# PLASTIC ZONE SPREAD AND SEISMIC RELIABILITY OF R. C. FRAMES

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## SUMMARY

The seismic fragility of reinforced concrete (r.c.) frames is evaluated as the conditional frequency of failure at a given value of the seismic intensity. The more accurate mechanical model compatible with the acceptable computational effort should be adopted in estimating these structural fragilities. For this purpose, an idealization of the frame taking into account also the finite size of the plastic zones is developed. The constitutive law to be introduced in the calculations is the cross-section moment-curvature relationship. The approach is implemented for a general purpose computer code: the results obtained for a four-storey three-span r.c. frame are compared with the fragilities estimated by using a model which concentrates the inelastic deformations in potential plastic-hinges.

## INTRODUCTION

Previous papers by the authors and associates (Refs. 1 to 4) have treated the problem of determining the seismic reliability of reinforced concrete frames allowing for the uncertainty of both ground motion details and mechanical and geometrical characteristics of the structure. In the same framework, the authors have separated the contribution to the system fragility (Refs. 5 to 7) of different sources of uncertainty. In particular, the influence of the uncertainty on the cross-section failure criterion has been studied with special attention (Refs. 1 and 3). The local failure criterion has been defined in terms of two damage indicators: a ductility factor that penalizes large displacements, and a parameter expressing inelastic-energy dissipation which penalizes inelastic cycles also at low displacement levels. The uncertain limit value of a function of these indicators was derived from statistical analysis of experimental data (Refs. 8 and 3).

Research still in progress has led to the formulation of a sophisticated procedure for the assessment of the seismic reliability of complicated structural systems (Ref. 9). This procedure,founded on the use of experimental design techniques, is very accurate in the evaluation of the statistics and in the estimations of the probabilistic properties of the fragility parameters; an analogous degree of sophistication in the mechanical idealization of the structure, however, should also be adopted. For this purpose improvements in the computational tools have

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been recently pursued, at least with reference to the non-linear dynamic analysis of r.c. frames. The axial force-bending moment interaction has been considered (Refs. 10, 11 and 4) and the dynamic analysis has been made able to proceed after one or more elements have failed (Refs. 12 and 13).

All these improvements have been introduced in computer codes which discretize the frame in perfectly-elastic members, while inelastic deformations concentrate at their ends (potential plastic hinges). The effect of the finite size of the plastic regions was therefore neglected. A different approach requiring a slightly increased computational effort has been developed in Refs. 14 to 16. It takes into account the spread of the plastic zones by subdividing each element in three parts of finite size: an elastic central region and two inelastic zones of finite size at the ends.

This model, allowing for the diffusion of the plasticity along the element, is introduced here in the calculation of seismic fragilities in order to investigate the approximation reached by idealizations which concentrate inelastic deformations. For this purpose the damage indicators are derived from the calculated response and the probabilistic failure criterion of Ref. 3 is used to establish the structural fragility. Comparisons are developed for the three-span four-storey frame which has been already analysed in Ref. 3 with the inelastic deformations concentrated at the member ends.

## MECHANICAL MODELS

The mechanical idealization of a plane frame usually adopted in non-linear dynamic analysis regards the frame as the assemblage of perfectly elastic elements with a hinge at each of its ends. The two hinges at the two ends of the element represent the flexural inelastic behaviour of the member. For reinforced concrete frames, the hysteretic moment-rotation constitutive law of these hinges is assumed to follow the modified Takeda model (Fig. 1). The bilinear primary curve of Fig. 1 is completely defined by the yielding moment $M_y$ (to be computed) and the slope of the second branch (to be derived from experimental data). The flexural hinges, in fact, are initially (first branch) infinitely stiff, so that they do not affect the behaviour of a member before yielding. Once they yield, the flexibilities of the plastic hinges are added to the rotation flexibility matrix of the elastic member (Ref. 18)

$$\underline{f} = \begin{bmatrix} f_{ii} & f_{ij} \\ f_{ji} & f_{jj} \end{bmatrix} = \begin{bmatrix} \frac{\ell}{3EJ}+\frac{1}{K_i} & -\frac{\ell}{6EJ} \\ -\frac{\ell}{6EJ} & \frac{\ell}{3EJ}+\frac{1}{K_j} \end{bmatrix} \tag{01}$$

$K_i$ and $K_j$ being the stiffnesses of the flexural hinges (i denoting the first end of the element and j the second end). Note that in this way the stiffness matrix of an element is modified only when there is a change of stiffness in one of the two hinges, so that the global stiffness matrix is not necessarily modified at each step.

