A STUDY ON APPLICATION OF THE COMPONENT MODE METHOD TO THE STRUCTURES CONSTRUCTED FROM COMPONENTS WITH DIFFERENT DAMPING

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SUMMARY

This paper presents the ways of apllying the Component Mode Method to the structures constructed of the components which have different dampings. It is shown that, for such structures whose equations of motion can not be transformed into the uncoupled form by the classical modes, the component mode method can give a simple but sufficiently rational treatment. It is also shown from a simple numerical example how the damping properties of a total system are related to those of its components.

INTRODUCTION

Generally in the modal analysis, the damping of a structure is determined by the coefficients of the classical normal modes of the structure in order to orthogonalize its equation of motion. This treatment is reasonable only for structures with the damping which satisfies the Caughey's condition. Because there are many uncertainties involved in the damping of structures, such as the effects of ground-structure interactions, it seems to be acceptable for the purpose of engineering problems to introduce the damping which is determined for the overall system.

Because, for a structure with distinct damping properties in its components, the method mentioned above is not available. The direct integral method and the complex eigenvalue approach are generally used. In this case, however, it becomes a difficult problem to determine the damping matrix of the structure.

The component mode method has been developed in order to avoid troubles relating to the storage capacity and the run time of computers and to conduct the analysis of each component independently.[1,2] The method is also effectively applied to the analyses of structures with distinct dampings in the components, since the dampings can be evaluated by the

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component modes which are necessarily calculated in the process of the method. If the damping of each component is proportinal to the stiffness, then the modal damping based on the component modes,'component-mode damping',can be determined. Accordingly, by introducing the component-mode damping, it will be possible to perform the reasonable analyses of such structures.

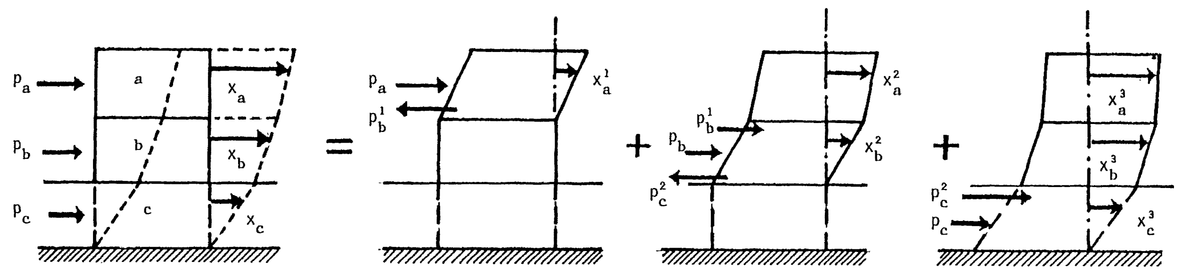
In this paper, the treatment of the damping using the component mode method is explained. The treatment of component modes, by which the damping of each component is determined, and the formation of the damping matrix, in the process of the component mode method, are shown. A method to evaluate the complex eigenvalue problems of large size structural models is shown as well. By a numerical example, using the idea of the component mode method, the relation between the dampings of a total system and its components is examined.

TREATMENT OF SUBSTRUCTURES TO ASSESS COMPONENT MODES

The substructure method using component modes, the component mode method, can be divided into several methods, according to the types of the adopted component modes. In the dynamic analyses of a structure subjected to earthquake motions, however, it is necessary to introduce appropriate components and their modes which can evaluate the dynamic properties of the structure effectively. In this paper, among the several types of the component modes, those which are shown by Benfield and Hruda[3] are adopted.

In Fig.1, the treatment of components and its meanings are shown schematically. In this figure, a structure is modeled to be composed of three components, a, b, and c. By the principle of superposition, the equilibrium equation (1) of the complete structure is divided into three equations, (2), (3), and (4). In the divided equations, there are two kinds of component displacements, that is, displacement which generates restoring force in the corresponding component and displacement which does not generate such force. In this paper, the former ones, x_a^1, x_b^2, and x_c^3, are said to be 'independent displacements'. From the independent displacements $\bar{X}$, the relative displacements X of the structure can be determined uniquely as shown in Eq.(5). The meaning of transformation matrix R, which relates $\bar{X}$ to X, and its formation are made clearer to consider the fact that the block LDL^t factorization of K matrix can be performed by R as shown in Eq.(7).

