

# DYNAMIC RESPONSE OF PILE SUBJECTED TO LATERAL LOADING.

## A PARAMETRIC STUDY

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### SYNOPSIS

In the present study the behaviour of a dynamically loaded pile is investigated by means of a numerical procedure which is based on the Finite Element Method and a step by step integration in the time domain.

The soil-pile interaction has been simulated by shear coupled, non-elastic axial springs, with an exponential  $p-y$  function. Unilateral conditions due to gapping and degradation have also been taken into consideration.

Finally, in the presented parametric study, the influence of shear coupling and pile head-cap-superstructure interaction on the pile response have been investigated for the simple case of a fully embedded flexible pile.

### INTRODUCTION

During the last years, an increasing demand for a rational investigation of the dynamic behaviour of piles has motivated numerous approaches (ref. 1,2,3). Those derived directly from the classical "beam on elastic - Winkler - foundation" hypothesis are the most suited to sophisticated parametric studies. In several recent publications, the behaviour of Winkler-type independent horizontal springs has been represented by non-linear reaction-deflection curves (ref. 4), and in a small number of these, unilateral effects such as soil-pile separation are taken into consideration.

The experimental data of lateral loading tests given in a number of early and recent studies (ref. 5, 6), show that the present state of knowledge cannot provide reliable values of the horizontal subgrade reaction constants  $K_h$  or  $n_h$  for various soil and pile types, under dynamic or static loading conditions.

Additionally, several recent computational studies are focused on the significant dependence of deformation and bending moment distributions along the pile shaft on the values of the horizontal subgrade reaction coefficients. It has also been found that for strong applied excitation and relatively low soil resistance the uncoupled spring model may lead to an un-

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realistic increase of gap widths and soil degradation near the surface. In some cases, large pile-head displacements and system instability may develop, thus leading to failure (ref. 7).

In this study, a shear coupling has been introduced between the non-linear axial springs, so that initial displacements can be transferred to neighbouring springs. A similar non-linear, Pasternak-type model has been proposed by Georgiadis and Butterfield (8). The results of a parametric study are presented in the last part of this paper.

## METHOD OF ANALYSIS

### Modelling the Soil Behaviour

In the present study the virgin loading is simulated by the following exponential relationships :

$$p_u = p_\infty(1-d \cdot e^{-\epsilon x}) \quad (1)$$

$$p = p_u(1-\exp(-b|y|)) \quad (2)$$

where:  $x$  = the depth from ground surface,  $p_u$  = ultimate soil resistance,

$$p_\infty = p_u(x = \infty), \quad d = \frac{p_\infty - p_u(x=0)}{p_\infty}, \quad e = -\ln\left(\frac{p_\infty - p_u(x=1)}{p_\infty \cdot d}\right)$$

The initial slope of the  $p$ - $y$  curve is  $p_u \cdot b$ .

On unloading-reloading a gap of width  $g$  is developed between the soil and the pile especially near the ground surface, where the overburden pressure is not large enough to confine the soil mass around the pile.

Unloading is approximated as linear, depended on the level of the applied load as well as on the number of cycles  $n$ .

$$p = a \cdot (y-g) = p_u \cdot b \left[1 - \exp\left(-\gamma - \frac{\delta}{n}\right)\right] (y-g), \quad (3) \quad (\gamma, \delta \text{ coefficients})$$

The rest of the loading curves are characterized by an initial linear part which coincides with the preceding unloading. In the case where this linear reloading branch approaches the first  $p$ - $y$  curve, reloading follows this virgin exponential curve. A more general model of the soil behaviour has been presented by the authors elsewhere (ref. 7)

### Numerical Formulation and Solution of the Problem

For the numerical treatment of the problem under investigation, the general theory of forced damped vibrations of beams on elastoplastic foundations is considered and a double discretization, in space and time, is applied (Clough and Penzien, 1975), (ref. 9).

