RELIABILITY OF NONLINEAR SYSTEMS WITH UNCERTAIN PARAMETERS AND RANDOM SEISMIC EXCITATION

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SUMMARY

An approach to assess the reliability of nonlinear systems under earthquake loading is presented. Monte Carlo simulation techniques are used to obtain probabilistic descriptions of the uncertainties on the seismic excitation at a given site during given time intervals and the uncertainties about the properties of the structural systems. The reliability of R/C simple frame systems located in Mexico City soft soil zone was evaluated. The results show the extreme sensitivity of the estimated failure probabilities to the distributions of the maximum ground intensities and to the upper bound of the latter.

INTRODUCTION

Significant contradictions are found when trying to reconcile the results of seismic risk studies with the performance of actual structures in seismic regions. These inconsistencies arise largely from our lack of information about reliability of random structural systems subjected to random earthquakes.

Determination of the reliability of nonlinear systems under seismic loading requires the performance of the following steps: a) statistical prediction of earthquake intensities and detailed ground motions, b) modelling of the dynamic behaviour of the structural system and c) determining the cumulative probability distribution of the peak structural response, and obtaining the integral of its product by the probability density function of structural capacity.

Concerning step a), in order to estimate the expected seismic loads for a specific site during a period of time, it is necessary to take into account the uncertainties related to the occurrence, location and magnitude of future earthquakes in the region where the site is located (Ref.1). Detailed characteristics of ground motion during each event can be defined statistically in terms of variables such as intensity, duration and frequency content. In relation to step b), the modelling of the structural system should include the nonlinear behaviour of the system when subjected to strong motions. The dynamic response mentioned in step c) depends on the space and time characteristics of the load as well as on the dynamic properties (inertia , damping, stiffness, strength) of the system considered, some of which have to be dealt with as non-deterministic (Ref.2).

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In this paper an approch for the assessment of the seismic reliability of nonlinear systems is presented. The method includes in a unified form the steps mentioned above; this is achieved by using concepts of reliability theory, techniques of step by step response and seismic risk analyses and Monte Carlo simulation. The approach is applied to evaluate the reliability of simple reinforced concrete (R/C) frames located in Mexico City soft soil zones.

RELIABILITY OF R/C FRAMES UNDER SEISMIC LOADS

The structural systems considered in this work are ductile R/C frames as the one shown in Fig. 1. It is assumed that the behaviour of the frame's columns when subjected to flexural cyclic loading is of the elastoplastic-hysteretic type (Fig. 2). The system's seismic excitation on the frame is represented by the horizontal component of the ground acceleration, $\ddot{u}_g$ (Fig. 1). It is also assumed that the frame has a single mode of failure due to the seismic loading; this mode corresponds to the collapse of the frame associated to the simultaneous formation of plastic hinges at the ends of the columns (Fig. 1).

It can be shown (Ref.3) that if S represents the random actions on the structural system and R its random resistance, and if it is assumed that S and R are statistically independent, the probability of failure(P_f) of the idealized frame can be estimated by the following expression:

$$P_f = P(S>R) = \int_0^\infty \bar{F}_S(\alpha) f_R(\alpha) d\alpha \qquad (1)$$

where P (S>R) means the probability that S will be larger than S at a certain time during the lifetime of the structure; $\bar{F}_S(.)$ is the complementary accumulated probability distribution of S and f_R (.) is the probability density function of R.

For the case under consideration S will be defined as the ratio of the peak deformation to the yield deformation and R equals the ductility that the system is capable of developing before failure.

Definition of $f_R(.)$

Let us introduce an auxiliary variable δ=R-1, assumed to possess a lognormal distribution. By definition the minimum value of interest of R is unity. It can be shown (Ref.4) that the mean value and the standard deviation of the natural logarithm of δ, which are the parameters of $f_\delta(.)$, are given by the following expressions:

$$m_{\ln\delta} = \ln\left(\bar{\delta}/(V_\delta^2 + 1)^{1/2}\right) \qquad (2)$$

$$\sigma_{\ln\delta} = (\ln(V_\delta^2 + 1))^{1/2} \qquad (3)$$

where $\bar{\delta}$ and V_δ are the mean value and the coefficient of variation of δ, respectively. $\bar{\delta}$ and V_δ can be obtained from the mean value and the coefficient of variation of R (ref.5). The $f_\delta(.)$ functions corresponding to a characteristic value (Ref.3) of R equal to 4 and V_R=0.05,0.15,0.25,0.40 are depicted in Fig. 3.