A different, more accurate model should be introduced if the finite size of the plastic regions has to be taken into account. In this case one considers a constant moment-curvature relationship along the element and describes it by the hysteretic model of Fig. 1. The primary curve is here completely defined by three parameters: the yielding moment $M_y$, the first slope $(EJ)_e$ and the second slope $(EJ)_2$. These quantities can be all calculated from the elementary material properties of steel and concrete (Refs.19 and 15). Any element of length $\ell$ is then subdivided into three regions (Refs. 15 and 16) in order to compute the tangent stiffness matrix of a general frame member:

i) the inelastic region of length $x_i$ at node i, with average stiffness $(\overline{EJ})_i$ ;

ii) the inelstic region of length $x_j$ at node j, with average stiffness $(\overline{EJ})_j$ ;

iii) the central region of length $(\ell - x_i - x_j)$, having the initial elastic stiffness $(EJ)_e$.

In this scheme, the flexibility coefficients are given by:

$$f_{ii} = \frac{1}{3(EJ)_e \ell^2} \left[ (Q_j - 1)x_j^3 - (Q_i - 1)(\ell - x_i)^3 + Q_i \ell^3 \right]$$

$$f_{jj} = \frac{1}{3(EJ)_e \ell^2} \left[ (Q_i - 1)x_i^3 - (Q_j - 1)(\ell - x_j)^3 + Q_j \ell^3 \right] \tag{02}$$

$$f_{ij} = - \frac{1}{3(EJ)_e \ell^2} \left[ (Q_j - 1)x_j^2(1.5\ell - x_j) + (Q_i - 1)x_i^2(1.5\ell - x_i) + \frac{\ell^3}{2} \right]$$

where $Q_i = (EJ)_e/(\overline{EJ})_i$ and $Q_j = (EJ)_e/(\overline{EJ})_j$.

The lengths x and the stiffness ratios Q of the plastic regions depend on the current branch of the moment curvature diagram. In the elastic field one has $x_{k1} = 0$, $Q_{k1} = 0$ (k = i, j). For inelastic loading along the second branch of the primary moment curvature diagram, the length of the plastic region can be determined by:

$$x_k = \frac{M_k - M_y}{M_i + M_j} \tag{03}$$

where $M_y$ is the yielding moment and $M_k$ is the current value of the bending moment at the end k (k = i,j). Within the plastic region the stiffness ratio is $Q_{k2} = (EJ)_e/(EJ)_2$.

In Refs. 15 and 16 the unloading $(EJ)_3$ and reloading $(EJ)_4$ flexural stiffnesses of the moment-curvature relationship are defined according to the scheme of Fig. 2. During the unloading and the reloading, $x_k$ remains the maximum plastic region length reached in any previous loading cycle, while $Q_{k3}$ (unloading) and $Q_{k4}$ (reloading) can be given by the approximate expressions (Ref. 15):

$$Q_{k3} = (EJ)_e/(\overline{EJ})_3 = c\left(\frac{(EJ)_e}{(EJ)_3} - 1\right) + 1 \tag{04a}$$

$$Q_{k_4} = (EJ)_e/(\overline{EJ})_4 = c\left(\frac{(EJ)_e}{(EJ)_4} - 1\right) + 1 \qquad (04b)$$

c being an empirical constant coefficient. Values between 0.5 and 0.75 of c have been found to give the most accurate results in Ref. 15.

The model of Eq. (01) has been introduced by Litton (Ref. 18) into the general purpose non-linear analysis computer program DRAIN-2D (Ref. 20). The model of Eq. (02) has been incorporated in the same computer code by Roufaiel and Meyer (Ref. 17) and, independently, by the authors of this paper. The computer routine written by the authors runs taking account of the axial-force under static loads in the yielding moment determination and in the P-$\Delta$ effect evaluation. However, the bending moment-axial force interaction of Ref. 11 was also introduced as an option. The code can stop when any element fails (brittle failure) or proceed the analysis until an ultimate displacement is reached (progressive failure (Ref. 12)). Moreover, some local effects in the moment-curvature relationship (as pinching or strength degrading) are not taken explicitly into account, but they can be easily incorporated. This numerical procedure requires a slightly increased computational effort in comparison with the computer time spent by using the plastic hinge model. The increase is justified by the need of updating the stiffness matrix also at the end of the steps during which one or more $x_k$ increase without changes in $Q_k$.