Because, the restoring force with deformation of a certain component is generated only by the corresponding independent displacement, the component modes which can represent the deformation properties of the component sufficiently must be determined for such a displacement. Accordingly, if the proportinal damping to the stiffness is considered, the independent displacement will provide a proper coordinate to evaluate the damping of the component.



$$KX = \begin{bmatrix} k_{aa} & k_{ab} & 0 \\ k_{ba} & k_{bb} & k_{bc} \\ 0 & k_{cb} & k_{cc} \end{bmatrix} \begin{Bmatrix} x_a \\ x_b \\ x_c \end{Bmatrix} = \begin{Bmatrix} p_a \\ p_b \\ p_c \end{Bmatrix} = P \quad (1)$$

$$\begin{bmatrix} k_{aa} & k_{ab} & 0 \\ k_{ba} & k_{bb} & k_{bc} \\ 0 & k_{cb} & k_{cc} \end{bmatrix} \begin{Bmatrix} x_a^1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} p_a \\ p_b^1 \\ 0 \end{Bmatrix}, \quad (2)$$

$$\begin{bmatrix} k_{aa} & k_{ab} & 0 \\ k_{ba} & k_{bb} & k_{bc} \\ 0 & k_{cb} & k_{cc} \end{bmatrix} \begin{Bmatrix} x_a^2 \\ x_b^2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ p_b - p_b^1 \\ p_c^2 \end{Bmatrix}, \quad (3)$$

$$\begin{bmatrix} k_{aa} & k_{ab} & 0 \\ k_{ba} & k_{bb} & k_{bc} \\ 0 & k_{cb} & k_{cc} \end{bmatrix} \begin{Bmatrix} x_a^3 \\ x_b^3 \\ x_c^3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ p_c - p_c^2 \end{Bmatrix} \quad (4)$$

principle of superposition

condensing eqs.

$$X = R\overline{X}$$

$$X = \begin{Bmatrix} x_a \\ x_b \\ x_c \end{Bmatrix} = \begin{Bmatrix} x_a^1 + x_a^2 + x_a^3 \\ x_b^2 + x_b^3 \\ x_c^3 \end{Bmatrix} = \begin{bmatrix} I & t_{ab} & t_{ab}t_{bc} \\ 0 & I & t_{bc} \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} x_a^1 \\ x_b^2 \\ x_c^3 \end{Bmatrix} \equiv R\overline{X} = R \begin{Bmatrix} \overline{x}_a \\ \overline{x}_b \\ \overline{x}_c \end{Bmatrix} \quad (5)$$

$$t_{ab} = -k_{aa}^{-1}k_{ab}, \quad t_{bc} = -(k_{bb} - k_{ba}k_{aa}^{-1}k_{ab})^{-1}k_{bc}$$

$$R^tKR = \begin{bmatrix} \overline{k}_{aa} & 0 & 0 \\ 0 & \overline{k}_{bb} & 0 \\ 0 & 0 & \overline{k}_{cc} \end{bmatrix}, \quad R^{-1} = \begin{bmatrix} I & -t_{ab} & 0 \\ 0 & I & -t_{bc} \\ 0 & 0 & I \end{bmatrix}, \quad \begin{aligned} \overline{k}_{aa} &= k_{aa} \\ \overline{k}_{bb} &= k_{bb} - k_{ba}(\overline{k}_{aa})^{-1}k_{ab} \\ \overline{k}_{cc} &= k_{cc} - k_{cb}(\overline{k}_{bb})^{-1}k_{bc} \end{aligned}$$

$$\begin{aligned} \overline{k}_{aa}\overline{x}_a &= \overline{p}_a \\ \overline{k}_{bb}\overline{x}_b &= \overline{p}_b \\ \overline{k}_{cc}\overline{x}_c &= \overline{p}_c \end{aligned} \quad (6)$$

$$K = (R^t)^{-1}R^tKRR^{-1} = \begin{bmatrix} I & 0 & 0 \\ -t_{ab}^t & I & 0 \\ 0 & -t_{bc}^t & I \end{bmatrix} \begin{bmatrix} \overline{k}_{aa} & 0 & 0 \\ 0 & \overline{k}_{bb} & 0 \\ 0 & 0 & \overline{k}_{cc} \end{bmatrix} \begin{bmatrix} I & -t_{ab} & 0 \\ 0 & I & -t_{bc} \\ 0 & 0 & I \end{bmatrix} \quad (7)$$

the component modes are determined for $\overline{x}_a$, $\overline{x}_b$, and $\overline{x}_c$

the block LDL^t factorization of K matrix

Fig.1 Treatment of individual components and Independent displacement $\overline{X}$

MK AND MCK TYPE EIGENVALUE PROBLEMS BY THE COMPONENT MODE METHOD

In Fig.2, the flow chart to solve the MK type and MCK type eigenvalue problems by means of the component mode method is shown.

