RESPONSE SPECTRUM METHOD FOR THE ANALYSIS OF NONLINEAR MULTISTORY STRUCTURES

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SUMMARY

A procedure is developed to extend the use of the conventional response spectrum method to the analysis of nonlinear multidegree-of-freedom systems. The procedure is based on the transformation of the equation of motion for nonlinear systems into a linear equation by considering the nonlinear terms in the original equation as additional external forces. It is limited, for simplicity, to elastoplastic systems of the shear-beam type. Its accuracy is verified by a numerical comparison with step-by-step integration solutions.

INTRODUCTION

Under the current practice of earthquake-resistant design, a structure depends on its inelastic behavior to resist severe ground motions. This means that the earthquake analysis of a structure cannot be performed realistically until its true nonlinear characteristics are taken into account. Presently, however, a reliable nonlinear analysis of multidegree-of-freedom (MDOF) systems is only possible through a step-by-step integration, a procedure that is cumbersome and costly in most cases, and certainly unacceptable for preliminary designs. Moreover, such numerical procedures do not facilitate a direct visualization of the structural behavior, and provide no easy assessment of the sensitivity of the structural response to design changes and variations in the characeristics of the input motion.

In the search for simpler methods and attracted by the simplicity and rationality of the response spectrum approach in the analysis of linear structures, several investigators have tried to extend the use of this method to nonlinear systems (Refs. 1-2). In general, the trend has been the use of inelastic response spectra in combination with elastic modal analysis (Refs. 1 and 3), or the use of elastic response spectra in combination with equivalent linearization techniques (Refs. 3-5). Thus far, none of these procedures appears to be completely satisfactory (Refs. 6-9).

The purpose of this paper is to propose an alternative approach to justify the use of a modal analysis/response spectrum approach in the analysis of nonlinear systems, and to prove that such a procedure may indeed lead to accurate results. For the sake of clarity, however, the derivation presented herein is restricted to elastoplastic shear systems defined by their masses, spring constants and floor yield strengths. At all response levels, the associated damping forces are assumed to remain linear.

APPROACH

By the definition of response spectrum, it is easy to see that it is also possible to construct response spectra for nonlinear systems. That is, if a certain load-deformation type of behavior is established, by a step-by-step integration analysis one can obtain for a given ground disturbance graphs that relate the maximum response of a series of simple oscillators to their natural

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frequencies, damping ratios, and a parameter that describes their nonlinearity. Examples of these are the inelastic response spectra obtained by Riddell and Newmark (1979) for systems with elastoplastic, bilinear, and bilinear with stiffness degradation load-deformation curves. Figure 1 shows a sample of such nonlinear response spectra. Notice that in these spectra the parameter used to describe the nonlinearity of the oscillators is the ductility factor.

By means of a nonlinear response spectrum, then, one can use information based on the initial (elastic) properties of a nonlinear system, together with some information about its nonlinear characteristics, to determine its maximum response. This fact leads one to think that it might be equally possible to estimate the maximum response of a nonlinear MDOF system from nonlinear response spectra if one knows its initial properties (i.e., mode shapes, natural frequencies and damping ratios) and the load-deformation characteristics of each of its floors. Intuitively, it seems that such an approach is feasible since the information is complete, although a modal decomposition would be necessary given that a response spectrum only gives the response of SDOF systems.

The method herein proposed is thus developed by deriving first an expression to represent analytically the ordinates of a nonlinear response spectrum in terms of a natural frequency, a damping ratio, and a ductility factor. Afterwards, the equation of motion for nonlinear MDOF systems is transformed into a linear equation by expressing the restoring forces in the nonlinear systems in terms of an elastic component and a corrective one, and by considering this corrective component as an additional external force. Under such a technique, the transformed equation of motion is then subjected to a modal decomposition, after which it is in turn transformed into a series of independent equations, each representing the equation of motion of a nonlinear SDOF system. Lastly, the solution to each of these independent equations is related to the nonlinear response spectrum of the excitation by comparing such a solution with the analytic representation of a nonlinear response spectrum and by introducing a definition for modal ductility factors.

