HYSTERESIS MODELS OF REINFORCED CONCRETE FOR EARTHQUAKE RESPONSE ANALYSIS

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SUMMARY

The effect of various stiffness parameters on nonlinear earthquake response amplitude of single-degree-of-freedom systems is studied. When the hysteresis model responds well in an inelastic range, the level of yield resistance and the capacity of hysteresis energy dissipation are shown to be the most important parameters of the system. If the two parameters are chosen comparable, the response amplitudes and waveforms are not significantly affected by the detail differences in the various hysteresis models.

INTRODUCTION

It was more than twenty years ago when the second World Conference on Earthquake Engineering was held in Tokyo in 1960, where many research papers reported the elasto-plastic response of simple systems using then-developing digital computers, placing more emphasis on the development of numerical procedures. Since numerical methods were made easily available to experimental investigators in the late 1960's, many "realistic" hysteresis models have been developed by the researchers, leading a maze of models, each claiming the best fit to the observed curves, without understanding which hysteresis properties might have significant influences on the earthquake response. This paper studies the effect of different hysteresis and stiffness parameters on the earthquake response. Hysteresis models used are limited to those which simulate dominantly flexural behaviour of the reinforced concrete.

HYSTERESIS MODELS FOR REINFORCED CONCRETE

A typical load-deflection relation of the reinforced concrete is shown in Fig.1 (Ref.1), where the deformation was governed by flexure. The general hysteretic characteristics can be summarized as follows: (a) Stiffness changed with the flexural cracking of concrete and the tensile yielding of longitudinal reinforcement; (b) When repeated at the same newly attained maximum amplitude, the loading stiffness in the second cycle was noticeably lower than that in the first cycle, although the resistances at the peak displacement were alomst identical; (c) Average peak-to-peak stiffness decreased with maximum displacement amplitude; and (d) The hysteretic relations depended on a loading history. A hysteresis model must be able to provide the stiffness and resistance under any displacement history.

Many hysteresis models have been developed in the past for dominantly flexural behaviour of the reinforced concrete (Ref.2); notably (a) Ramberg-Osgood model (Ref.3), (b) Clough (Degrading Stiffness) model (Ref.4), (c) Degrading Trilinear model (Ref.5), (d) Takeda model (Ref.6), (e) (Degrading) Bilinear model (Ref.7), and others. The primary curve, load-deformation curve under monotonically increasing deformation, may be given by a "curve"(Ramberg-

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Osgood model), a "bilinear" line (Clough and Bilinear models), or a "trilinear" line (Degrading Trilinear and Takeda models) with stiffness changes at "cracking" and "yielding" points. Some hysteresis models are elaborate, including many hysteresis rules, while others are simple. Note that the complicatedness of a model requires a large memory, but that it does not necessarily lead to a longer computation time because a very limited number of hysteresis rules are referred to at a given loading situation.

The Clough model is modified to include the degradation in unloading stiffness Kr from the initial yield stiffness Ky with maximum displacement amplitude Dm beyond the yield displacement Dy;

$$Kr = Ky \times ABS(Dy / Dm)^{a} \qquad (1)$$

where "a" is called the unloading stiffness degradation index (USD index), and the value might vary from 0.0 to 0.5 for the reinforced concrete. This feature existed in the Takeda model.

STIFFNESS AND DAMPING PROPERTIES

The response of single-degree-of-freedom (SDF) systems is computed by a computer program "SDF" to study the effect of different stiffness properties and hysteresis models. The Newmark's design criteria (Ref.8) are adopted to determine the yield resistance of SDF systems; i.e., the linear acceleration response spectrum at a 5 percent of critical damping is divided either (a) by an allowable ductility factor of 4.0 if the yield period, period corresponding to secant stiffness at the yield point, is greater than 0.5 sec, or (b) by the square root of (2x4-1=7) if the period less than 0.5 sec. The yield displacement is determined from the yield resistance and stiffness; the post-yield stiffness is assumed to be 10 percent of the yield stiffness. For a trilinear primary curve, the uncracked stiffness is 2.0 times the yield stiffness; and the cracking resistance one-third the yield resistance. The parameters of the Ramberg-Osgood model are chosen so that the yield point and the resistance at the allowable ductility are the same as the other models.

