

MATERIAL AND GEOMETRIC NONLINEAR EARTHQUAKE ANALYSIS OF STEEL
FRAMES USING COMPUTER GRAPHICS

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SUMMARY

A promising analytical model for nonlinear dynamic analysis of steel frames is described. This approach is based on an updated Lagrangian formulation in conjunction with force-space, concentrated plasticity. Kinematic strain hardening behavior is modelled by the bounding surface concept. The analysis is implemented in a highly interactive, adaptive system using computer graphics and a super-minicomputer. Selected examples illustrate the effectiveness of the analysis strategy described.

INTRODUCTION

The main objective of the investigation reported is to establish a nonlinear dynamic computational procedure effective for the design of steel frames. This objective leads to four fundamental requirements: (1) an element formulation which accurately reflects structural and material behavior such as the Bauschinger effect; (2) a simple and concise procedure to evaluate the structural tangent stiffness matrix; (3) a flexible and efficient implementation in an interactive transient analysis system; and (4) interactive computer graphics to display current information regarding the analysis and results in real time.

A promising model that satisfies the above requirements is based on the common assumption of concentrated plasticity. However, a new aspect is that material nonlinearity is accounted for by a bounding-surface kinematic hardening model in force space.

MODEL DESCRIPTION

The proposed stiffness formulation is in some ways similar to the Porter and Powell elastic-perfectly plastic model (Ref. 1) but differs in that it includes the effect of strain hardening. A concise and simple formulation of the element stiffness matrix is given in detail in Ref. (2). However, it is appropriate here to mention some of the assumptions, principles, and strategies used in the current approach:

1) Concentrated plasticity. All the plastic deformations are confined to zero-length plastic zones at the ends of beam-column elements.
2) Force-space yield surface. At each stage of plastic deformation there is a unique yield surface (y.s.) in the force space ($F(S,a) = k$) so that strain hardening deformation takes place only for $F(S,a) > k$. $\{S\}$, $\{a\}$, and k are, respectively, the vector of element end forces, the vector of

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y.s. offset, and y.s. size.

3) Incremental displacement and force decomposition principles. During the plastic flow the total displacement vector $\{dq\}$ is decomposed to elastic and plastic displacement vectors, $\{dq\} = \{dqe\} + \{dqp\}$. The force vector $\{S\}$ is also decomposed into components tangent and perpendicular to the y.s., that is, $\{dS\} = \{dSt\} + \{dSn\}$.

4) Associated flow rule and Drucker's normality criterion. This assumes that the $\{dqp\}$ is perpendicular to the y.s., i.e., $\{dqp\} = [G]\{m\}$, in which $\{m\}$ is a two-element vector containing the magnitudes of the plastic deformation at the two element ends and $[G]$ is the y.s. gradient coefficients matrix.

5) Geometric nonlinearities. The force vector is related to the elastic displacement vector by the total stiffness matrix $[Kt]$ such that $\{dS\} = [Kt]\{dqe\}$, in which $[Kt]$ is the summation of the conventional elastic stiffness matrix $[Ke]$ and the geometric stiffness matrix $[Kg]$. The latter is based on an updated Lagrangian formulation and the assumption of large-displacement, small-strain behavior.

6) Hardening coefficient matrix $[Kh]$. It is assumed that $\{dSn\}$ is related to the plastic deformation matrix as: $\{dSn\} = [Kh]\{dqp\}$. $[Kh]$ is defined as a diagonal matrix of the plastic moduli coefficients and is given in a concise form. For example, neglecting shear effects, $[Kh]$ for a 2D beam-column element has the form

$$[Kh] = \text{diagonal } [Kapl \ 0 \ Kfpl \ Kap2 \ 0 \ Kpf2]$$

in which $Kapi$ and $Kfpi$ are the axial and flexural plastic moduli at the end i .