At first, the physical coordinates X of the equation of motion (1) are transformed by the matrix R. By the transformation, while $\bar{K}$ becomes block diagonal, coupling relations occur in the mass matrix $\bar{M}$. As explained in Fig.1, the coordinates $\bar{X}$ stand for the independent displacements of each component, only by which restoring forces are generated in the component. Because the eigenvalue analyses of individual components are performed for such independent displacements, obtained eigenvectors, ϕ_a, ϕ_b, and ϕ_c, represent well the deformation properties of the components. If the components are introduced to represent well the deformation properties of the structure subjected to earthquake motions, then the behaviors of the structures can be sufficiently approximated by the first several component modes. Furthermore, by selecting necessary component modes and/or by introducing the Wilson-yuan Ritz vectors,[4] a small number of fundamental component modes, which express the properties of each substructure efficiently, can be determined. Accordingly, by such formed matrix Φ, it is possible to reduce the degrees of freedom of the total system to a sufficiently small but meaningfull size.

By a coordinate transformation by the block diagonal modal matrix Φ, Eq.(3) is transformed into the Eq.(5). In Eq.(5)', K* is a diagonal matrix composed of the eigenvalues of each component. and M* is a matrix which is coupled by the transformation. From M*, information on the coupling relation of component modes between different components can be extracted. For example, from a block matrix $\phi_a^t \bar{m}_{aa} t_{ab} t_{bc} \phi_c$, the mass orthogonality relation between ϕ_a and the deformation mode $t_{ab} t_{bc} \phi_c$ in substructure a, generated by ϕ_c in substructure c, can be examined. It is possible to think that this mass orthogonality relation represents the participating effect of ϕ_c on ϕ_a.[5]

Because component modes ϕ_a, ϕ_b, and ϕ_c are determined for the independent displacements, that is the relative displacements which produce restoring forces for each corresponding component, the proportional damping to the stiffness can be appropriately introduced by the component modes. In Fig.2, C* is determined as the component-mode dampings, which are modal dampings concerning components. For the structures composed of several components with different proportional dampings, it is quite rational to introduce such a damping matrix C*, as is determined in the generalized coordinate, introduced by the component mode method. If the damping of the structures is considered as the Rayleigh damping, then C* becomes the form $\alpha M^* + \beta K^*$, in which α and β are arbitrary proportionality factors.

By the coordinate transformation shown in Eq.(4), the degrees of freedom of Eq.(5)', can be reduced to a small size. Therefore, the sizes of M*K* type and M*C*K* type eigenvalue problems are sufficiently small and their executions become quite easy. The resulting modal matrices U and U^c are the solutions of MK type and MCK type eigenvalue problems of the total system of a large size. Accordingly, by means of the component mode

