RELIABILITY OF RANDOMLY EXCITED AND STRONGLY DEFORMED HYSTERETIC OSCILLATORS

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SUMMARY

This paper develops a practical approach to estimating reliabilities of strongly deformed hysteretic structures. Accumulated absolute plastic deformations are considered. The response processes are divided into several intervals, each consisting of an elastic and a plastic step. Simulation shows the durations of the both steps have Wald and lognormal distributions respectively. The distribution of plastic deformation increments are derived by random vibration theories. The numbers of yielding events during a given period are delayed positive recurrent renewal processes. When the period is long enough, conservative values of the mean and the variance of accumulated plastic deformations are easily obtained.

INTRODUCTION

Even for linear structures of SDOF, to obtain the exact results of reliabilities under random dynamic load is very difficult. However, several effective approximate approaches have been developed (Ref. 's 1 and 2).

In the field of earthquake engineering, hysteretic structures have to be dealt with because of the importance of plastic deformation. It makes the problem more difficult. Caughey (Ref.3), Wen (Ref.4) and others develop the equivalent linearized approach. It can give reasonable results of response only when yieldings are much less than elastic deformation, because it uses viscous damping to represent the actual hysteretic damping.

Karnopp and Sharton (Ref. 5) developed two-state approach. Vanmarcke (Ref. 6), Grossmayer (Ref. 7), and Iyengars (Ref. 8) gave some improvements to it. The approach follows real response processes. However, all the authors assumed yielding less than elastic deformation, so they could use the envelope process.

Because the ductility design principle is more and more popular for designing earthquake-resistent structures, and ductility coefficient 4 to 8 is often used for fram structures. Such designed structures will suffer plastic deformation greater than elastic deformation. Mahin and Bertero (Ref. 9) indicated structures designed by Newmark-Hall inelastic response spectra with ductility coefficient 6, or by "Tentative Provisions for the Development of Seismic Regulations for Buildings" (Ref. 10) with Cd=6 and R=7 will have a number of yielding events and the cyclic ductility coefficient will be similar to or greater than the specified ones. Obviously, durations of the plastic steps of responses may be longer than those of elastic steps. Elastic stiffness is mush greater than plastic stiffness for any structure, therefore the responses can not be assumed narrow-band any more. Their approach will not be satisfactory ti this situation.

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This paper gives a improved two-state approach to estimating reliability of hysteretic structures strongly deformed under random excitation.

SIMULATION RESULTS OF DURATION OF ELASTIC AND PLASTIC RESPONSE STEPS

Symmetric elasto -plastic oscillators of SDOF under stationary Gaussian white noise input with zero mean are considered. The following equation holds:

$$\ddot{\eta}(n)+4\pi\zeta\dot{\eta}(n)+4\pi^2 f(n) = -\ddot{\eta}_g(n) \qquad (1)$$

where

$\eta(n)$ = dimensionless deformation, (= deformation/yielding level)
n = dimensionless time, (= $\omega_0 t/2\pi$)
ω_0 = elastic cyclic natural frequency
ζ = viscous damping ratio, from 0.02 to 0.08
$(\dot{})$ = derivative to dimensionless time n, (= d( ) / dn)
$\ddot{\eta}_g$ = dimensionless ground acceleration of Gaussian white noise of 0-mean
f(n)= dimensionless resistence of the oscillators, see Fig. (1):

$$f(n) = \begin{cases} \eta(n)-\varepsilon(n), & \text{for } |\eta(n)-\varepsilon(n)| \leq 1 \\ \text{sign } \eta(n), & \text{for } |\eta(n)-\varepsilon(n)| > 1 \end{cases} \qquad (2)$$