First the Finite Element Method is used for the spatial discretization of the pile-soil system. So, taking into account rotatory inertia effects, the pile is discretized into plane frame finite elements, having as generalized displacements two displacements and a rotation per node. On the other hand, each soil-pile element consists of a nonlinear spring and a dashpot, connected in parallel (fig. 1). These soil-pile elements activate a compressive force  $r(t)$  only when they come in contact with the corresponding pile nodes. Thus, if  $y(t)$  denotes the soil-pile element shortening deforma-

tion and  $g(t)$  the existing gap between pile and soil, the unilateral behaviour of the soil-pile interaction is given by the following conditions :

$$\text{If } y \leq g \quad \text{then} \quad r = 0 \quad (4a)$$

$$\text{If } y > g \quad \text{then} \quad r > 0 \quad (4b)$$

Taking into account shear coupling effects, the soil-pile element compressive force is :

$$r = C_s \dot{y} + p(y) + G.d^2y/dx^2 \quad (5a)$$

where:  $C_s$ ,  $\dot{y}$ ,  $G$  are the soil damping coefficient, the deformation velocity and the shear coupling constant, respectively. The term  $p(y)$  denotes the spring force and is given by either the nonlinear, exponential expression (2):

$$p(y) = p_u \cdot [1 - \exp(-b|y - g_0|)] \quad (5b)$$

( $g_0$  is the initial gap) for the virgin loading or reloading curved paths in the  $p$ - $y$  diagram (fig. 2), or the linear expression (3):

$$p(y) = a \cdot (y - g) \quad (5c)$$

for the unloading or reloading straight paths ( $g$  is the current gap).

After the above spatial discretization, the problem relations for the assembled soil-pile system are in matrix form :

$$\underline{M} \ddot{\underline{u}} + \underline{C} \dot{\underline{u}} + \underline{K} \underline{u} = \underline{f} + \underline{A}^T \underline{r}, \quad (6)$$

$$\underline{y} = \underline{A} \underline{u}, \quad (7)$$

$$\underline{u}(t=0) = \underline{u}_0, \quad \dot{\underline{u}}(t=0) = \dot{\underline{u}}_0, \quad \underline{g}(t=0) = \underline{g}_0 \quad (8)$$

where:  $\underline{M}$ ,  $\underline{C}$ ,  $\underline{K}$  are the mass, damping and stiffness matrix, respectively;  
 $\underline{g}$ ,  $\underline{r}$ ,  $\underline{y}$  are the gap, force and deformation vector of the soil-pile elements;  
 $\underline{f}$ ,  $\underline{u}$ ,  $\dot{\underline{u}}$ ,  $\ddot{\underline{u}}$  are the force and the relative displacement, velocity and acceleration vector of the pile nodes;  
 $\underline{A}$  is a kinematic transformation matrix; and  
 $\underline{u}_0$ ,  $\dot{\underline{u}}_0$ ,  $\underline{g}_0$  are the known vectors of initial displacements, velocities and gaps (usually  $\underline{g}_0 = \underline{0}$ ).

Thus, the problem is to find  $\underline{u}$ ,  $\dot{\underline{u}}$ ,  $\ddot{\underline{u}}$ ,  $\underline{r}$ ,  $\underline{y}$  that satisfy the relations (4) - (8), when  $\underline{u}_0$ ,  $\dot{\underline{u}}_0$ ,  $\underline{g}_0$  and  $\underline{f}$  are given. For an earthquake, the force vector  $\underline{f}$  represents inertia forces due to the prescribed acceleration history of the supports (Clough and Penzien, 1975).

Now, for the solution of the problem rels. (4) - (8) a time discretization is applied.

For reasons of accuracy, a small time step is usually required. Thus the explicit central difference method is preferred to other implicit time integration schemes, as it has been already discussed in our previous work (ref. 7).

#### EXAMPLE PROBLEM; A PARAMETRIC STUDY. CONCLUSION.

The procedure that was previously described has been tested for the case of a 12 m long and 0.3 m wide, flexible H-pile (steel), fully embedded in a clay deposit. Cyclic excitation was provided by means of a horizontal sinusoidal force :

$H_0 = H \cdot \sin \omega t$ , where:  $\omega$  = frequency,  $H$  = force amplitude.

This force was applied to the lumped mass at the pile head.