## SEISMIC RELIABILITY

### Failure criterion

The damage analysis developed in Ref. 17 is founded on the introduction of a flexural damage ratio as damage indicator. It is evaluated on the moment-curvature diagram. The dynamic analysis of a frame following the model of Eq. (02) works in terms of curvatures and therefore no additional calculation is necessary for the estimation of the damage indicator. However this flexural damage ratio appears to be very poor for seismic reliability purposes, because it takes into account the effect of large deformations, but not the effect of cycles at low displacement levels.

In Refs. 1 to 9, two quantities have been combined into a single measure of section damage R(t) (R(t) being a function of the time t from the beginning of the excitation). In particular in Ref. 3 the authors considered the data-based damage measure:

$$R(t) = \left[(1.10(D(t))^{0.38})^2 + (67.27\ \xi^{0.3625}(\theta_m(t))^{0.4138})^2\right]^{\frac{1}{2}} \qquad (05)$$

where D(t) is the dimensionless energy dissipated by inelastic rotation at one end of the member and $\theta_m(t)$ is the maximum inelastic rotation reached in (0,t) at the same member end. The first indicator penalizes inelastic cycles also at low displacement levels, while the second indicator penalizes large displacements. In Eq. (05) $\xi$ is the dimensionless neutral axis depth in the ultimate state. The value $R(t) = R^*$ at which failure occurs varies randomly from hinge to hinge and has been found to be accurately described by a random variable with lognormal distribution, mean 12.1 and variance 11.2.

Eq. (05) fits a larger quantity of experimental data than other damage measures. In addition to the results of few laboratory tests of r.c. specimens subject to cyclic loading (Ref. 8), in fact, a lot of laboratory (Ref. 21) and theoretical (Ref. 22) results are available on the relation between the ultimate rotation and appropriate physical quantities (as the neutral axis depth $\xi$, the transversal reinforcement ratio and the shear span ratio) for monotonic (non-cyclic) loading. Both these experimental results have been combined in Ref. 3 in order to provide Eq. (05) and the properties of the critical value $R^*$.

The damage measure of Eq. (05), however, requires the evaluation of the inelastic rotation of each member end at each time t in order to calculate its maximum value $\theta_m(t)$ and to evaluate the dissipated energy D(t). This task is very easy if one adopts the mechanical model of Eq. (01), whereas additional computations are required at the end of the analysis procedure founded on Eq. (02). Integration of the curvature along the finite length of the plastic zone is the direct approach to the problem but its use is discouraged by the drastic simplifications (Eq. (04)) introduced in the calculation of the member flexibility matrix. Therefore, a simplified procedure is formulated in the sequel in order to estimate the inelastic rotations of the member ends.

i) At time t the stiffness tangent matrix of the element is obtained as the inverse of the flexibility matrix $\underline{f}$ (Eq. (02)) and the global stiffness matrix of the frame is obtained by assembling the single member matrices.

ii) By the classical formulae of the numerical integration, the solution of the equation of motion is obtained at the time ($t+\Delta t$). From this solution the increments $\underline{\Delta\theta}^T = \{\Delta\theta_i, \Delta\theta_j\}$ of the rotations at the ends of the h-th element are easily derived from the node displacements.

iii) An increment of bending moment at the member ends

$$\underline{\Delta M} = \begin{Bmatrix} \Delta M_i \\ \Delta M_j \end{Bmatrix} = \underline{f}^{-1}\, \underline{\Delta\theta} \qquad (06)$$

is associated with the rotations $\underline{\Delta\theta}$. In a perfectly elastic elements these moments would be associated with an elastic rotation

$$\underline{\Delta\theta}_e = \underline{f}_e\; \underline{\Delta M} \qquad (07)$$

$\underline{f}_e$ being the member flexibility matrix calculated for $x = 0$ and $Q = 1$.

iv) The inelastic rotation increment is finally defined as

$$\underline{\Delta\theta}_p = \underline{\Delta\theta} - \underline{\Delta\theta}_e \qquad (08)$$

As $\underline{f}$ may change during the single time step, different alternative procedures are possible. They will be reported in a separate paper, but the first numerical experiments show a coincidence of the results in terms of damage indicators. It is worth noting, however, that $\underline{\Delta\theta}_p$ is not the inelastic rotation in the usual sense. In fact, $\Delta\theta_{pk}$ can also be different from zero at ends where plasticity has not developed, $\Delta\theta_k$ being the effect of the plastic zone spread at the other end.