ANALYTIC REPRESENTATION OF NONLINEAR RESPONSE SPECTRUM

Consider the SDOF system depicted in Fig. 2(a), which is defined by its mass M, damping constant C, initial stiffness K, yield strength F_y, and the load-deformation relationship shown in Fig. 2(b). The equation of motion for this system when subjected to a ground acceleration $\ddot{u}g(t)$ may be written as

$$M\ddot{x} + C\dot{x} + F_R(x) = -M\ddot{u}_g(t) \qquad (1)$$

where x denotes the displacement of the mass M, relative to the ground, and $F_R(x)$, a function of x that needs to be read from Fig. 2(b), represents the restoring force in the spring of the system. However, following the idea suggested by Geschwindner(1981) and Dungar(1982), one may note that for every point in the load-deformation curve the restoring force $F_R(x)$ can be written in terms of an elastic component and a reductive one as (see Fig. 3)

$$F_R(x)=Kx \qquad \text{if } x\leq x_y \qquad (2)$$
$$F_R(x)=Kx-Kx_y(x/x_y-1) \qquad \text{if } x\geq x_y \qquad (3)$$

Introducing a new variable μ_x defined as

$$\mu_x=x/x_y \qquad (4)$$

and noting that $Kx_y=F_y$, Eqs. 2 and 3 can be alternatively expressed as

$$F_R(x)=Kx-F_y(\mu_x-1) \tag{5}$$

Then, upon substitution of Eq.5 into Eq. 1 the equation of motion of the system in Fig. 2 can be written as

$$M\ddot{x} + C\dot{x} + Kx= -M[\ddot{u}_g(t)-F_y(\mu_x-1)/M] \tag{6}$$

or as

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x= -\ddot{u}_{eff}(t) \tag{7}$$

where $\omega=\sqrt{k/m}$ is the natural frequency of the system at a low response level, $\xi=C/2\omega M$ its damping ratio, and $\ddot{u}_{eff}(t)$ is interpreted as an effective ground acceleration defined as

$$\ddot{u}_{eff}(t)=\ddot{u}_g(t)-(F_y/M)(\mu_x-1) \tag{8}$$

It should be noticed that since μ_x is a function of x and x is in turn a function of t, $\ddot{u}_{eff}(t)$ is a function of time only.

Thus, the vibrational motion of a nonlinear SDOF system can be viewed as that of a linear one with the initial properties of the nonlinear, but subjected to an effective ground acceleration as given by Eq.8. If by the time being it is assumed that the time variation of μ_x is known in its entirety (i.e., from the step-by-step integration of Eq.1), then from Eq. 7 the displacement response of the nonlinear system results as

$$x(t)=-(1/\omega) \int_0^t e^{-\xi\omega(t-\tau)}[\ddot{u}_g(\tau)-(F_y/M)(\mu_x(\tau)-1)]\sin \omega'(t-\tau)d\tau \tag{9}$$

where $\omega'=\omega\sqrt{1-\xi^2}$, from which the maximum displacement response is simply the maximum value in the time-history of x(t). Notice, however, that this maximum represents an ordinate in the nonlinear response spectrum of $\ddot{u}_g(t)$ corresponding to a natural frequency ω, a damping ratio ξ, and a ductility factor $\mu=|x_{max}/x_y|=\max|\mu_x|$. That is, if $SD(\omega,\xi,\mu)$ denotes such an ordinate, one may write

$$SD(\omega,\xi,\mu)=\left|(1/\omega) \int_0^t e^{-\xi\omega(t-\tau)} [\ddot{u}_g(\tau)-(F_y/M)(\mu_x-1)]\sin \omega'(t-\tau)d\tau\right|_{max} \tag{10}$$

This expression can be considered the analytical representation of a nonlinear response spectrum.