An interesting aspect of the parametric study is the computed variation of pile-head deflections and bending moments with respect to the shear constant  $G$ . The value of  $G$  was varied within the rational range of 0, (Winkler hypothesis), to  $1.5 p_u \cdot b$ .

The results of the parametric study for three different groups of data are shown in the following figures :

Case 1: Figure 3 shows that the soil-pile system seems to be in a stable state for the whole range of  $G$ -Values. The ratio of maximum pile-head movements after six cycles for the characteristic values of  $G=0$  and  $G=p_u \cdot b$ , is approximately equal to 1.3. The detailed data are shown in the same figure.

Case 2: The force amplitude in this case is doubled and the system seems to reach an unstable state, (figure 4). The ratio of maximum head movements after one and six loading cycles, as in Case 1, ( $G=0$ ,  $G=p_u \cdot b$ ), is equal to 1.15 and 1.6 respectively.

Case 3: In the third -and more impressive- case, the force amplitude is similar to the first case but the pile stiffness is about 10 times lower. The results show that while for  $G$  greater than  $0.5 p_u \cdot b$  the system remains in a stable state, for lower values of the coupling constant the system develops instability and gradual failure takes place (figure 5). Figure 6 shows the deflection along the pile shaft for two representative  $G$  values.

Another aspect of this work has been the investigation of the role of the pile-head, cap and superstructure connection and interaction. This problem was studied by altering the force amplitude, together with the top lumped mass of the system. A first indicative observation of this study was that a reduction of the top lumped mass and rotational inertia, resulted in a significant increase of the corresponding acceleration. There was no change in the order of the maximum deflections of the system.

In conclusion it can be said that shear coupling of the horizontal springs has a significant influence on the pile response. This is particularly true when axial springs coefficient  $K_n$  has low values near to the ground surface and increases with depth.

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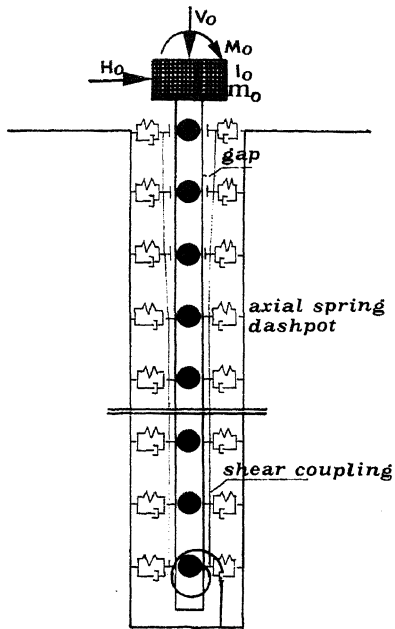


Fig.1

Model of pile-soil system.  
(Non linear, non elastic axial springs shear coupled, viscous dashpot).

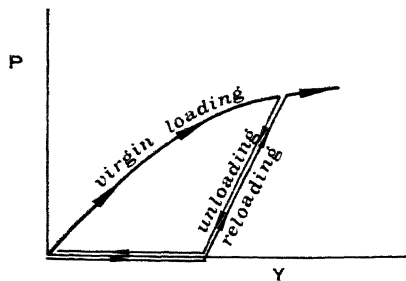


Fig.2

Unilateral soil stress-strain behaviour in loading-unloading (remaining gaps).