Computation of $F_S(.)$

In order to compute the probability distribution of the ductility demands associated to the R/C frame systems mentioned above, it is necessary to obtain samples of the seismic responses of the systems subjected to the horizontal accelerations recorded at the soft soil zone of Mexico City (the site where the frames are supposed to be located). In order to get those samples, Monte Carlo simulation techniques are used to generate sets of sample accelerograms at the site (for the time interval of interest) and families of structural systems which reflect the uncertainties on its parameters.

If it is assumed that the occurrence of earthquakes at the site of interest follows a Poisson process such that the intensities and detailed ground motion time histories of any two different events are statistically independent and identically distributed, it can be shown that the probability distribution function of the maximum response can be estimated by means of Monte Carlo simulation of sets of mechanical properties of structures and the corresponding response of each structure to random earthquake records generated by sampling from the distribution of the maximum intensity for the time interval of interest. In order to determine structural responses from ground motion intensities and detailed time histories, step by step methods of dynamic analysis (Ref.5) are used.

GENERATION OF ACCELEROGRAMS FROM THE DISTRIBUTION OF MAXIMUM EARTHQUAKE INTENSITIES FOR GIVEN TIME INTERVALS

The methodology proposed in this paper to generate samples of ground motion time histories at the soft soil zone of Mexico City for a period of time can be sinthezised in these steps: a) select a set of recorded accelerograms which include the main amplitudes, duration and frequency content; b) compute the probability distribution of the maximum ground intensities at the site for a given interval of time; c) generate the sample of expected accelerograms at the site by scaling the ordinates of the accelerograms of a). The scaling factors take the seismic risk at the site into account, which is reflected through the probability distribution obtained in b).

The steps mentioned above were applied as follows:

1) Eight accelerograms recorded during March 14, 1979 (m=7.6) and October 24, 1980 (m=6.5) earthquakes were selected from a 20 year record at the site. The chosen accelerograms have peak ground accelerations which vary from 30 to 100 cm/s^2, the latter being the maximum recorded at the site; their duration varies between 30 and 80 s and their corresponding seudovelocity spectra show a frequency content typical of Mexico City soft soil (Ref.6).

2) As the seudovelocity spectra show good correlation with the local seismic amplification effects of the mentioned type of soil, its maximum ordinate for a 0.10 percentage of critical damping, $S_v(0.10)$, was taken as representative of the maximum ground motion intensity at the site. In order to compute the probability distribution of $S_v(0.10)$ for 50 and 100 year time intervals, it was assumed that the occurrence of earthquakes in the region where the site is located can be represented by a Poisson process; therefore, the probability, P, that a particular S_v is exceeded can be expressed by

$$P(S_v) = \exp\ (-\nu(S_v)T_0) \quad (4)$$

where T_0 is the interval of time and the rate of occurrence ν was evaluated with the expression

$$\nu(S_v) = \nu_0(S_v^{-r} - S_{v_1}^{-r}) \quad (5)$$

In Eq.(5) ν_0 and r depend on the seismicity of the region and S_v is the maximum S_v which may occur at the site. In Fig. 4 the $P(S_v)$ obtained from the seudovelocity spectra associated to the 20 year record sample are presented; it was assumed that S_v =200 and 412 cm/s, and T_0 =50 and 100 years. The values of S_v were associated to large events at close epicentral distances which would produce peak ground accelerations of 250 and 500 cm/s^2, respectively (Ref.4).

3) Values of $S_v(0.10)$ were simulated from the distribution of maximum intensities for 50 and 100 years. For each simulated S_v one of the eight accelerograms mentioned above was randomly chosen and scaled so as to give place to the corresponding value of $S_v(0.10)$.