MODAL DECOMPOSITION OF EQUATION FOR NONLINEAR MDOF SYSTEMS

If elastoplastic behavior is assumed for all the floors of a MDOF system, but each characterized by its own initial stiffness and yield strength, its vibrational motion can also be represented by the equation of motion of an elastic system subjected to an effective ground acceleration. To derive such an equation, consider, without any loss of generality, the three-degree-of-freedom system shown in Fig. 4 (a), whose springs have the load-deformation characteristics presented in Fig. 4(c). According to the free-body diagrams shown in Fig.4(b), the equations of motion for each of the masses of the system are

$$\begin{aligned} &M_1(\ddot{x}_1+\ddot{u}_g)+C_1\dot{x}_1+F_{R1}-C_2(\dot{x}_2-\dot{x}_1)-F_{R2}=0 \\ &M_2(\ddot{x}_2+\ddot{u}_g)+C_2(\dot{x}_2-\dot{x}_1)+F_{R2}-C_3(\dot{x}_3-\dot{x}_2)-F_{R3}=0 \\ &M_3(\ddot{x}_3+\ddot{u}_g)+C_3(\dot{x}_3-\dot{x}_2)+F_{R3}=0 \end{aligned} \tag{11}$$

where M_1, M_2, M_3 are such masses; C_1, C_2, C_3 its damping constants; F_{R1}, F_{R2}, F_{R3} the restoring forces in its springs as given by Fig. 4(c); and

x_1, x_2, x_3 the relative displacements of the masses M_1, M_2, M_3, respectively. But according to the procedure employed in the analysis of the SDOF system, the restoring forces can be written as

$$F_{R1}=K_1x_1-F_{y1}(\mu_{d1}-1)$$
$$F_{R2}=K_2(x_2-x_1)-F_{y2}(\mu_{d2}-1) \tag{12}$$
$$F_{R3}=K_3(x_3-x_2)-F_{y3}(\mu_{d3}-1)$$

where $\mu_{d1} = d_1/d_{y1}$, $\mu_{d2}=d_2/d_{y2}$, and $\mu_{d3} = d_3/d_{y3}$, in which d_1, d_2, d_3, and d_{y1}, d_{y2}, d_{y3}, are defined in the load-deformation curves in Fig. 4(c). Consequently, the equations of motion of the system can be expressed as

$$M_1\ddot{x}_1+(C_1+C_2)x_1+C_2x_2+(K_1+K_2)x_1+K_2x_2=-M_1\ddot{u}_g+F_{y1}(\mu_{d1}-1)-F_{y2}(\mu_{d2}-1)$$
$$M_2\ddot{x}_2-C_2x_1+(C_2+C_3)x_2-C_3x_3-K_2x_1+(K_2+K_3)x_2-K_3x_3=$$
$$-M_2\ddot{u}_g+F_{y2}(\mu_{d2}-1)-F_{y3}(\mu_{d3}-1) \tag{13}$$
$$M_3\ddot{x}_3-C_3x_2+C_3x_3-K_3x_2+K_3x_3=-M_3\ddot{u}_g+F_{y3}(\mu_{d3}-1)$$

which in matrix form can be put into the form

$$[M]\{\ddot{x}\}+[C]\{x\}+[K]\{x\}=-[M]\{\ddot{u}g\}+[\mu_d-1]\{Fy\} \tag{14}$$

where

$$[\mu_d-1]=\begin{bmatrix}(\mu_{d1}-1) & -(\mu_{d2}-1) & 0\\ 0 & (\mu_{d2}-1) & -(\mu_{d3}-1)\\ 0 & 0 & (\mu_{d3}-1)\end{bmatrix} \tag{15}$$

$$\{Fy\}=\{Fy1 \quad Fy2 \quad Fy3\}^T \tag{16}$$

and [M], [C], and [K] are the system's mass, damping, and stiffness matrices, respectively, based on initial properties. $\{x\}$ is the displacement vector and $\{\ddot{u}g\}$ is the ground acceleration vector.