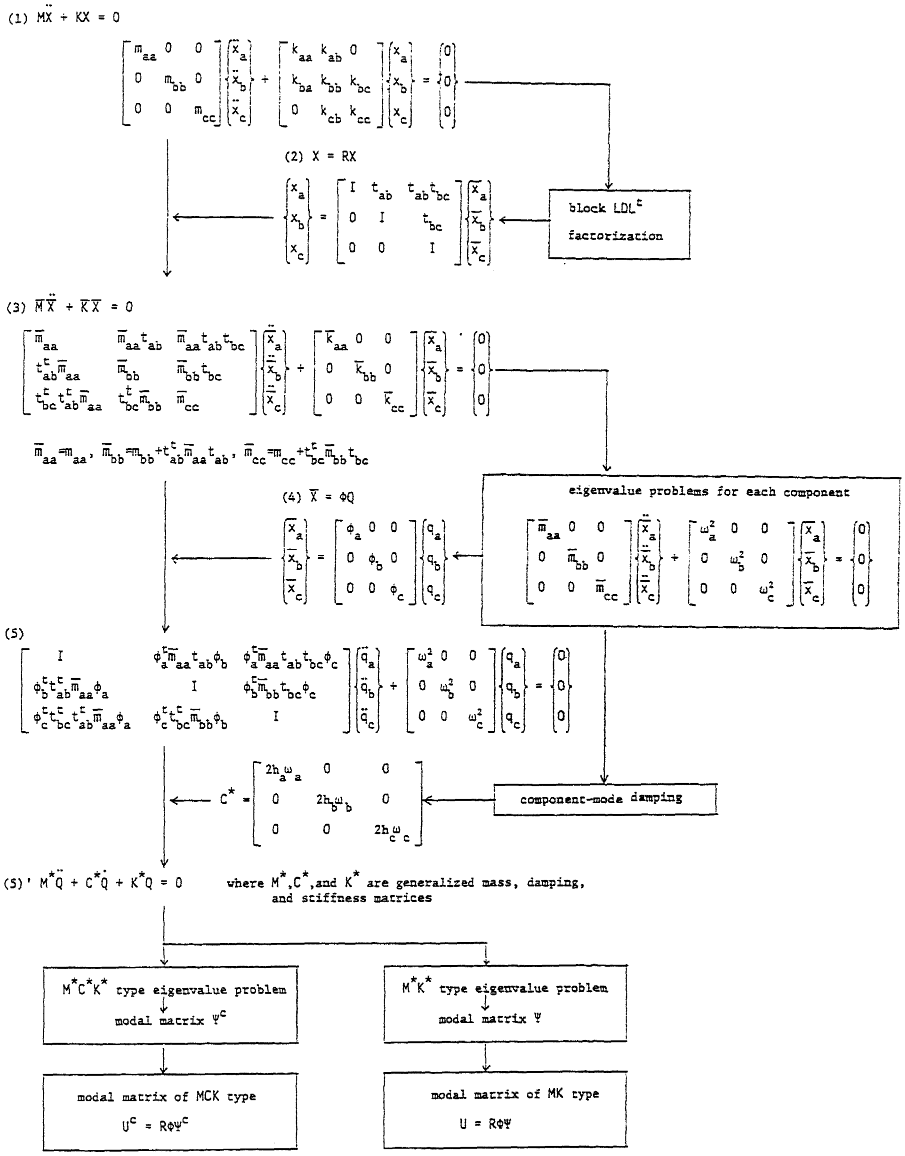


Fig.2 Flow Chart to Solve MK type and MCK type Eigenvalue Problems and Component-Mode Damping

method, the large size complex eigenvalue problems of structures with nonproportional dampings can easily and rationally be executed with the procedures shown in Fig.2.

A SIMPLE EXAMPLE TO EVALUATE THE DAMPING OF THE GROUND-STRUCTURE SYSTEM BY THE COMPONENT MODE METHOD

It is quite a difficult problem to assess the real damping properties of structures, because there are many uncertain factors affecting the damping. In general, the modal damping of structures with fixed foundations is said to be less than 0.01 and the value becomes greater according to the increase in their interactions with grounds.[7]

In Fig.3, the variations of the 1st modal damping coefficient h, of the ground-structure systems, with the change of the degree of the interactions are simulated. As shown by Eq.(1) in the figure, the systems are modeled as two degrees of freedom systems, in the generalized coordinate defined by the component mode method. In Eq.(1), hs and hg are ' component-mode dampings', and are assumed to be 0.01, for the structure components with fixed interfaces, and 0.1, for the ground components.

In the figure, two parameters, M_0 and α,are introduced to represent the degree of the ground-structure interactions. As shown in the figure, M_0 represents a modal coupling between ϕ_s and ϕ_g, and also represents the ratio of the masses, m_{ss} and m_{gg}, in the case of two mass-spring systems. If $M_0=0$, then m_{gg} tends to be infinity, and the 1st modal damping coefficient h of the total system tends to be h_g. On the other hand, α, which reveals the ratio of the 1st natural frequency Ω_1 of the total system, to the 1st natural frequency ω_s of the structure component, provides information on the stiffness relation between two components,i.e., structure and ground components. If $\alpha=1$, then $\omega_g=\infty$. This means that the ground is perfectly rigid and that h is equal to h_s. If $\alpha=0$, then $\omega_g=0$. And this means that the ground is flexible and that h tends be the value of h_g.

From the above, it follows that the variation of h, due to the change of ground-structure interactions, can be well simulated by the parameters M_0 and α. For example, the increase in the ground mass m_{gg}, which results in the smaller M_0, gives the greater deviation of h from h_s, and the decrease in the stiffness of ground, which result in the smaller α, and also gives the greater deviation of h from h_s.

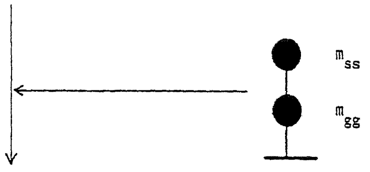
By the consideration of the modal damping of the simple two degrees of freedom systems, the relation between the dampings of a total system and its components is well explained. On the other hand, the meanings of the generalized mass M*, coupled by the transformation of the component mode method, are also clarified.