The viscous damping coefficient is assumed to vary proportional to the instantaneous stiffness so that energy dissipation by damping should decrease with increasing hysteretic energy dissipation. The stiffness proportional damping was found more desirable in analytically reproducing the shake table response of model structures (Ref.9). The damping factor of a system is chosen to be 5 percent of the critical at the yield stiffness.

EFFECT OF STIFFNESS PARAMETERS

The effect of stiffness parameters (cracking and yield resistance levels, initial and post-yielding stiffnesses, and USD index) of the Takeda model on ductility demand, maximum calculated displacement divided by the yield displacement, of SDF systems is studied. Since the response of short-period systems is sensitive to some parametric values, the response of systems with yield periods of 0.14 and 1.13 sec are computed under the N21E component of Taft (1952) earthquake record.

EFFECT OF INITIAL STIFFNESS AND CRACKING RESISTANCE: The initial stiffness of the reinforced concrete is approximately 1.5 to 4.0 times the yield stiffness, and cracking force level from 0.1 to 0.7 times the yield resistance. The ductility demand is not affected by the variation in the initial stiffness (Fig.2) and the cracking force level (Fig.3), as long as the ductility demand is greater than 4.0. Note that the two parameters might influence the maximum response amplitude if the ductility demand is less than or around unity. The hysteretic energy dissipation capacity of a Degrading Trilinear model is sensitive to the choice of a cracking point relative to the yielding point, and the response is significantly affected by the parameters.

EFFECT OF YIELD RESISTANCE LEVEL: The level of yield resistance is one major factor that influences the amplitude of maximum response. Figure 4 shows the variation of maximum response with yield resistance. With increasing yield resistance, the ductility demand is significantly reduced, especially for systems with short periods. Note that the value of yield displacement increases proportional to the yield resistance. Consequently, the maximum displacement amplitude does not decrease so much as the ductility demand does.

EFFECT OF POST-YIELD STIFFNESS: The strain hardening after yielding influences the hysteresis energy dissipation capacity; i.e., for a high post-yield stiffness, a large strain energy is stored at a given displacement. Maximum response decreases with an increasing post-yielding stiffness (Fig.5), noticeably in short-period systems and insignificantly in long-period systems. The response amplitude decreases rapidly when the post-yield stiffness increases 4/80 to 16/80 times the yield stiffness.

EFFECT OF UNLOADING STIFFNESS DEGRADATION INDEX: The USD index controls the fatness of a hysteresis loop and also the plastic residual deformation. Maximum response increases with an increasing value of the index (Fig.6), and this tendency is remarkable for short-period systems. The system's capacity to dissipate kinetic energy is known to have a conspicuous influence on the maximum response of a short period structure. The effect of the USD index is insignificant on the response amplitude when the maximum occurs at the initial part of the earthquake. However, the index generally influences the maximum response, response waveform, residual displacement, and hysteresis shape.

EFFECT OF HYSTERESIS MODELS

The effect of hysteresis properties are examined by using five hysteresis models: (a) (Degrading) Bilinear model, (b) Clough model, (c) Takeda model, (d) Degrading Trilinear model, and (e) Ramberg-Osgood model. The yield period is varied from 0.1 to 1.6 sec. Because the fatness of a hysteresis loop influences the response amplitude, the USD index values of 0.0 and 0.5 are used in applicable models. The hysteresis energy dissipation index (Eh) is introduced to express the capacity to dissipate hysteretic energy dW per cycle at peak resistance Fm and displacement Dm,