Thus, as in the case of the SDOF system, the nonlinear response of a MDOF system can be interpreted as the response of an associated linear system subjected to the effective inertia forces:

$$\{F_I\}_{eff}=-[M]\{\ddot{u}_g\}+[u_d-1]\{F_y\} \tag{17}$$

Accordingly, Eq.14 accepts a modal decomposition. By expressing the displacment vector as

$$\{x\}=\sum_{r=1}^{3}\{\Phi\}_rY_r(t) \tag{18}$$

where $\{\Phi\}_r$ is the rth mode shape of the system when its initial properties are considered, and $Y_r(t)$ is an unknown function of time, after applying orthogonality conditions Eq. 14 simplifies to the following series of independent equations:

$$\ddot{Y}_r+2\xi_r\omega_rY_r+\omega_r^2Y_r=-(1/M_r^*)\,[\{\Phi\}_r^T[M]\{\ddot{u}_g\}-\{\Phi\}_r^T[\mu_d-1]\{F_y\}] \tag{19}$$

where M_r^* denotes the rth generalized mass.

Note once again that Eq. 19 is the equation of motion of a linear SDOF system subjected to the effective acceleration:

$$\ddot{u}_{eff}=-\{\Phi\}_r^T[M]\{\ddot{u}_g\}+\{\Phi\}_r^T[\mu_d-1]\{F_y\} \tag{20}$$

which can be alternatively expressed as

$$\ddot{u}_{eff}=\ddot{u}_g-F_{yr}(\mu_{dr}-1)/M_r^* \tag{21}$$

where $\mu_{dr}-1$ is the scalar

$$\mu_{dr}-1=(1/\alpha_r\ F_{yr})\ \{\Phi\}_r^T[\mu_d-1]\{F_y\} \tag{22}$$

in which α_r represents the rth participation factor of the system. Hence, the solution of Eq. 19 in terms of Duhamel integral results as

$$Y_r=-(\alpha_r/\omega_r)\int_0^t e^{-\xi\omega_r(t-\tau)}[\ddot{u}_g-F_{yr}(\mu_{dr}-1)/M_r^*]\sin\omega_r'(t-\tau)d\tau \tag{23}$$

where ω_r' signifies the rth damped natural circular frequency.

By comparing Eq. 23 with Eq.9, one can conclude that Y_r represents α_r times the nonlinear solution of a SDOF system with initial natural frequency ω_r, damping ratio ξ_r, and a μ_{dr} value given by Eq. 22. Consequently, the maximum value of Y_r can be obtained from the nonlinear response spectrum for $\ddot{u}_g(t)$. In other words, the maximum value of Y_r is given by

$$|Y_r|_{max}=\alpha_r\ \ SD\ (\omega_r,\xi_r,\mu_r^*) \tag{24}$$

where $\mu_r^*=\max|\mu_{dr}|$ and should represent the ductility factor associated to the rth mode of the system. According to Eq. 22, this modal ductility factor is then defined as

$$\mu_r^*=\max|1+(1/\alpha_r F_{yr})\{\Phi\}_r^T\ [\mu_d-1]\ \{F_y\}| \tag{25}$$

It is apparent, thus, that the response of a nonlinear MDOF system can be determined by a conventional modal analysis. The displacement response is given by Eq. 18 and, similar to the linear case, the maximum can be approximated by

$$\{X\}_{max}=\{\sum_{r=1}^{N}[\alpha_r\{\Phi\}_r\ SD(\omega_r,\xi_r,\mu_r^*)]^2\}^{1/2} \tag{26}$$

where $SD\ (\omega_r,\xi_r,\mu_r^*)$, $r=1,2,\ldots,N$, are ordinates in the inelastic response spectrum of the excitation under consideration.

It should be noticed that because the response of the nonlinear system is given by the analysis of a linear one subjected to an effective ground acceleration, the approximate rules to combine the modal responses of nonlinear systems should be the same.