From the above concepts, expressions for the plastic deformation magnitudes and the hardening reduction stiffness matrix $[Kp]$ are obtained (Ref. 2):

$$\{m\} = ([G]^T[[Kt] + [Kh]][G])^{-1}[G]^T[Kt]\{dq\} \quad (1)$$

$$\{dS\} = [[Kt] - [Kt][G]([G]^T[[Kt] + [Kh]][G])^{-1}[G]^T[Kt]]\{dq\} \quad (2)$$

or

$$\{dS\} = [[Kt] + [Kp]]\{dq\} \quad (3)$$

The plastic deformation magnitudes $\{m\}$ serve as indicators for unloading if negative values are obtained. In such a case, the force point is no longer constrained to remain on the loading surface and structural stiffnesses are reformed to reflect elastic unloading.

As mentioned above $[Kh]$ is a diagonal matrix that contains plastic moduli coefficients for the element's two ends. In the current investigation the following formulae are used for the plastic axial and flexural moduli at the i th end of an element:

$$Kapi = Bli(EA/L) \quad \& \quad Kfpi = B2i(6EI)/(L(2+Mi/Mj)) \quad (4)$$

in which Mi and Mj are the updated bending moments at the element ends.

Equation 4 is derived from the virtual work principle and is based on the assumption that the curvature distribution along the beam is mainly a function of the end moments ratio since, in this work, all loading is applied only at the joints.

At initiation of plastic behavior, the B_i 's must be infinite to achieve a smooth transition from the elastic into the plastic range. On the other hand, the B_i 's are zero for the elastic-perfectly plastic case. In the latter case the formulation will lead essentially to the Porter and Powell approach which has been used by Orbison and others (Ref. 3).

PLASTICITY MODEL

The approach selected for detailed development because it provides a smooth hardening model and relatively good computational efficiency is the bounding surfaces approach originally proposed by Dafalias and Popov (Ref. 4). In the current work this model is extended to model behavior in force space.

In this model the elastic region is represented by the interior of a loading surface F that represents force interaction at the start of yield. This surface is always enclosed by a second surface called the bounding surface which may also move in force space (Fig. 1).

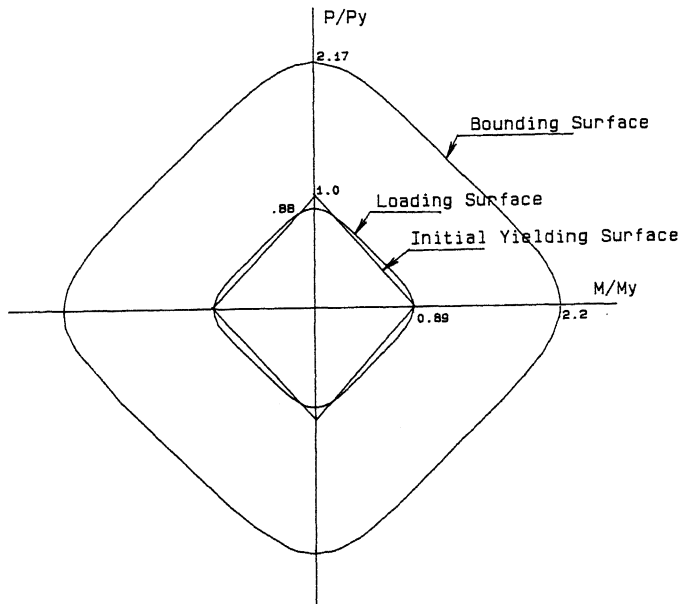


Fig. 1 Use of the Same Single-Equation Surface to Idealize Both the Loading and the Bounding Surfaces.

The loading surface may translate according to an appropriate hardening assumption. In the current investigation, the direction of the translation is specified, as suggested by Mroz (Ref. 5), as the unit vector along the line connecting two points, {S} and {S'}, one on each of the surfaces, characterized by identical outward unit normals.