$$Eh = dW \,/\, 2\pi\, Fm\, Dm \qquad (2)$$

The values Eh are calculated at a displacement ductility of 4.0 for the models; (a) Degrading Bilinear model (Eh=0.33 for a=0.0; Eh=0.19 for a=0.5), (b) Clough model (Eh= 0.21 for a=0.0; Eh=0.11 for a=0.5), (c) Takeda model (Eh=0.23 for a=0.0; Eh=0.14 for a=0.5), (d) Degrading Trilinear model (Eh=0.11), and (e) Ramberg-Osgood model (Eh=0.28). The ductility demands of the models are shown in Fig.7 in two groups; Eh value greater than 0.20 (fat-loop models) and less than 0.20 (thin-loop models).

The reliability of the Newmark's design criteria is examined using four different earthquake motions: the El Centro (1940) NS and EW records and the Taft (1952) N21E and S69E records. For the criteria to be exact, the ductility demand should be equal to the allowable value of 4.0. Although the ductility demand falls close to the allowable value in a wide range of yield periods for fat-loop models under the El Centro (NS) motion (Fig.7), the other three motions caused ductility demands, especially from thin-loop models, much greater than the allowable value. The design criteria are not satisfactory in a very short-period range and for thin-loop models. The effect of hysteresis energy dissipation needs to be included in the design criteria.

The distribution of maximum response with periods is different from one earthquake to another, showing irregular shapes, although the resistance of each model is determined for individual earthquake motion. On the other hand, the distribution of maximum response with periods is similar from one model to another for a given earthquake motion. In a fat-loop group, the Takeda and Clogh models give comparable maximum response amplitudes; the regular bilinear model, in general, demands less ductility than the Clough and Takeda models probably because the Eh value of the Bilinear model is larger than those of the two models. In a thin-loop group, the four models developed comparable ductility demands. In other words, maximum response amplitudes are not as sensitive to detail difference in hysteretic rules of the models, but rather are influenced by more basic characteristics of hysteresis loops, such as the shape of a primary curve and the fatness of a hysteresis loop.

The response waveforms of different hysteresis models under the El Centro (NS) 1940 motion are compared in Fig.8 for the two groups. The yield period is arbitrarily chosen to be 0.4 sec, and the yield resistance level is reduced to 60 percent of the standard model. The hysteresis relations obtained are shown in Fig.9.

In fat-loop systems, the Ramberg-Osgood, Clough and Takeda models show similar hysteresis relations. The maximum and second largest displacements are observed at the same instances for the four models. The Takeda model shows a short-period oscillation at 1.0 sec, since the model has a trilinear primary curve. The Bilinear model oscillates in a period shorter than the other models, between 2.5 and 4.5 sec, attributable to the non-degrading nature of stiffness. The Bilinear and Ramberg-Osgood models developed residual displacment in the negative direction at 7.0 sec, whereas the Clough and Takeda models developed positive residual displacement. The latter two response waveforms are very similar each other.

Thin-loop models show displacement waveforms distinctly different from those of fat-loop models, oscillating regularly in larger amplitudes and in longer periods, which points out the importance of the energy dissipating properties on the response. The Clough, Takeda, and Degrading Trilinear models produced waveforms very similar one another, while the Degrading Bilinear models exhibited a different waveform. The Clough and Takeda models developed very similar hysteresis relations although the Takeda model had a trilinear primary curve. This may be attributable to the fact that a large-amplitude oscillation occurred at an early stage of the earthquake motion. In other words, the behaviour of the Takeda and Clough models could be different if a small oscillation continues for a long duration, or if the yielding does not occur during an earthquake.

CONCLUSIONS

Some stiffness and hysteresis properties of a model have a strong influence on earthquake response amplitude; i.e., (a) the level of yield resistance and the fatness of a hysteresis loop significantly influence the maximum amplitude, (b) post-yielding stiffness has a medium influence, and (c) initial stiffness and cracking force level have little influence for a ductility demand higher than 4.0. The effect of these parameters is distinct in short period systems. If the stiffness properties of a primary curve and hysteresis energy dissipation capacity are comparable, the response amplitudes and waveforms cannot differ appreciably from a hysteresis model to another. A complicated hysteresis model should not be penalized because the complicatedness does not require a longer computation time.