MODAL DUCTILITY FACTORS

It is convenient to express the modal ductility μ_r^* in terms of the story ductilities of the system. To do this, one may note first that un upper bound to μ_r^* is $[1+\max|(1/\alpha_r\ F_{yr})\{\Phi\}_r^T[\mu_d-1]\{F_y\}|]$, where the maximum possible difference between the real value of μ_r^* and this upper bound is 2. Similarly, it is easy to see that an upper bound to this upper bound is obtained when all the μ_{dr} are substituted by their corresponding maximum values, i.e., the story ductility factors μ_i. In this manner, if a less conservative approximation is adopted, the modal ductility factors can be calculated by

$$\mu_r^* = 1+|(1/\alpha_r F_{yr}) \sum_i d\Phi_i(r)(\mu_i-1) F_{yi}| \tag{27}$$

where $d\Phi_i(r)=\Phi_i - \Phi_{i-1}$ represents the difference between the displacements of the ith and (i-1)th masses in the rth mode, and hence in matrix notation it can be expressed as

$$\mu_r^* = 1+|(1/\alpha_r F_{yr})\{(\mu_i-1)F_{yi}\}^T\{d\Phi\}_r| \tag{28}$$

where $\{d\Phi\}_r$ is the vector of modal distortions

$$\{d\Phi\}_r=\{\Phi_1 \quad \Phi_2-\Phi_1 \quad \Phi_3-\Phi_2\}_r^T \tag{29}$$

and

$$\{(\mu_i-1)F_{yi}\}=\{(\mu_1-1)F_{y1} \quad (\mu_2-1)F_{y2} \quad (\mu_3-1)F_{y3})\}^T \tag{30}$$

Note that since Eq. 27 is a function of the story ductilities μ_i, which in turn depend on the response under calculation, an iterative procedure is necessary to determine such modal ductility factors.

ILLUSTRATIVE EXAMPLE

To illustrate the application of the foregoing procedure, consider the system shown in Fig. 5 when subjected to the first 18.2 seconds of the S16E component of Pacoima Dam, 1971, earthquake . Elastoplastic behavior is assumed for each of its floors with the indicated initial stiffnesses and yield strengths. The corresponding natural frequencies mode shapes, and damping ratios on the basis of the initial stiffnesses are given in Table 1.

Assume now the following vector of floor ductilities: $\{\mu\} = \{2.37 \quad 4.63 \quad 8.23\}^T$. For this set of ductility factors Eq. 30 yields $\{(\mu_i-1)F_{yi}\}^T = \{479.5 \quad 544.5 \quad 361.5\}^T$. Then, since according to Eq. 29 the three $\{d\Phi\}_r$ vectors result as: $\{d\Phi\}_1 = \{0.47689 \quad 0.33146 \quad 0.83556\}^T$; $\{d\Phi\}_2= \{0.35883 \quad 0.02388 \quad -1.07127\}^T$; $\{d\Phi\}_3= \{0.16440 \quad -0.35537 \quad 0.23570\}^T$; using the definition given by Eq. 28 the three modal ductilities are: $\mu_1^*=3.03$; $\mu_2^*=2.35$; and $\mu_3^*= 1.59$. For these modal ductilities and from the nonlinear response spectra reported in Ref. 10 for Pacoima Dam, 1971, earthquake, the following spectral displacements are obtained: SD(1.0,0.02,3.03)=24.6cm; SD(1.749,0.035,2.35)=9.7cm; SD(3.218,0.064, 1.59)=4.3cm. Consequently, the maximum displacements of the system are

$$\{x\}_{max}=[\sum_{r=1}^{3} (\alpha_r\{\Phi\}_r SD(\omega_r,\xi_r,\mu_r^*))^2]^{1/2}=\{12.3 \quad 20.3 \quad 41.0\}^T \text{ cm}$$

With this set of maximum displacement one can compute now a new set of story ductility factors and proceed as before until the assumed ductility factors are close to the calculated ones to the degree of accuracy desired. For instance, with the above calculated displacements one obtains the story ductilities $\{\mu\} = \{2.44 \quad 3.70 \quad 5.75\}^T$, which are close to the assumed ones. Notice by inspection of Eq.27 that the modal ductilities are not very sensitive to changes in the story ductility factors. Thus, one may expect a rapid convergence in most cases.

