A STUDY ON MODAL COUPLING ANALYSIS OF STRUCTURES BY THE COMPONENT MODE METHOD

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SUMMARY

In this paper, the conception of Two-Stage eigenvalue analysis, based on the component mode method is developed. Based on this conception, the Story-Mode coupling analysis, applicable to multistory buildings is developed as well. The idea of modal couplings obtained from the analyses is examined theoretically and numerically. The eigenvalue analysis of a building model with 3174 degrees of freedom is performed.

INTRODUCTION

The substructure method using component modes, the component mode method, has been developed in order to avoid the troubles relating to the storage capacity and the run time of computers and to perform the analysis of each substructure independently. In general, the substructure method provides information on the interactions among substructures, which is not obtained from the method treating the complete structures. Accordingly, the authors have developed the component mode method as one which enables reserchers to assess the dynamic properties of a structure based on information about the interactions between the substructures.

In this paper, the component mode method, which is devided into several methods according to the treatment of substructures, is used to develop the Two-Stage eigenvalue analysis method. In this method, several fundamental modes of the substructures are selected for the purpose of not reducing truncation errors but evaluating the interactions between substructures effectively.[1,2,3] As an application of the method, the idea of the Story-Mode coupling analysis for multistory buildings is developed. Furthermore, how the interactions between substructures can be observed from the analysis is explained. As a numerical example, the eigenvectors of a three dimensional building model with 3174 degrees of freedom, calculated by means of the Story-Mode coupling analysis are illustrated.

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TWO-STAGE EIGENVALUE ANALYSIS

The idea of the Two-Stage Eigenvalue Analysis based on the component mode method is shown in Fig.1.

In this figure, a total structure is devided into five substructures. There are several treatments of the substructures. In a given substructure, the treatments can be classified depending on whether the displacements of other substructure are made fixed or free, and whether the forces of other substructure are zero or not, as well as other factors.

In the first stage of the analysis, the eigenvalue analyses of the substructures are performed to extract several fundamental modes, such that they represent the properties of the substructures effectively. Furthermore, if necessary, the Ritz function approach,[4] which facilitates the evaluation of the arbitrary deformation modes of each substructure, can be used as the fundamental modes. The component modes are determined by the mass normalizing fundamental modes of substructures. By synthesizing the necessary component modes of each substructure and by considering the way of the treatment of substructures, a transformation matrix L, which relates the physical coordinates X to the generalized coordinates Q, can be determined. In the formation of L, the degrees of freedom can be sufficiently reduced by selecting necessary component modes in each substructure. The generalized mass, M^*, and stiffness, K^*, are orthogonalized within each substructure. That is the diagonal block of M^* and K^* are diagonal. Because the component modes are selected and determined to represent the deformation properties of the component well, from M^* and K^*, the coupling relations of component modes between different substructures can be extracted.

In the second stage, the eigenvalue analysis of the generalized equation, which is locally orthogonalized by the transformation matrix L, is performed to orthogonalize the equation globally. Since the size of the eigenvalue analysis is reduced to a considerably smaller size in the first stage, the execution of the analysis will be very easy. Accordingly, not only M^*K^* type but $M^*C^*K^*$ type eigenvalue problems can be done easily. As for the damping of structures, it seems to be quite rational to determine the damping matrix for the generalized coordinates, since they are formulated by the fundamental modes of each substructure. The obtained modal matrices G and G^c, combined with the transformation matrix L, represent the MK type and the MCK type modal matrices of the total system.

As mentioned above, the eigenvalue problem of the total system can be performed by the eigenvalue analyses of the individual substructures and a eigenvalue analysis of the generalized system whose degrees of freedom are sufficiently reduced to be a small size, by selecting necessary component modes. Accordingly, by the two-stage eigenvalue analyses, large size eigenvalue analyses can be conducted economically and information on the coupling relations of components can be extracted.