SIMULATION OF THE NONLINEAR SHEAR SYSTEMS

As mentioned before, the behaviour of the columns of the frame system is characterized by two parameters: the initial lateral stiffness K and yield shear force V_y (Fig. 2). It can be shown (Ref.4) that K is a function of the elasticity modulus of the concrete, E, the inertia moment of the columns cross-sections, I, and the length of the columns, L; and that V_y depends on a formulae error parameter ϕ, the yielding stress of the reinforcing steel, f_y, the effective depth of the column cross section, d, the concrete strength, f'_c, the cross section of the reinforcing steel, A_s, and the width of the column cross section, b. Accordingly to Ref.7, the following parameters can be considered as random: E, I, Φ, f_y, f'_c and d while L, b and A_s can be assumed as deterministic. Therefore, K and V_y are also random variables. It can be shown (Ref.4) that second-moment probability descriptions of K and V_y can be obtained in terms of its parameters. If it is assumed that K and V_y are statistically independent and lognormally distributed, it is possible to simulate pairs of values (K_i, V_{y_i}) with i=1...M which represent M structural systems.

APPLICATION EXAMPLE

The methodology described in the previous paragraphs was applied to estimate the probability of failure, P_f, of R/C frames with initial fundamental period of 0.5, 0.8, 1.5, 2.5 and 3.5 s and a percentage of critical damping of 0.05. The latter value is being recommended in Ref.4. The values of the parameters required to simulate the structural systems are given elsewhere (Ref.4); M was taken as 40.

The results obtained are shown in Table 1; from the table, it can be observed that the P_f of the rigid systems(periods of 0.5 and 0.8 s) on soft soil are small compared with the flexible ones, whose periods correspond to the dominant ground periods typical of this type of soil (Ref.6). The influence of S_v (the upper bound of the ground intensity) on the values of P_f is shown for the structural system with a period of 2.5 s. The influence of T (the interval of time) on P_f is also shown on the mentioned table.

CONCLUSIONS

1) The methodology proposed in this paper to estimate the reliability of simple R/C frames under seismic loading allows to incorporate with relative ease the uncertainties on the structural properties and those about the seismic excitation.

2) The probabilities of failure of the considered systems show the extreme sensitivity to the coefficient of variation of the structural resistance, to the probability distribution of the maximum ground intensity at the site of interest for a given period of time, and to the upper limit of the ground intensity.

3) A systematic study of the nominal reliabilities of realistic structural systems designed in accordance with different requirements and their comparison with observed performance of actual systems is mandatory in order to improve the usefulness of seismic risk studies.

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T_c (years)	50				100			
T (s) \ V_μ	0.05	0.15	0.25	0.40	0.05	0.15	0.25	0.40
0.5	0.8158×10^{-9}	0.3695×10^{-9}	0.3250×10^{-9}	0.5316×10^{-9}	0.2452×10^{-5}	0.1380×10^{-5}	0.0970×10^{-5}	0.0755×10^{-5}
0.8	0.2335×10^{-8}	0.1078×10^{-8}	0.1335×10^{-8}	0.4437×10^{-8}	1.1700×10^{-5}	0.6130×10^{-5}	0.4255×10^{-5}	0.3540×10^{-5}
1.5	3.69×10^{-3}	2.63×10^{-3}	2.03×10^{-3}	1.53×10^{-3}	27.47×10^{-3}	23.35×10^{-3}	20.49×10^{-3}	17.48×10^{-3}
2.5	20.11×10^{-3}	16.20×10^{-3}	13.78×10^{-3}	11.46×10^{-3}	40.50×10^{-3}	33.37×10^{-3}	28.80×10^{-3}	24.30×10^{-3}
3.5	26.60×10^{-3}	24.30×10^{-3}	22.66×10^{-3}	20.84×10^{-3}	27.46×10^{-3}	25.0×10^{-3}	21.50×10^{-3}	21.51×10^{-3}
2.5*		0.74×10^{-7}				0.242×10^{-4}		

TABLE 1. FAILURE PROBABILITIES OF STRUCTURAL SYSTEMS WITH NATURAL PERIODS T, COEFFICIENT OF VARIATION OF THE STRENGHT V_μ AND TIME INTERVALS T_c, ALL CASES FOR S_{v1} = 412 cm/s EXCEPT THE ONES WITH * WHICH IS FOR S_{v1} = 200 cm/s