COMPARATIVE ANALYSIS

To test the validity of the proposed procedure, the maximum displacement response of the 3-DOF system considered above is calculated with this procedure and compared with the solutions obtained by a numerical integration method. Three earthquake acceleration ground motions are considered: (a)first 15.18 seconds of the E-W component of El Centro, 1940 record; (b) first 26.67 seconds of the N86E component of the Olympia, 1949 record; and (c) first 18.2 seconds of the S16E component of Pacoima Dam, 1971 record. The comparison is presented in Table 2.

SUMMARY AND CONCLUSIONS

An approximate procedure has been presented to utilize the response spectrum method in the analysis of nonlinear multistory structures. The procedure involves the computation of mode shapes and natural frequencies on the basis of initial properties, the computation of modal ductility factors, and the use of nonlinear response spectra defined in terms of initial natural frequencies, damping ratios and ductility factors. The procedure is iterative since the required modal ductility factors depend on the ductility demands for a given earthquake excitation. Systems of the shear-beam type with elastoplastic behavior are considered.

The method extends the simplicity and the convenience of the response spectrum method to the analysis of nonlinear systems, and appears to give results consistent with solutions obtained by numerical integration. Nonetheless, the procedure needs to be tested for a wider variety of structures and ground disturbances, and expanded to include flexible systems with a more sophisticated hysteretic model. It is believed that the concept herein introduced is a general one, and that thus such an extension should present no major difficulties.

REFERENCES

1. Newmark, N.M., 'Current Trends in the Seismic Analysis and Design of High-Rise Structures', Chap. 16 in Earthquake Engineering, Ed. by R.W. Wiegel, Prentice-Hall, 1970.
2. Shibata, A. and Sozen, M.A., 'Substitute-Structure Method for Seismic Design in R/C', J. Struc. Div., ASCE, Vol. 102, No.ST1, Jan. 1976.
3. Tansirikongkol, V. and Pecknold, D.A., 'Approximate Modal Analysis of Bilinear MDF Systems Subjected to Earthquake Motions', SRS No. 449, Univ. of Illinois, Urbana, Ill., Aug. 1978.
4. Caughey, T.K., 'Equivalent Linerization Techniques', J. Acoustical Soc. Amer., Vol. 35, No. 11, Nov. 1963.
5. Iwan, W.D., 'Multi-Degree-of-Freedom Yielding Systems', J. Engrg. Mech. Div., ASCE, Vol. 94, No. EM2, Apr. 1968.
6. Clough, R.W., Benuska, K.L. and Wilson, E.L., 'Inelastic Earthquake Response of Tall Buildings', Proc. 3rd. World Conf. Earthq. Engrg., New Zealand, 1965, Vol. II.
7. Anagnostopoulos, S.A., Haviland, R.W. and Biggs, J.M., 'Use of Inelastic Spectra in Aseismic Design', J. Struc. Div., ASCE, Vol. 104, No. ST1, Jan. 1978.
8. Veletsos, A.S. and Vann, W.P., 'Response of Ground-Excited Elasto-Plastic Systems', J. Struc. Div., ASCE, Vol. 97, No. ST4, Apr. 1971.
9. Hadjian, A.H., 'A Re-Evaluation of Equivalent Linear Models for Simple Yielding Systems', Earthq. Engrg. and Struc. Dyn., Vol. 10, 1982.
10. Riddell, R. and Newmark, N.M., 'Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes', SRS No. 468, Univ. of Illinois, Urbana, Ill. Aug. 1979.
11. Geschwindner, L.F., Jr., 'Nonlinear Dynamic Analysis By Modal Superposition', J. Struc. Div., ASCE, Vol. 107, No. ST12, Dec. 1981.
12. Dungar, R., An Imposed Force Summation Method for Non-Linear Dynamic Analysis', Earthq. Engrg. and Struc. Dyn., Vol. 10, 1982.