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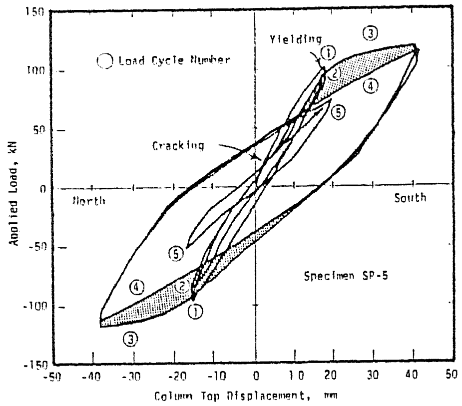


Fig.1: Hysteresis

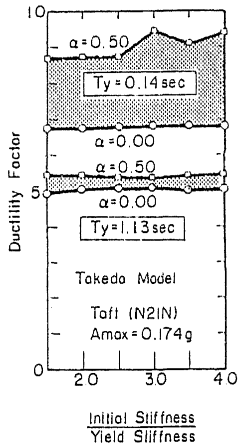


Figure 2

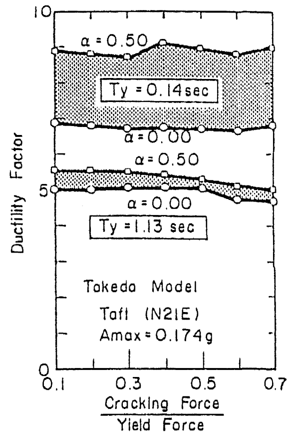


Fig.3: Crack Level

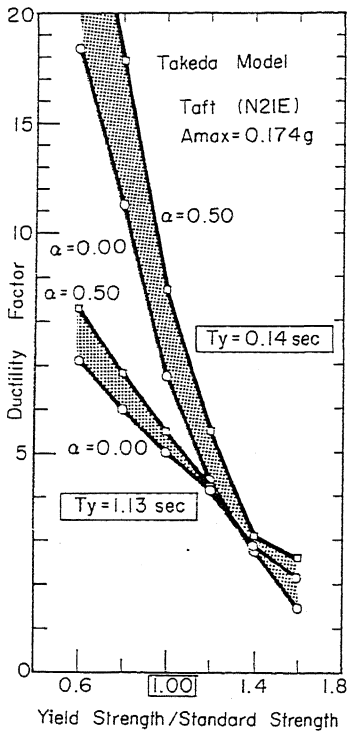