$\varepsilon(n)$ = dimensionless plastic deformation

Shinozuka's triangle series expression with 200 terms is used to generate the input:

$$\ddot{\eta}_g(n) = -\sum_{k=1}^{200} a_k \cos\left(\frac{2\pi\omega_k n}{\omega_0} + \phi'_k\right) \qquad (3)$$

where

ϕ'_k = random numbers uniformly distributed in (0, 2π)
$\omega_k = \omega_1+(k-1\ /\ 2)\Delta\omega$, and $\Delta\omega = (\omega_u-\omega_1)/200$
ω_1 and ω_u = the lower and the upper band of the cyclic frequency of the input, (=$0.01\omega_0$, $20\omega_0$, respectively)
$a_k = \frac{0.8}{a_0\ \omega_0}\sqrt{\pi^3(\omega_u-\omega_1)\ \zeta}$, and $\sigma_0 = \sqrt{\frac{\pi\ S_0}{2\zeta\ \omega_0{}^3}}$
a_0 = (yielding level)/σ_0, from 0.4 to 1.5 in order to get ductility 48
σ_0 = variance of stationary elastic deformation response
S_0 = the power spectrum density of the input

In Eq.1, there are only two parameters of structures and input, ζ and a_0. For each of 35 different pairs of their values, over 300 groups of samples of first excursion time t_e (i.e. duration of elastic steps), duration of plastic steps t_p are taken. The method of moment and the method of maximum likelihood are used and the following conclusions are found:

1) The histogram of elastic step durations can be best fitted by Wald distribution (Ref,11), see Fig. 2 for an example among 35 curves, among the 6 common positive half axis distributions. The probabilistic density of duration of the elastic steps, $P_e(n)$, equals

$$P_e(n) = \left(\frac{r}{2\pi n^3}\right)^{\frac{1}{2}} \exp\left[\frac{-r(n-c)^2}{2c^2 n}\right], \quad n > 0 \qquad (4)$$

where the constants r and c have the following relations to ζ and a_0 obtained by regression analysis, see Fig. 3:

$$r = 0.6317a_0-1.795\zeta+0.421 \qquad (5)$$

$$c= 0.3704a_0-2.948a_0^2+1.664a_0^3-10.69\zeta+205.8\zeta^2-22.93a_0\zeta-0.2802 \qquad (6)$$

Both the histograms and outcrossing rate (it is the upper band of the probability density of first excursion time) appear multimodal. It can be believed that $p_e(n)$ is multimodal. Therefore, all of the unimodal distributions, including those derived from the maximum entropy principle (ref. 12), can not fit it well. So the chi-square test indicates Eq. 4 is significantly different from the histograms. However, the first peaks are much greater than all of the others. After several steps, all the small peaks in the densities will have negligible influence to the next excursion time. According to Mahin and Bertero (ref. 9), structures designed with ductility coefficient of $\mu = 4$ and excited by the design-level ground motion will have 10 to 20 or more yielding events, if they do not collapse. Consequently, if a distribution function fits the major peak very well, and passes the average levels of the remains, it should work well for estimating reliability in this case. Then it can be avoided to use more difficult methods such as Kernel density fitting technique making the results too complicated to be used.

2) The histograms of plastic durations are fitted by the beta distribution, see Fig. 4 for an example among the 35 curves, at significant level 0.05. So the probability density of plastic durations, $P_p(n)$, is

$$p_p(n) = \frac{n^{p-1}(b-n)^{q-1}}{b^{q+p-1}\ \beta(p,q)}, \quad n > 0,\ p,q > 0 \tag{7}$$

where

$\beta(p,q)$ = the beta function.

The relations of the constants b,p and q to ζ and a_0 are from regression analysis, see Fig. 5:

$$b = -0.06524a_0 + 0.02152a_0^{-2} - 3.796\zeta - 9.77\zeta^2 + 12.68\zeta a_0^{-1} - 35.91\zeta^2 a_0^{-2} + 0.2115 \tag{8}$$

$$p = 5.481a_0 - 2.443a_0^{2} + 25.28\zeta - 87.09a_0\zeta + 49.83a_0^{2}\zeta - 0.8875 \tag{9}$$

$$q = 3.184a_0 - 2.421a_0^{2} + 11.17\zeta - 363.7\zeta^2 + 31.01a_0\zeta + 2.263 \tag{10}$$

PLASTIC DEFORMATION INCREMENTS

For the dimensionless absolute plastic deformation increments of the nonstationary plastic responce steps, y(n), the following equation holds:

$$\ddot{y}(n) + 4\pi\zeta\dot{y}(n) = -[\ \ddot{\eta}_g(n) + 4\pi^2\ ], \quad \dot{y}(n) > 0 \tag{11}$$

with the initial conditions:

$$\begin{cases} y(0) = 0 \\ \dot{y}(0) = v_0, \quad v_0 > 0 \end{cases} \tag{12}$$

where v_0 = the dimensionless impact velocity, a random variable.