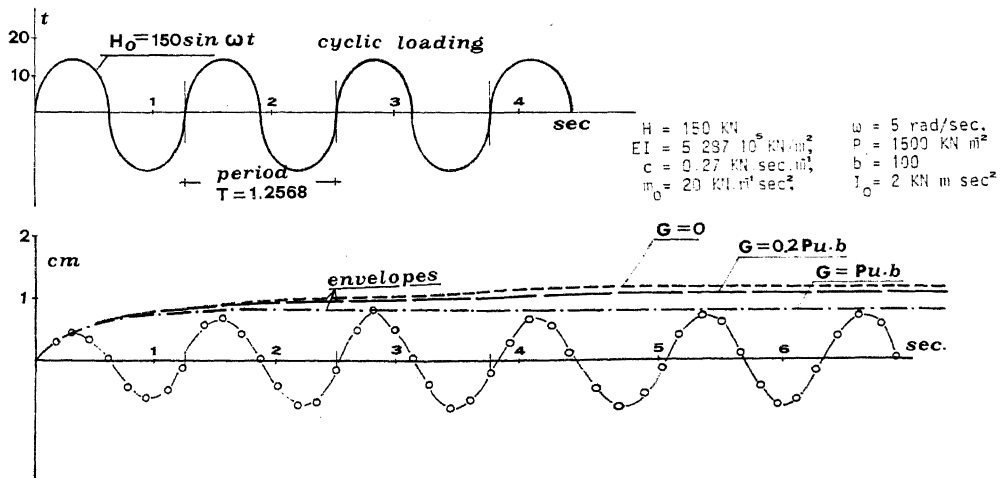


Fig.3 Pile head maximum computed deflection with time for different spring shear coupling conditions (Stable state).

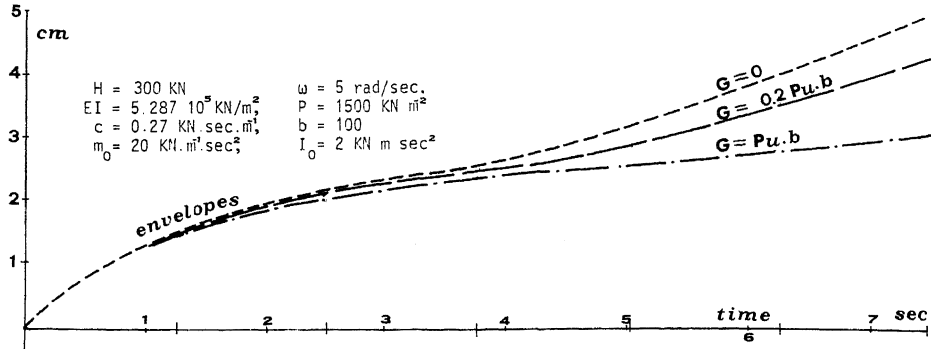


Fig.4 Pile head maximum computed deflection with time for different springs shear coupling conditions (Instability).

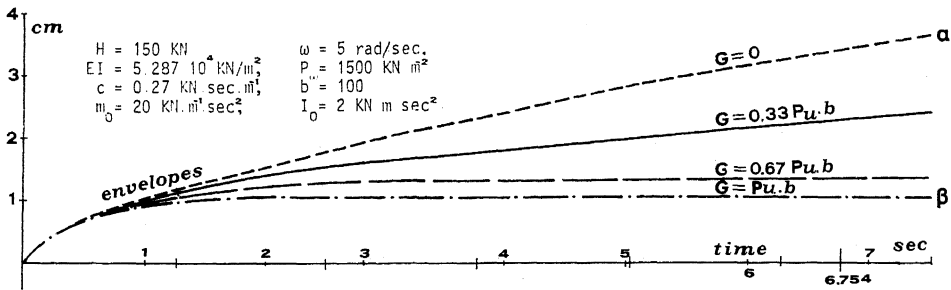


Fig.5 Pile head maximum computed deflection with time for different springs shear coupling conditions (For  $G$  greater than  $0.5 P_{ub}$  the system remains stable).

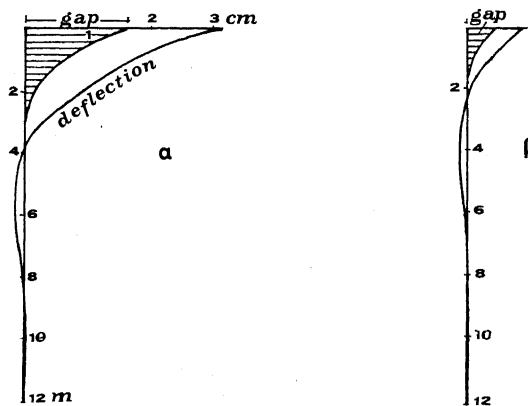


Fig.6 Deflections and gaps along pile shaft for unstable (curve  $\alpha$ ) and stable (curve  $\beta$ ) representative cases. ( $\alpha$  and  $\beta$  correspond to curves  $\alpha$  and  $\beta$  of Fig.5, at time 6.754 sec).