Fig.4: Yield Level

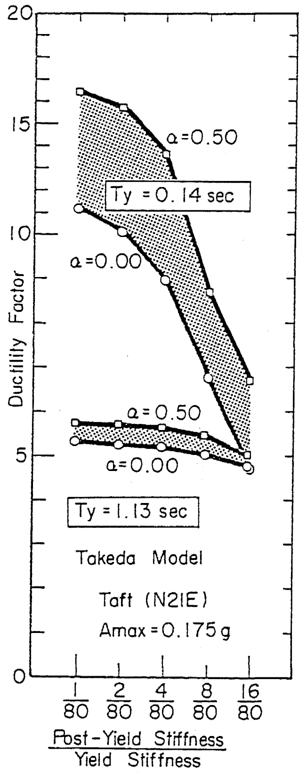


Fig.5: Post-Yield Stiffness

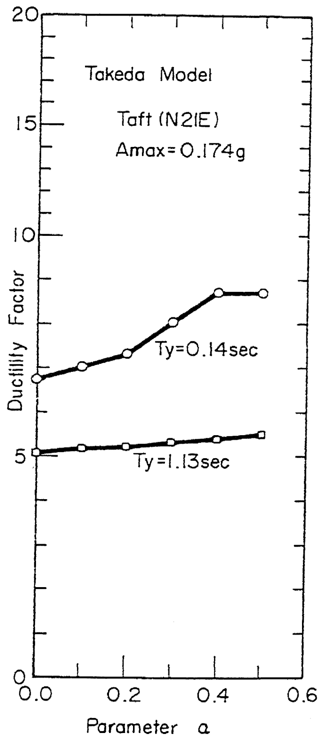


Fig.6: USD Index

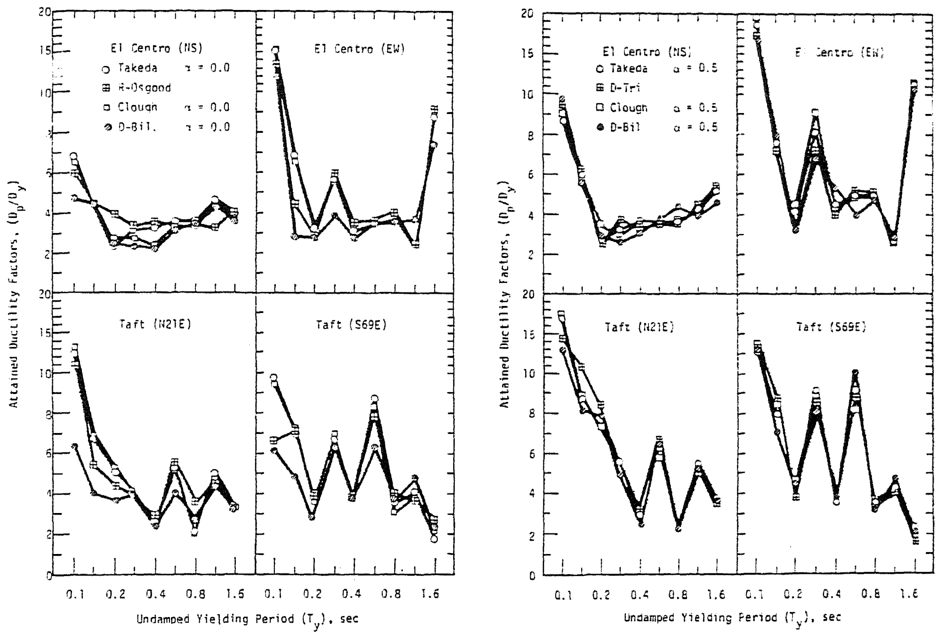
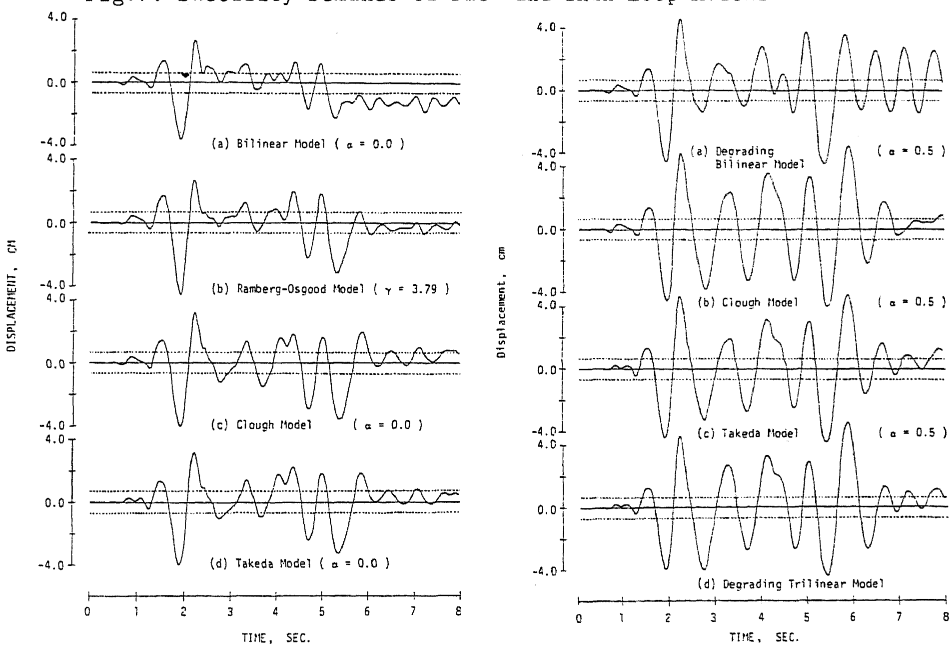
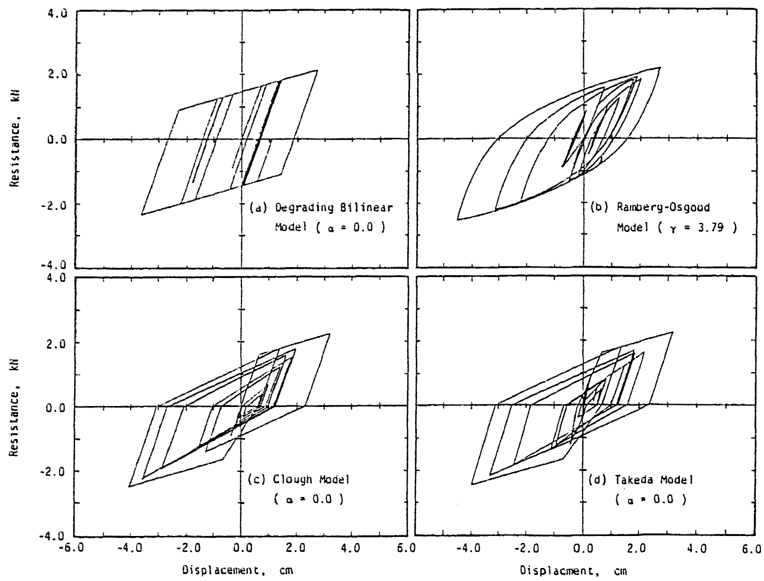