Eg.'s (11 and 12) are linear. The input $\ddot{\eta}_g$ is Gaussian. So, the smaller the variance $\sigma_{\dot{y}}(n)$ is, the closer to Gaussian y(n) will be. Then, approximate probability density of y(n), $p_y(y;n)$, can be derived analytically.

The joint density function of y(n) and $\dot{y}(n)$ is

$$p_{y\dot{y}}(u,v;n) = C\exp\{-A(u-E[y(n)])^2 + B(u-E[y(n)])\} \tag{13}$$

where

$$A = \tfrac{1}{2}[1-\rho_y(n)]^{-2}\sigma_y^{-2}(n), \quad \text{and} \quad B = \frac{\rho_y(n)(v-E[\dot{y}(n)])}{(1-\rho_y(n))^2\sigma_y(n)\sigma_{\dot{y}}(n)} \tag{14,15}$$

$$C = \frac{1}{2\pi\sigma_y(n)\ \sigma_{\dot{y}}(n)}\exp\{\ \frac{-(v-E[y(n)])^2}{2(1-\rho_y(n))^2\sigma_y^2(n)}\ \} \tag{16}$$

$$E[y(n)] = \frac{1}{4\pi\zeta}(E[v_0] + \frac{\pi}{\zeta})(1-e^{-4\pi\zeta n}) - \frac{\pi n}{\zeta} \qquad (17)$$

$$E[y(n)] = E[V_0]e^{-4\pi\zeta n} - \frac{\pi}{\zeta}(1-e^{-4\pi\zeta n}) \qquad (18)$$

$$\sigma_y{}^2(n) = \frac{\sigma^2_{v_0}}{16\pi\zeta^2}(1-e^{-4\pi\zeta n})^2 + \zeta^{-2}a_0^{-2}(2\pi\zeta n - \frac{3}{4} + e^{-4\pi\zeta n} - \frac{1}{8}e^{-8\pi\zeta n}) \qquad (19)$$

$$\sigma_y{}^2(n) = \sigma_{v_0}^2 e^{-8\pi\zeta n} + 4\pi^2 a_0^{-2}(1-e^{-8\pi\zeta n}) \qquad (20)$$

$$Cov[y(n),\dot{y}(n)] = \frac{\sigma_{v_0}^2}{4\pi\zeta}e^{-4\pi\zeta n}(1-e^{-4\pi\zeta n}) + 2\pi\zeta^{-1}a^{-2}(\tfrac{1}{2}-e^{-4\pi\zeta n}+\tfrac{1}{2}e^{-8\pi\zeta n}) \qquad (21)$$

$$\rho_y(n) = \frac{Cov[y(n),\dot{y}(n)]}{\sigma_y(n)\sigma_{\dot{y}}(n)} \qquad (22)$$

When $\dot{y}(n) = 0$, $y(n)$ is the absolute yielding increment of the plastic step, Δ_y. The probability density conditional on $\dot{y}(n) = 0$ is

$$p_{y(n)|\dot{y}(n)=0}(u;n) = \frac{p_{y(n)\dot{y}(n)}(u,0;n)}{\int_0^\infty p_{y(n)\dot{y}(n)}(u,0;n)du} \qquad (23)$$

The marginal density of Δ_y is

$$p_\Delta(u) = \int_0^\infty p_{y(n)|\dot{y}(n)=0}(u;n)\ p_p(n)\ dn \qquad (24)$$