It is possible for the loading surface to contact the bounding surface but not intersect it, and the two surfaces move together while in contact. However, if elastic unloading takes place, the loading surface detaches from the bounding surface and moves in an inward direction, while the bounding surface is assumed stationary. It is possible to allow for gradual change of surface sizes. The bounding surface may also be allowed to move but with a slower rate. However, in the present implementation the bounding surface is assumed fixed except when in contact with the loading surface, and both surfaces are kept at their original size.

The plastic moduli depend on the distance d between {S} and {S'} as suggested by Dafalias (Ref. 4) and shown in Fig. 2:

$$B1 = K11(1 + K12 \frac{d}{d_{in} - d}) \quad (5)$$

$$B2 = K22(1 + K21 \frac{d}{d_{in} - d}) \quad (6)$$

in which d_{in} is the value of d at the most recent initiation of yield. $K11$, $K12$, $K21$, and $K22$ are coefficients to be specified by the user according to the available experimental data. The updating of d_{in} is essential, since it accounts for the effects of the recent past history.

The surface used in the current approach was developed at by Orbison (Ref. 3) to model plastic hinge formation of steel cross sections,

$$1.15 p^2 + m^2 + 3.65 p^2 m^2 = k \quad (7)$$

in which $p = (P - a1)/P_y$ is the ratio of the axial force to the yield load, $m = (M - a2)/M_y$ is the ratio of the strong axis bending moment to the corresponding yield moment, and $a1$ and $a2$ are the current offsets of the surface in the P and M directions, respectively.

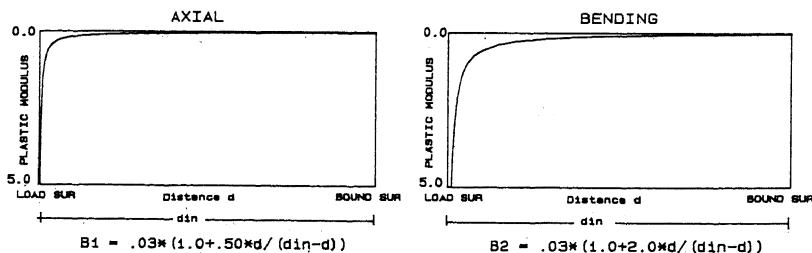


Fig. 2 Plastic Modulus vs. Distance from Bounding Surface (d).

Figure 1 shows the application of Eq. (7) to model both the loading and the bounding surfaces by scaling it to two different sizes. It is shown that, although the equation shape is based on the assumption of complete plastification of the cross section, it gives an acceptable approximation to the initial yielding surface when appropriately scaled.

ANALYSIS IMPLEMENTATION

The above model was implemented in a two-dimensional dynamic nonlinear analysis program, initially developed by Gattass (Ref. 6) and expanded by the first author. This program performs static, dynamic, modal, and buckling analysis with real-time graphical feedback. The program uses updated Lagrangian procedures, a skyline solver, and dynamic memory allocation.

Implicit direct integration analysis is used to solve the coupled equations of motion for the system described above. The time increment size is controlled during the analysis so a change of the element hardening (onset of yielding or contact with the bounding surface) does not occur within the increment itself.

The analysis program is incorporated in a comprehensive interactive graphic analysis and design system which provides an environment with considerable flexibility for the performance of nonlinear analysis (Ref. 6). In this environment, analysis may be performed either in a time-sharing mode or assigned to a batch queue for execution at a later time. The interactive graphics techniques to control the flow of the analysis and to monitor results are based on a hardware configuration that consists of a vector refresh display terminal, a digitizing tablet, an editing terminal, and a super-minicomputer.

The program consists of various menu pages such as the one shown in Fig. 3. The structural response is monitored in 3 different viewports, and the user may switch information from one to another. The user interacts with the program by pointing to the commands on the menu either to move to another menu or to perform a required procedure. Special parameter pages permit user specification and checking of the experimental data of eqs. (5) and (6) and all the other parameters needed for performing nonlinear analysis. For the batch analysis mode, a provision is made to save and display the result output files with full advantages of the graphics feedback originally developed for real-time analysis. More details of the program are given in Ref. (2).

