i & x_i^M A_i m_i & J_{oi} \end{bmatrix} \tag{10}$$

The mass matrix of the entire structure is assembled from each of the floor mass matrices.

For split-level buildings, the mass of the interconnecting walls can be neglected because they are very small compared to that of the entire building. Then there is no coupling between the mass matrices of the two independent parts, and the mass matrix of the entire coupled structure is the sum of that for the two independent parts.

Modal Analysis

Based on the computer model developed herein, a modal analysis can be conducted using the classical dynamic equilibrium equations. If damping is neglected and only the free vibration response is required, these equations reduce to the standard eigenvalue problem to determine mode shapes and natural frequencies. We used the modal analysis to analyze a two-story building.

Model Building

A two story house was selected as an example. It corresponds to a building studied by the Applied Technology Council (Ref. 1). The floor plans of the two-story are shown in Fig. 4. The structural model in Fig. 1 corresponds to this building.

Wall Stiffness

The walls for each building are plywood (exterior) and gypsum wallboard (interior) sheathed, typical in light-frame timber construction. The load-horizontal displacement curve is nonlinear for this type of construction. Thus, a linear sensitivity approach is used in which the initial stiffness (k_{max}) and secant stiffness (k_{min}) depend on sheathing materials, framing species and grade, and nail size and spacing. Approximations are made for k_{max} and k_{min} by relating wall strength and stiffness to lateral nail strength and stiffness. A previous study (Ref. 10) related wall panel racking strength, R, to lateral nail strength of the corner nail, r. This work was extended (Ref. 5) to relate wall panel stiffness, k, to the lateral nail stiffness of the corner nail. The total lateral stiffness of a wall is the summation of the individual panel stiffnesses.

The walls are assumed to be constructed with 3/8-inch plywood exterior sheathing with 6d nails spaced at 6-inch intervals along the perimeter and 12 inches along the interior framing and 1/2-inch gypsum wallboard with 1-1/2-inch ring shank nails spaced at 8-inch intervals. Using minimum and maximum lateral nail stiffness from Ref. (8) and the wall panel stiffness relationship from Ref. (5) results in k_{min} = 10,800 lb/ft per foot of shear wall and k_{max} = 54,000 lb/ft per foot of shear wall.

Results

The lowest natural frequency for the two-story building is 1.79 Hz and 4.0 Hz corresponding to k_{min} and k_{max}. Mode shapes corresponding to the minimum stiffness values, for the first three frequencies, are given in Fig. 5. These responses are based on assuming the stiffness of the walls per unit length is the same on each level and for the base connection.

A split-level building was analyzed in two different forms in a companion report (Ref. 5). First, it was assumed that the two parts of the house are independent. In the second analysis the connections between the two parts were taken into consideration, and the whole building was analyzed as one single structure.

SUMMARY AND CONCLUSIONS

Timber structures are susceptible to lateral-torsional displacement damage during earthquakes. Particularly vulnerable are split-level houses and two-story buildings with large openings on the first floor. This study describes a model which can be used with classical modal analysis to yield natural frequencies and mode shapes for coupled lateral-torsional motion.

The dynamic response of a two-story building has been determined numerically. The building has nonsymmetrical mass and stiffness centers resulting in coupled lateral-torsional vibrations.

The structural model and analysis method is suitable for the dynamic modal analysis of single- and multi-story light-frame timber buildings of shear wall and diaphragm construction. Since the degree of freedom of the model is low, the analysis does not require considerable calculation and computer time; however, the results obtained are realistic (compared to observed natural frequencies) and give reliable information about the dynamic response.

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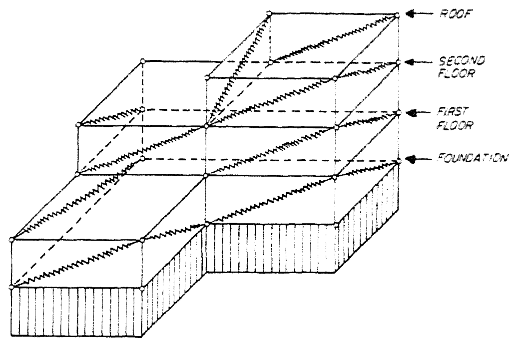

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Figure 1.--Structural model of a two-story shear-wall building. Lowest level is a "dummy" wall representing foundation-wall connection.