In Eq.'s (13 to22) $E[v_0]$ and $\sigma_{v_0}^2$, the mean and the variance of v_0, are derived [illegible] the elastic responce equation, Eq. (1), and the initial conditions:

$$\begin{cases} |\eta(0)| = 1 \\ \dot{\eta}(0) = 0 \end{cases} \qquad (25)$$

The joint density function of the elastic deformation and velocity, η and $\dot{\eta}$, is

$$p_{\eta\dot{\eta}}(x,v;n) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\{\frac{-1}{2(1-\rho^2)}[\frac{(x-E_1)^2}{\sigma_1{}^2} - \frac{2\rho(x-E_1)(v-E_2)}{\sigma_1\ \sigma_2} + \frac{(v-E_2)^2}{\sigma_2{}^2}]\} \qquad (26)$$

where

$$E_1 = E[\eta(n)] = e^{-2\pi\zeta n}[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(2\pi n\sqrt{1-\zeta^2}) + \cos(2\pi n\sqrt{1-\zeta^2})] \qquad (27)$$

$$E_2 = E[\dot{\eta}(n)] = -2\pi e^{-2\pi\zeta n}\frac{1}{\sqrt{1-\zeta^2}}\sin(2\pi n\sqrt{1-\zeta^2})] \qquad (28)$$

$$\sigma_1{}^2 = Var[\eta(n)] = \frac{1}{a_0^2}\{1-\frac{e^{-4\pi\zeta n}}{1-\zeta^2}[1-\zeta\cos(4\pi n\sqrt{1-\zeta^2})+\zeta\sqrt{1-\zeta^2}\sin(4\pi n\sqrt{1-\zeta^2})]\} \qquad (29)$$

$$\sigma_2{}^2 = Var[\dot{\eta}(n)] = \frac{4\pi^2}{a_0^2}\{1-\frac{e^{-4\pi\zeta n}}{1-\zeta^2}[1-\zeta\cos(4\pi n\sqrt{1-\zeta^2} - \zeta\sqrt{1-\zeta^2}\sin(4\pi n\sqrt{1-\zeta^2})]\} \qquad (30)$$

$$\rho = \frac{\zeta e^{4\pi\zeta n}}{a_0{}^2\sigma_1\sigma_2(1-\zeta^2)}[1-\cos(4\pi n\sqrt{1-\zeta^2})] \qquad (31)$$

When $|\eta(n)| = 1$. $\dot{\eta}(n)$ is the impact velocity, v_0. The density conditional on $|\eta(n)| = 1$ equals

$$p_{\dot{\eta}||\eta(n)|=1}(v_0;n) = \frac{p_{\eta\dot{\eta}}(1,v_0;n) + p_{\eta\dot{\eta}}(-1,-v_0;n)}{\int_0^\infty [p_{\eta\dot{\eta}}(1,z;n) + p_{\eta\dot{\eta}}(-1,-z;n)]dz},\quad v_0>0 \qquad (32)$$

Note that the distribution densites, Eq.'s(26 and 32) are also conditional on that the duration of elastic steps, t_e, equals n. The marginal density of v_0 can be derived from Eq. 4:

$$p_{v_0}(v) = \int_0^\infty p_{\dot{\eta}||\eta(n)|=1}(v\ ;n)\ p_e(n)\ dn \qquad (33)$$

The mean and the variance of the impact velocity, v_0, are

$$E[v_0] = \int_0^\infty \frac{\int_0^\infty z[p_{\eta\dot{\eta}}(1,z;n) + p_{\eta\dot{\eta}}(-1,-z;n)]dz}{\int_0^\infty [p_{\eta\dot{\eta}}(1,z;n) + p_{\eta\dot{\eta}}(-1,-z;n)]dz} p_e(n)\ dn,\qquad v_0>0 \qquad (34)$$

$$\sigma_{v_0}^2=\int_0^\infty \frac{\int_0^\infty z^2[p_{\eta\dot\eta}(1,z;n)+p_{\eta\dot\eta}(-1,-z;n)]dz}{\int_0^\infty [p_{\eta\dot\eta}(1,z;n)+p_{\eta\dot\eta}(-1,-z;n)]dz} P_e(n)dn-(E[v_0])^2, \qquad v_0>0 \tag{35}$$