EXAMPLES

Two abbreviated examples are selected to indicate the effectiveness of the program developed from the above ideas. For both examples, the hardening parameters are chosen as follows: $K_{11} = 0.03$, $K_{12} = 0.5$, $K_{21} = 2.0$, and $K_{22} = 0.03$. Figure 2 shows the variations of the plastic moduli along the distance between the loading and the bounding surfaces. Geometric nonlinearity is considered for all incremental analyses. Zero values are assigned for damping parameters. The CPU times given are for

a VAX 11/780.

A single-bay, one-story portal frame is subjected to vertical and horizontal concentrated loads as shown in the upper left monitor of Fig. 3. These loads follow a cyclic sinusoidal load history with an amplitude of 4.0 kips and a period of 5 seconds. A light-weight steel cross section of W14x22 was used for both the beam and columns. The analysis was performed in 200 time steps with an average time of 0.081 seconds and a total CPU time of 90 seconds. Figure 3 also shows the force-point trace of the lower end of the right column at the end of the analysis. The upper monitors portray the bending moment diagram and the relationship of the

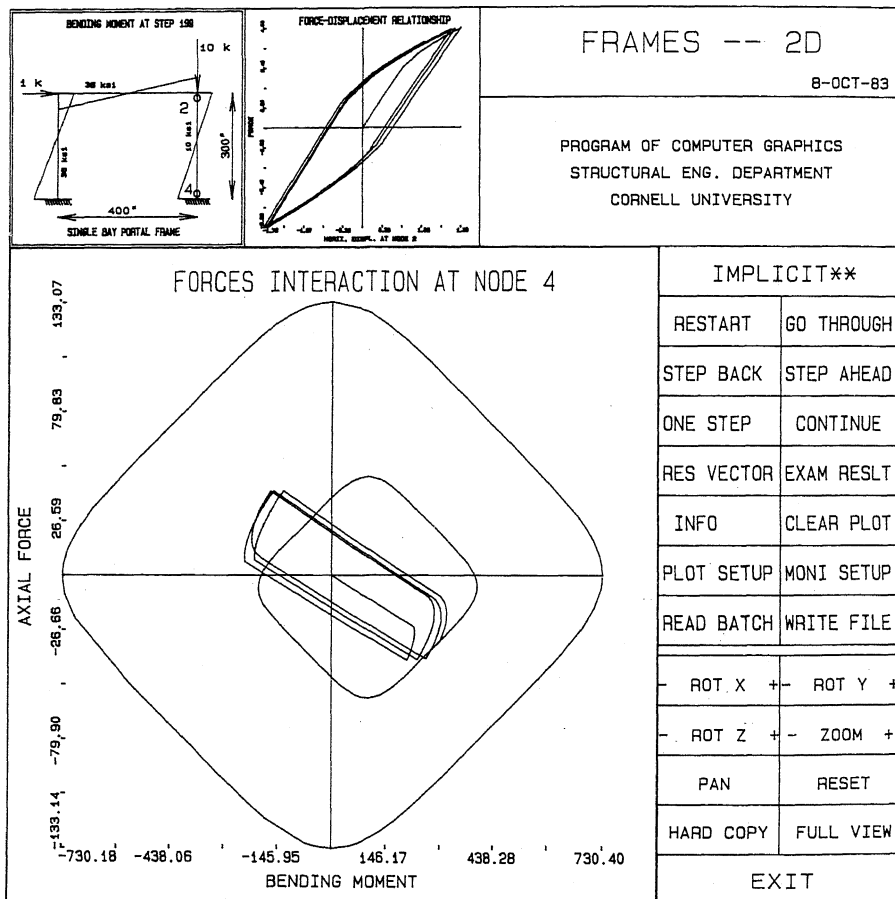


Fig. 3 Analysis Information for Example 1.

applied force vs. the horizontal displacement of the upper right node.