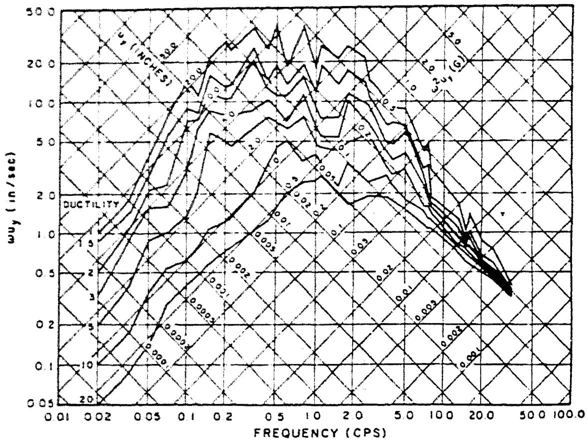

FIG. 1. NONLINEAR RESPONSE SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH 2% DAMPING; EL CENTRO, 1940, COMP. E-W

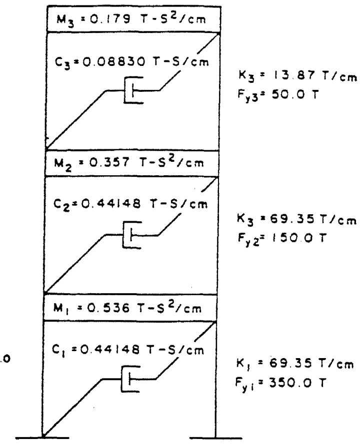

FIG. 5. ELASTOPLASTIC THREE-DEGREE-OF-FREEDOM IN NUMERICAL EXAMPLE

TABLE 1. NATURAL FREQUENCIES, DAMPING RATIOS, AND MODE SHAPES

MODE	1	2	3
FREQUENCY (HZ)	1.300	1.749	3.218
DAMPING RATIO	0.020	0.035	0.064
MODE SHAPE	0.47689 0.80835 1.64391	0.35883 0.38271 -0.68856	0.16440 -0.19397 0.04473

TABLE 2. APPROXIMATE AND EXACT MAXIMUM DISPLACEMENTS (CM)

EQ:	EL CENTRO 1940, E-W		OLYMPIA 1949, N86E		PACOIMA 1971, S16E	
FLOOR	APPR.	EXACT	APPR.	EXACT	APPR.	EXACT
1	4.2	4.2	3.4	2.6	12.3	15.3
2	6.7	6.6	5.4	4.6	20.3	24.5
3	13.6	14.7	11.0	8.8	40.9	45.9

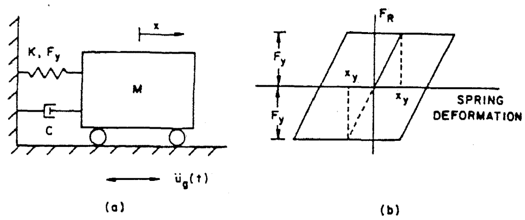

FIG. 2. ELASTOPLASTIC SINGLE-DEGREE-OF-FREEDOM SYSTEM

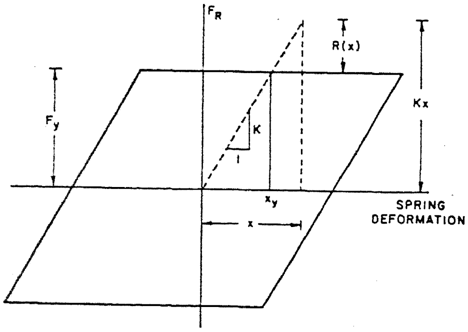

FIG. 3. LOAD-DEFORMATION CURVE

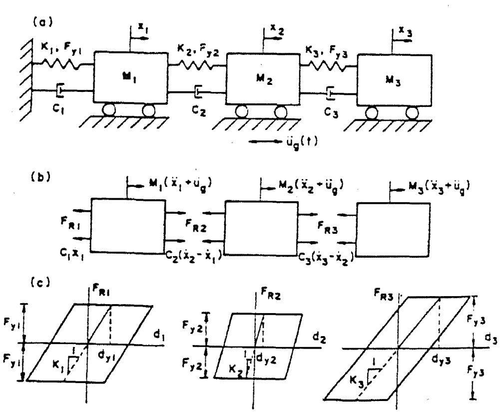

FIG. 4. ELASTOPLASTIC THREE-DEGREE-OF-FREEDOM SYSTEM