nalytically integrating the numerator and the denominator in Eq.'s (34 and 35) gives the following results:

$$\frac{\int_0^\infty z[p_{\eta\dot\eta}......]dz}{\int_0^\infty [p_{\eta\dot\eta}......]dz} = \frac{D[Q_4(1)+Q_4(-1)]}{Q_3(1)\Phi[DQ_1(1)]+Q_3(-1)\Phi[DQ_1(-1)]} \tag{36}$$

$$\frac{\int_0^\infty z^2[p_{\eta\dot\eta}......]dz}{\int_0^\infty [p_{\eta\dot\eta}......]dz} = \frac{D[Q_0(1)+Q_0(-1)]}{Q_3(1)\Phi[DQ_1(1)]+Q_3(-1)\Phi[DQ_1(-1)]} \tag{37}$$

$$D = \frac{1}{\sigma_2\sqrt{2(1-\rho^2)\pi}}, \qquad Q_1(k) = \frac{\sigma_2}{\sigma_1}\rho(1-kE_1)+kE_2 \tag{38,39}$$

$$Q_2(k) = \exp\{\frac{-1}{1-\rho^2}[\frac{E_2^2}{2\sigma_2^2}+\frac{k\rho E_2(1-kE_1)}{\sigma_1\ \sigma_2}+\frac{(1-kE_1^2)}{2\sigma_1^2}]\} \tag{40}$$

$$Q_3(K) = \exp\{\frac{(1-kE_1)^2}{\sigma_1^2}[\sigma_2^2\rho^2-\frac{1}{2(1-\rho^2)}]\} \tag{41}$$

$$Q_4(k) = \frac{Q_2(k)}{D^2}+\frac{\sqrt{2\pi}}{D}\{Q_1(k)+Q_3(k)\Phi(DQ_1(k))\} \tag{42}$$

$$Q_0(k) = \frac{Q_1(k)Q_2(k)}{D^2}+\frac{\sqrt{2\pi}}{D}\{[Q_1^2(k)+\frac{1}{D^2}]Q_3(k)\Phi[DQ_1(k)]\} \tag{43}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_\infty^x \exp(\frac{-t^2}{2})\ dt \tag{44}$$

Substituting Eq.'s (36 to 44) and Eq.'s (4) in Eq.'s (34 and 35) can give $E[v_0]$ and $\sigma_{v_0}^2$.

The denominator in Eq. (23) can be analytically integrated:

$$\int_0^\infty p_{y\dot y}(u,0;n)du = \frac{1}{\sqrt{2\pi}\ \sigma_y(n)}\exp\{\frac{-(E[\dot y(n)])^2}{2\sigma_y^2(n)}\}\ \Phi(Z_1) \tag{45}$$

where

$$Z_1=\frac{E[y(n)]}{\sigma_y(n)\sqrt{1-\rho_y^2(n)}} - \frac{E[y(n)]}{\sigma_y(n)\sqrt{1-\rho_y^2(n)}}\ \rho_y(n) \tag{46}$$

Then, from Eq. (23)

$$p_{y(n)\ \dot y(n)=0}(u;n)= -\frac{\exp\{\frac{-1}{2(1-\rho_y^2(n))\sigma_y(n)}[\frac{\rho^2(n)(E[\dot y(n)])^2}{\sigma_y(n)}+Z_2]}{\sigma_y(n)\Phi(Z_1)\ \sqrt{2\pi(1-\rho_y^2(n))}} \tag{47}$$

where

$$Z_2 = \frac{(u-E[y(n)])^2}{\sigma_y(n)}+\frac{2E[y(n)](u-E[y(n)])}{\sigma_y(n)\ \sigma_{\dot y}(n)}\ \rho_y(n), \qquad u>0 \tag{48}$$

With substituting Eq.'s (17 to 22) into Eq. (47), and using Eq. (7), one can integrate Eq. (24) numerically. Then, the mean and the variance of Δ_y can be found numerically through $p_\Delta(u)$, Eq. (24):

$$E[\Delta_y] =\int_0^\infty y p_\Delta(y)\ dy \tag{49}$$

$$\sigma_{\Delta_y}^2 =\int_0^\infty y^2 p_\Delta(y)\ dy - (E[\Delta_y])^2 \tag{50}$$