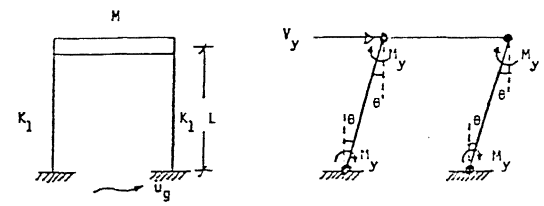

FIG 1 STRUCTURAL SYSTEM AND COLLAPSE MECHANISM

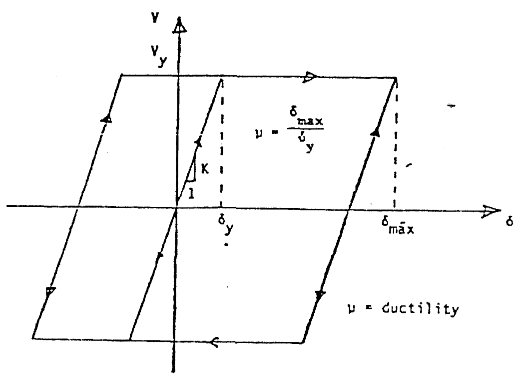

FIG 2 R/C COLUMNS ELASTOPLASTIC HYSTERETIC BEHAVIOUR

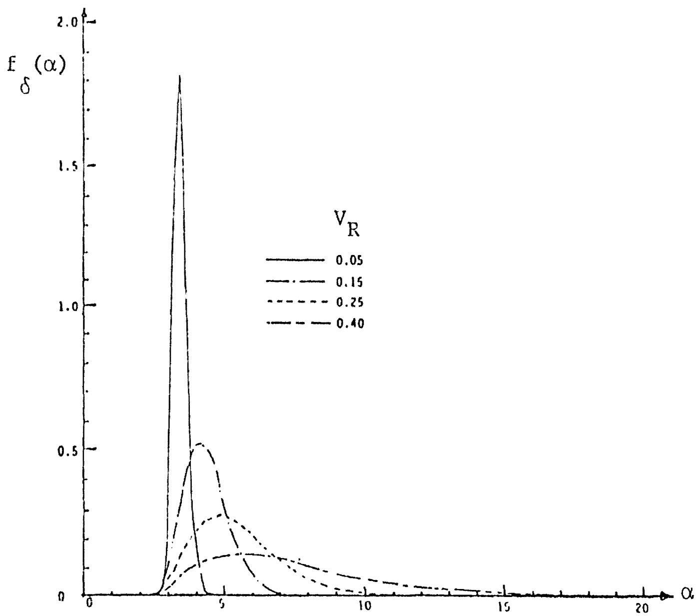

FIG 3 PROBABILITY DENSITY FUNCTIONS OF AVAILABLE DUCTILITY FOR SEVERAL COEFFICIENTS OF VARIATION OF THE STRENGHT V_R FOR $R = 4$ AND $\delta = R - 1$

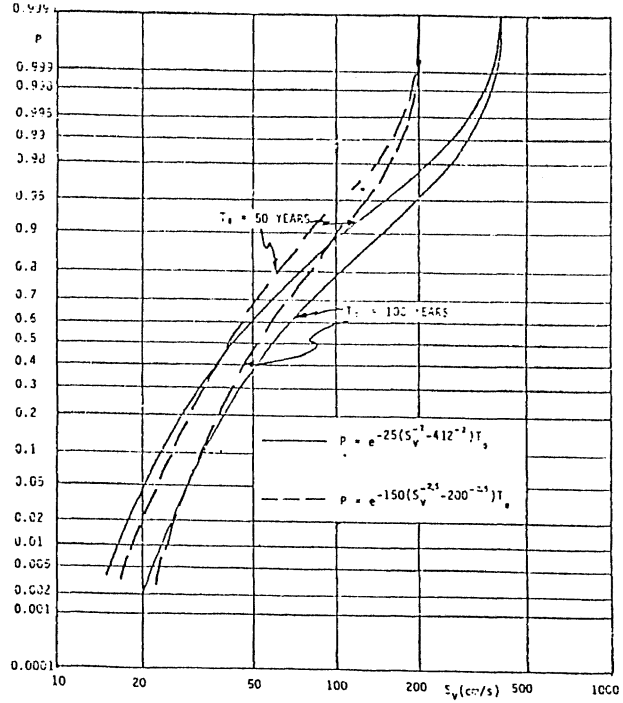

FIG 4 CUMULATIVE PROBABILITY DISTRIBUTIONS OF THE PEAK VALUES OF THE SEUDOVELOCITY SPECTRA S_v (0.10) OF MEXICO CITY SOFT SOIL