A four-story two-bay building is shown in Fig. 4. A concentrated mass of $0.1 \text{ k-sec}^2/\text{in.}$ is attached to each joint. The structure is subjected to the first 5.0 seconds of the N-S component of the 1940 El Centro earthquake. The moment history of the lower end of the fourth story central column is shown in Fig. 5 for both elastic and nonlinear cases. The CPU time for the linear analysis was 35 seconds for 62 steps of 0.1 seconds each, while the nonlinear analysis was performed in 102 seconds with 112 steps. Plastification was observed in eleven different locations.

CONCLUSION

The approach described in this extended abstract incorporates, in the authors' view, a promising tool to provide a clear insight into nonlinear dynamic behavior of steel frames. The concise formulation of the tangent stiffness matrix along with the bounding surface approach which employs realistic and admissible single-equation yield surfaces permit effective and reasonably fast nonlinear analysis. The implementation in an interactive computer graphics environment has an obvious advantage in that it allows the user to trace and understand structural nonlinear response.

Acknowledgement

Research presented herein was made possible by the Grants No. PRF-7815357 and No. CEE-8117028 from the U. S. National Science Foundation to both the Department of Structural Engineering and Program of Computer Graphics. Appreciation is extended to project leader W. McGuire, Program Director D. Greenberg, and colleagues C. Pesquera and M. Gattass.

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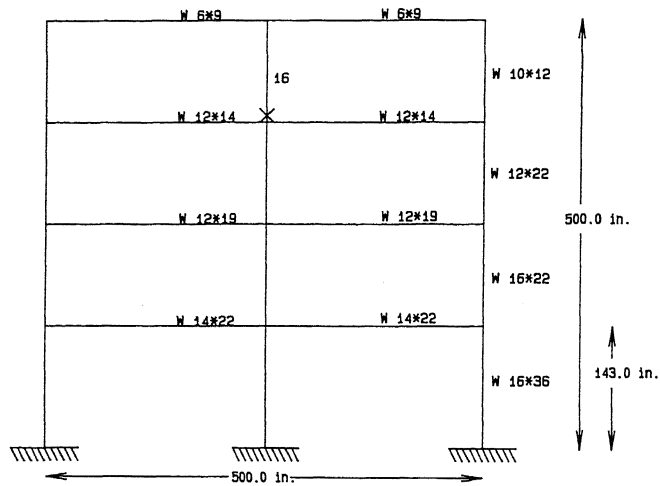


Fig. 4 Four-Story Frame.

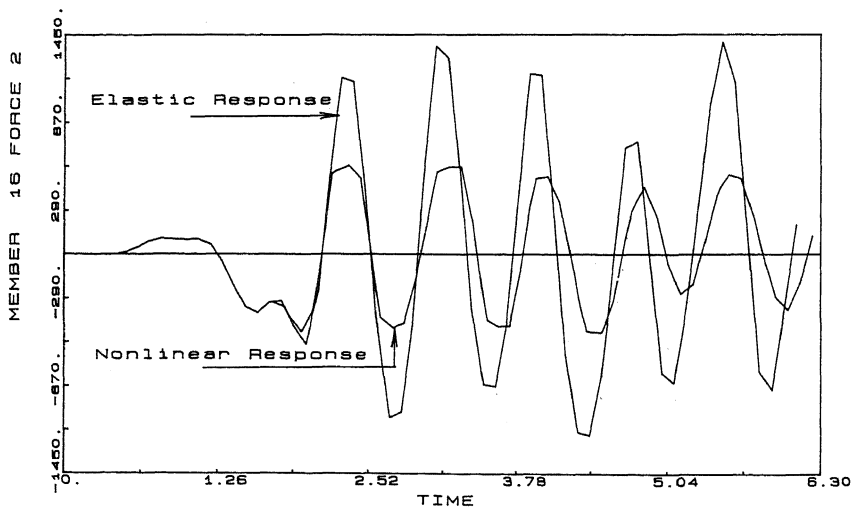


Fig. 5 Moment History at the Lower End of Member 16.