Fragility Curves

Consider a frame excited by a given loading history. The dynamic analysis of the frame provides the displacements at each integration step. Then, Eq. (08) gives the value of the inelastic deformation at the member ends and the damage measure R(t) can be determined by Eq. (05) at each step.

Let the duration of the time loading history be T and let the load amplification factor be Y. As Y is increased, R(T) increases: the carrying capacity of the structure is the minimum value $Y_R$ of Y for which

$$Y = Y_R \Rightarrow R(T) = R^* \text{ at least in one element end} \quad (09)$$

If the structure characteristics, the time loading history and the member resistances $R^*$ are assumed to be random, also $Y_R$ is a random variable. According to the discussion of Ref. 1, the fragility curve of the structure can be regarded, under sufficiently wide hypotheses, as the cumulative distribution function of $Y_R$. This distribution function can be estimated by simulating a sample of mechanical problems, whose analysis provide a sample of $Y_R$. The cumulative frequency of this sample gives an approximation of the searched fragility. The results of a previous hazard analysis and the fragility curve can then be combined in the well-known convolution integral of the classical reliability theory in order to estimate the mean failure rate of the structure, i.e. its seismic reliability if the stochastic excitation is a ground motion.

## A NUMERICAL EXAMPLE

In order to have the possibility of comparing the fragility curves evaluated by the mechanical model summarized in Eq. (02) with the fragilities calculated with concentrated inelastic rotations, the numerical example has been developed for the reinforced concrete frame already used in previous studies (Refs. 8 and 3) and designed by Lai in Ref. (23).
Three sets of random parameters are considered:

i) the random parameters necessary to describe the motion(duration;central frequency and damping of the Kanai-Tajimi spectral density function);
ii) the vector of equicorrelated random parameters of resistance $R^*$ at the critical sections;
iii) the random parameters which define the behaviour of the mechanical system (mass, stiffness, damping ratio, hardening and the vector of yielding moments).

The coefficient c in Eq. (04) has been assumed to be 0.6.
A sample of 20 values of $Y_R$ (the minimum value of the earthquake intensity which causes the failure of the structure) has been obtained corresponding to 20 randomly simulated ground motions and sets of behaviour parameters. For each simulated set of parameters and for the corresponding simulated ground motion, the structure has been analysed using the DRAIN-2D algorithm with finite size plastic zones at the end sections of otherwise

linear elastic members. At each of the analysis step the program calculates the quantity R(t) of Eq. (05) for each critical section. If R(t) is greater than the simulated value of $R^*$, the element fails and hence the frame fails. In order to accurately estimate $Y_R$, it has been necessary in each of the 20 simulation cases to perform several calculations with different values of Y.

According to Ref. 1 and 3, an average spectral pseudovelocity has been used as seismic intensity measure:

$$Y = \frac{1}{\ln\gamma_\omega} \int_{\ln\omega_s - \ln\gamma_\omega}^{\ln\omega_s} S_v(\omega_s, \beta_s)\, d(\ln\omega) \tag{10}$$

where $S_v(\omega_s, \beta_s)$ is the pseudovelocity response spectrum ordinate for $(\omega_s, \beta_s)$; $\omega_s$ is the first natural frequency of the elastic system, $\beta_s$ is the associated damping ratio. The averaging interval parameter $\gamma_\omega$ has been taken equal to 2.