TOTAL ACCUMULATED ABSOLUTE PLASTIC DEFORMATION

Different steps generate different Δy_i, i =1,2,3,...... They are from the same equations, Eq.'s (11 and 12). Therefore, Δy_i's have identical and independent distributions (abbreviated to i.i.d.). For the same reason t_e's and t_p's both are i.i.d. At time T, the accumulated absolute plastic deformation D(T) is

$$D(T) = \sum_{i=1}^{N(T)} \Delta y_i \tag{51}$$

where N(T) = the number of plastic steps in the period (0,T).

N(T) is a delayed positive recurrent renewal process, because the first elastic step has zero deformation at its initial time, and all the subsequent steps have Eq. (25) as their initial conditions.

The correlation analysis shows t_e and t_p correlate weakly to each other. Furthermore, t_p depends stochastically upon v_0, and joint desity of v_0 and t_e is Eq. (32). It can be said that there is no functional relation between t_e and t_p. Therefore, t_e and t_p are approximately independent. Then the mean and the variance of the durations of a elastic and a consecutive plastic step t_i are

$$\mu = E[t_i] \cong E[t_e] + E[t_p] \tag{52}$$

$$\sigma_{t_i}^2 = Var[t_e+t_p] \cong Var[t_e] + Var[t_p] \tag{53}$$

From theories of random processes (Ref. 13), when T is long enough, the mean and the variance of N(T) are

$$E[N(T)] \cong T/\mu \tag{54}$$

$$Var[N(T)] \cong \sigma_{t_i}^2 T/\mu^3 \tag{55}$$

and probability of (N(T)<j) goes asymptotically to Gaussian as T increases. For j>10, the density of the duration of the first elastic step has negligible influence to distribution of N(T). Based on the law of large numbers, D(T) can become Gaussian asymptotically as j increases. N(T) can be assumed independent from Δy_i's. Then, the conservative estimators of the mean and the variance of D(T) are

$$E[D(T)] \cong E[\Delta y]\ E[N(T)] \tag{56}$$

$$Var[D(T)] \cong \sigma_{\Delta y}^2\ E[N(T)] + (E[\Delta y])^2 Var[N(T)] \tag{57}$$

EXAMPLES

A symmetric elasto-plastic oscillator of SDOF with the natural period T_0= 1 Sec. and damping ratio ζ = 0.04 is taken as an example. The intensity of the input makes the variance of stationary elastic deformation equal to a/0.8 and a/0.6, a - yielding level of deformation. Its band-width is from 0.1 to 200 Hz

Fig. 6(a and b) shows the relations of the mean and the variance of D(T) to dimensionless time T for a_0=0.8; Fig. 6(c and d) is for a_0=0.6. The range of the abscessa covers 10 to 25 plastic steps or so. The solid curves are given by the present method; the dash curves are the simulation results. The means by the two methods are close to each other. But the variances by the present method are greater than that by simulation. In Fig.6(a and c), the dash-and-dot curves are the equal probability curves, i.e. the curves whose points can be exceeded at the same probability for the results by both the present method and the simulation. For the case of a_0=0.8, the probability is approximately 10%, and the dimensionless accumulated absolute plastic deformation is about 11. Note that D(T) closely relates to the energy ductility coefficient, and it is much greater than the convenient ductility coefficient.

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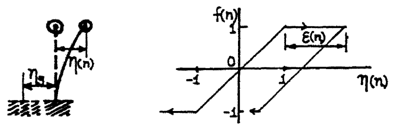


Fig. 1, An Elasto-plastic Oscillator under Ground Motion Loads.

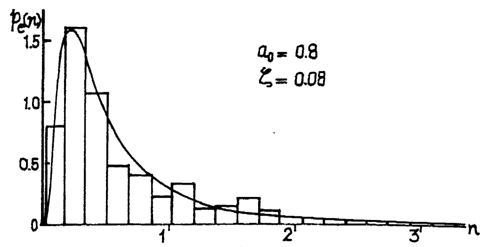


Fig. 2, Histogram of Elastic step Duration, t_e.