Equation of motion in generalized coordinate (subscripts s and g mean structure and ground respectively)

$$\begin{bmatrix} I & M_0 \\ M_0 & I \end{bmatrix}\begin{Bmatrix}\ddot{q}_s\\ \ddot{q}_g\end{Bmatrix} + \begin{bmatrix} 2h_s\omega_s & 0 \\ 0 & 2h_g\omega_g\end{bmatrix}\begin{Bmatrix}\dot{q}_s\\ \dot{q}_g\end{Bmatrix} + \begin{bmatrix}\omega_s^2 & 0\\ 0 & \omega_g^2\end{bmatrix}\begin{Bmatrix}q_s\\ q_g\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \qquad (1)$$

Parameters to express the degree of interaction between ground and structure:

1. $M_0 = \phi_s^t \bar{m}_{ss} t_{sg} \phi_g$, $\phi_s^t \bar{m}_{ss} \phi_s = 1$, $\phi_g^t \bar{m}_{gg} \phi_g = \phi_g^t m_{gg} \phi_g + \phi_g^t t_{sg}^t m_{ss} t_{sg} \phi_s = 1$



$$M_0^2 = \frac{m_{ss}}{m_{ss} + m_{gg}}$$

M_0 ; modal coupling ⟶ the ratio of mass

$0 \leq M_0 \leq 1$ (if $M_0=1$ then $m_{gg}=0$, if $M_0=0$ then $m_{gg}=\infty$)

2. $\alpha = \frac{\Omega_1}{\omega_s}$, Ω_1^2; eigenvalue of the first mode of the total system

$0 \leq \alpha \leq 1$ (if $\alpha=1$ then $\omega_g=\infty$, if $\alpha=0$ then $\omega_g=0$)

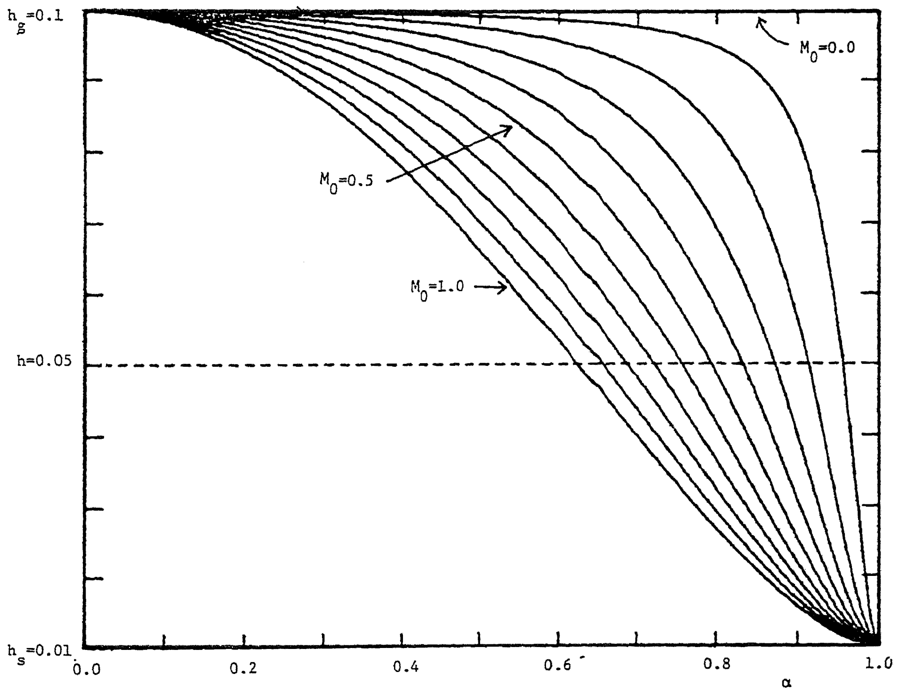


Fig.3 Variation of the modal damping h of total system with the parameters α and M_0 which are obtained from the component mode method

CONCLUSIONS

This paper presents the ways of applying the component mode method to the structures with different damping properties in their components. The following conclusions are obtained.

Based on the component mode method shown in Fig.2, it will be possible to execute the eigenvalue analyses , including the complex eigenvalue problems of a structure with large degrees of freedom. Furthermore, in the process of analyses, the interactions between the components of the structure can be understood as the coupling relation of the component modes.

The generalized coordinate introduced by the component mode method is based on the independent displacement, by which only the restoring force with deformation is generated in the component. Therefore, it is rational to determine the damping, which is stiffness proportional in each component, in the generalized coordinate.

In a simple numerical example, the relation between the dampings of a total system and its components is sufficiently explained by the parameters introduced by the component mode method.

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