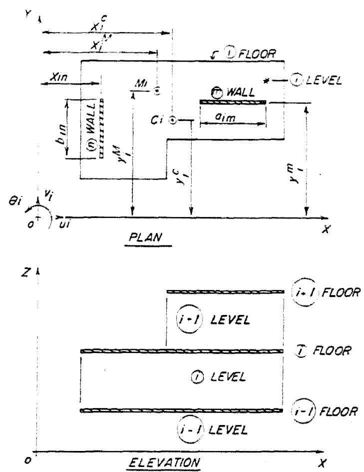

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Figure 2.--General arrangement and nomenclature for a two-story building.

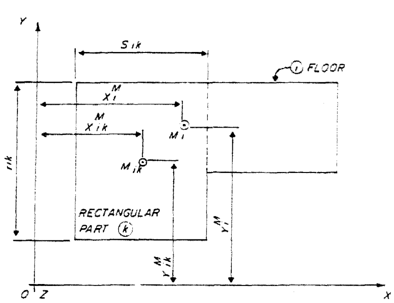

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Figure 3. --General arrangement and nomenclature for a typical floor.

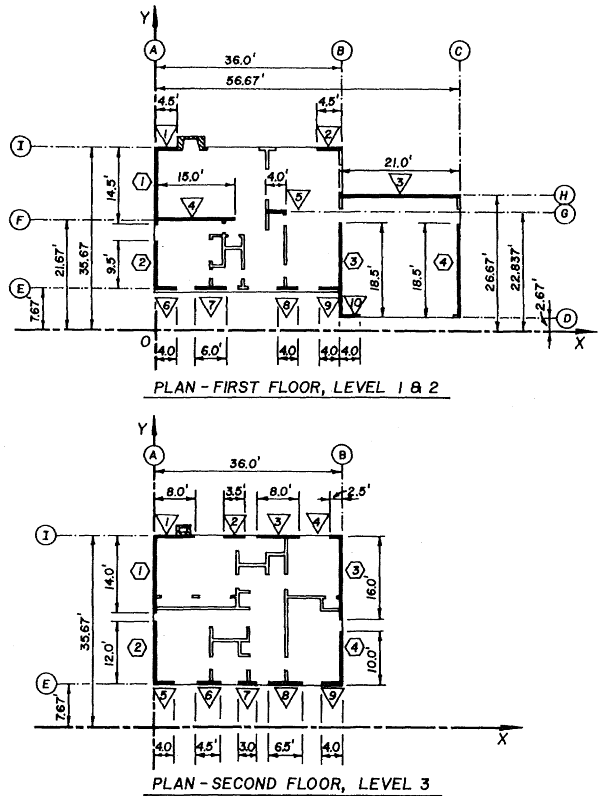

Figure 4.--Floor plans of example two-story building.

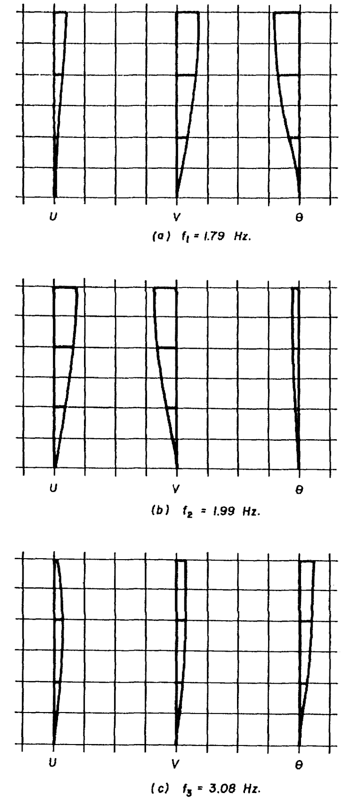

Figure 5.--Free vibration mode shapes of two-story building with respect to the mass center of the first floor, for k_{min} = 10,800 lb/ft. Only the first three modes are shown.