Fig.7: Ductility Demands of Fat- and Thin-Loop Models

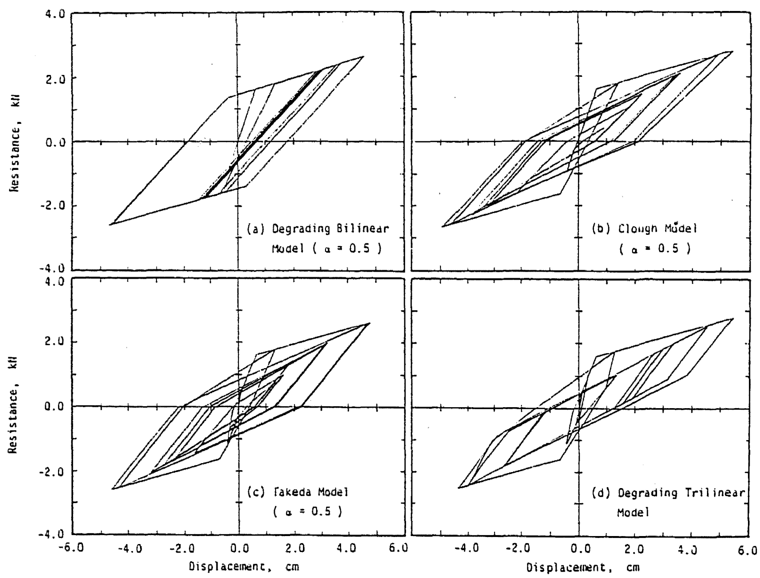


(a) Fat-Loop Models (b) Thin-Loop Models

Fig. 8: Response Waveforms of Different Hysteresis Models



(a) Fat-Loop Models



(b) Thin-Loop Models

Fig.9: Hysteresis Relations of Different Models