The empirical distribution of $Y_R$ obtained by this simulation is shown in Fig. 3, where it is compared with a lognormal fit. The points $Y'_R$ obtained in Ref. 3 with the inelastic deformations concentrated at the member ends and the relevant lognormal fit are also reported in Fig. 3. The abscissa is the ratio $Y_R/\grave{E}[Y'_R]$, where $\grave{E}[Y'_R]$ is the sample average of $Y'_R$. The sample means $\grave{E}[\;]$, the sample coefficients of variation $\grave{\delta}$, and the empirical coefficients of variation $\grave{\delta}^*$ associated with the fits of Fig. 3 are:

$$\begin{array}{lll} \grave{E}[Y_R] = 91.700 \text{ cm/s} & \grave{\delta}_{Y_R} = 0.333 & \grave{\delta}^*_{Y_R} = 0.403 \\ \grave{E}[Y'_R] = 96.225 \text{ cm/s} & \grave{\delta}_{Y'_R} = 0.312 & \grave{\delta}^*_{Y'_R} = 0.424 \end{array} \tag{11}$$

Figure 3 shows a substantial coincidence of the fragilities obtained by using the procedures of Eqs. (01) or (02). For a single simulated problem, the procedure taking into account the finite size of the plastic zones has given results in terms of $Y_R$ generally lower than the values obtained by the plastic hinge approach. In some few cases, however, $Y_R$ has been found greater than $Y'_R$; a larger dissipation (i.e. higher values of the damage indicators), in fact, is obtained for the beams and this influences the damage accumulation in the column which are the failing elements. In other cases these higher values of the damage indicators are common to beams and columns and $Y_R < Y'_R$. Regarding to the sample central values of the distributions of $Y_R$, the mean value of $Y_R$ (Eq.(11)) is lower than $\grave{E}[Y'_R]$ of about 5%, while a higher coefficient of variation has been found.

## CONCLUSIONS

A procedure able to calculate the fragilities of reinforced concrete frames without neglecting the spread of plastic zone has been formulated and implemented for a general purpose program.

Comparison of the results obtained by this procedure and a plastic hinge idealization do not give large discrepancies, even if the concentration of the inelastic deformation is proved to be slightly unconservative. The main feature of the method is that the moment-curvature constitutive law can be derived by elementary material data and geometric section properties, whereas the moment-rotation relationship has to be derived from experimental data which may not be available in some practical situations.

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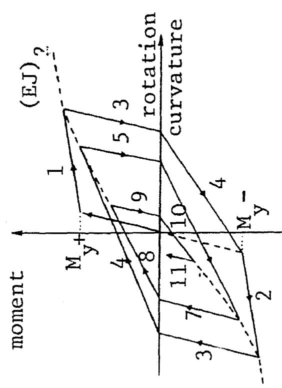


Fig. 1 - Takeda model for hysteretic constitutive laws

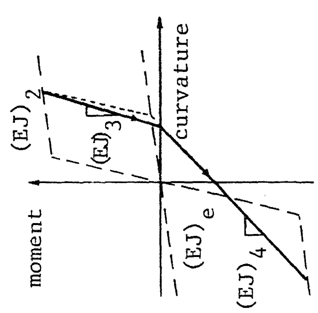


Fig. 2 - Unloading and realoading stiffnesses

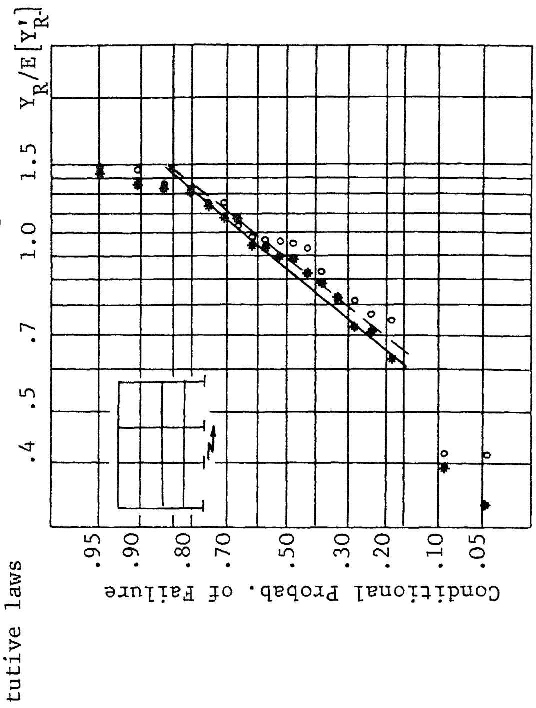


Fig. 3 - Fragility curves for the four-storey three-span r.c. frame: o plastic hinge; * model taking account of the finite size of the plastic hinges. The dashed and solid straight lines are the corresponding lognorm.fits

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