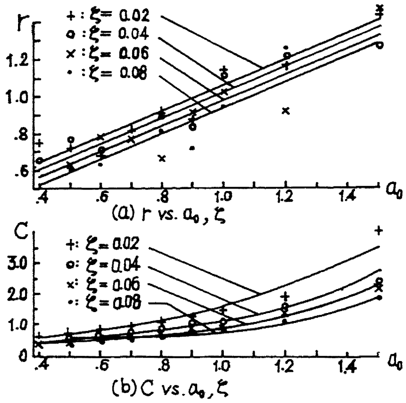


Fig. 3, Regression of parameters of Density Function of Elastic Duration.

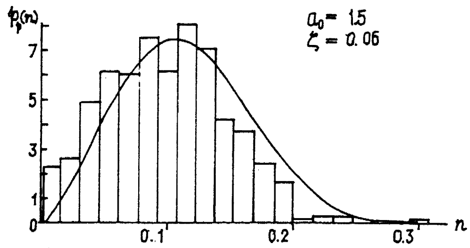


Fig. 4, Histogram of Plastic step Duration, t_p.

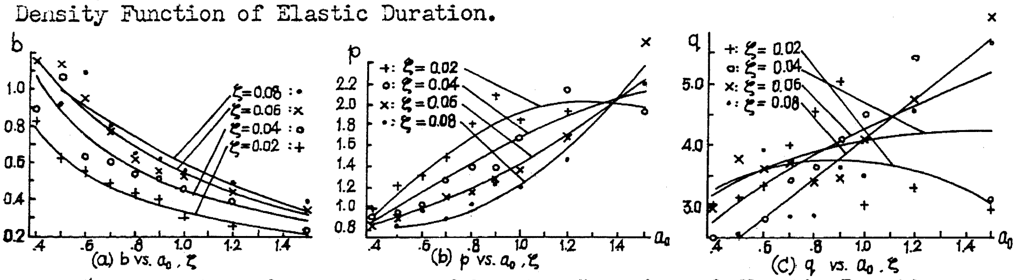
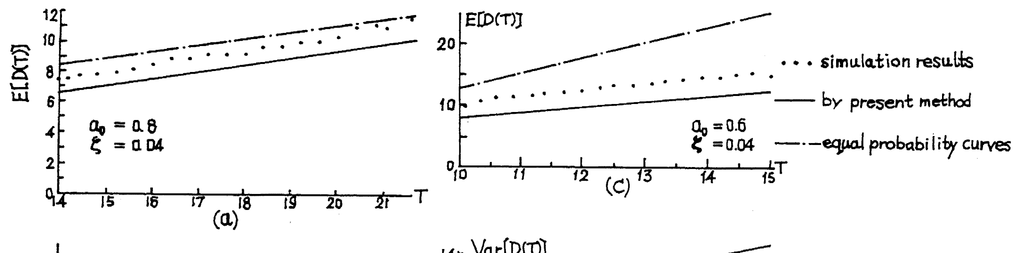


Fig. 5 Regression of parameters of Density Function of Plastic Duration, p_p.



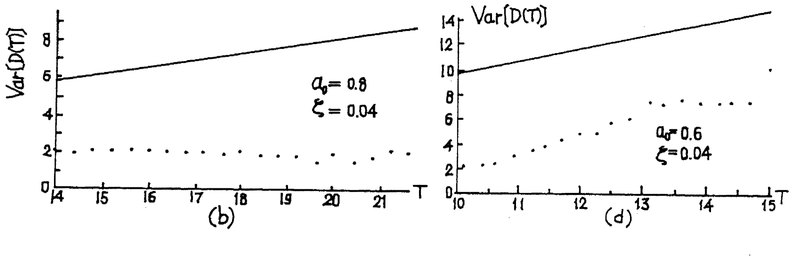


Fig. 6 Comparason of Reliability by Present Method with Simulation Results.