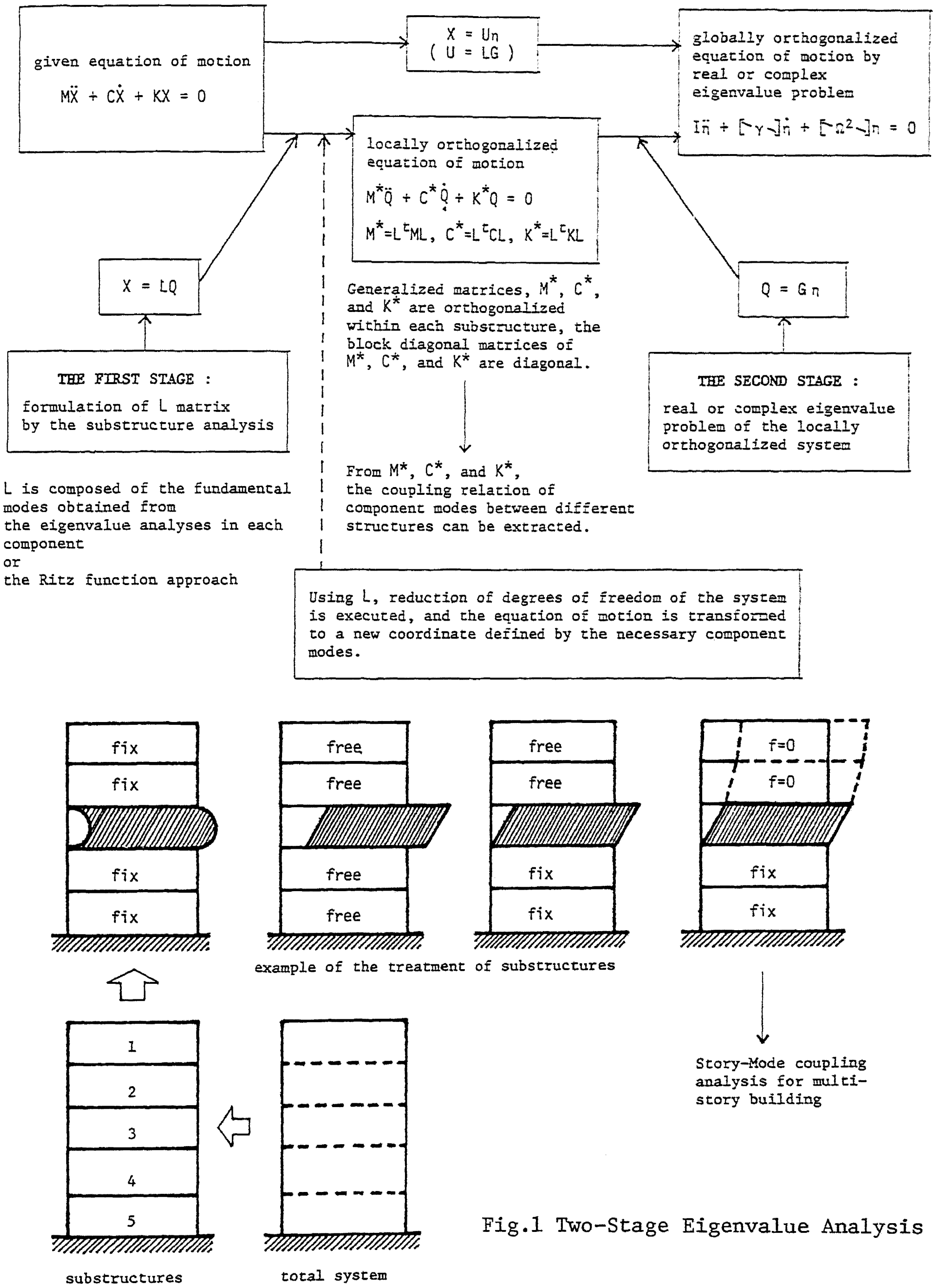


Fig.1 Two-Stage Eigenvalue Analysis

STORY-MODE COUPLING ANALYSIS

As an application of the two-stage eigenvalue analysis, the story-mode coupling analysis method for multistory buildings is developed in Fig.2.

For an N-story building, if the component mode method is applied, it is quite rational to divide the building into N components. In Fig.2, therefore, N components are introduced. In this case, since a story becomes a component, all information obtained from the two-stage eigenvalue problem can be used for story-modes coupling analysis as well. Therefore, the component modes become story-modes, the coupling relation of component modes becomes the coupling relation of story-modes, and so on.

The treatment of each component, that is each story, is shown in Fig.2 schematically. This treatment is based on the method by Benfield and Hruda[3]. Without introducing the complicated idea of the interface, the procedure of the component mode method by them can be easily be extracted to the N-story model by introducing a transformation matrix R. It is evident from the figure, the transformation matrix R is determined in the process of the block Gaussian elimination of the stiffness matrix K. A block in this case is a story. The relative displacement X is transformed to $\bar{X}$ by R. The component modes are calculated for the displacement $\bar{X}$. In this paper, $\bar{X}$ is said to be an independent displacement. The meaning of $\bar{X}$ is understood by the figure. That is $\bar{X}_s$ is the displacement which generates a restoring force with deformation in the s-th component. As shown in the figure, the restoring force with deformation in the s-th component is generated only by the independent displacement $\bar{X}_s$. Therefore, the deformation properties of s-th component can be determined by $\bar{X}_s$ sufficiently, and thus $\bar{X}_s$ gives a proper coordinate to determine the component mode ϕ_s.

L matrix in the two-stage eigenvalue problem becomes the form $L=R\Phi$, where Φ is a block diagonal matrix composed of story modes. By the transformation by L, while the generalized stiffness matrix K^* becomes diagonal, the generalized mass matrix M^* is modally coupled. By M^*, the coupling relations of story-modes between stories can be understood. For example, from $\phi_s^t \bar{m}_{ss} \phi_{sN}$, the participating effects of ϕ_N on ϕ_s can be realized.

Because the independent displacement $\bar{X}_s$ represents the deformation properties of story s, it will be possible to approximate the $\bar{X}_s$ by first several story-modes. Therefore, the generalized mass and stiffness matrices become a small size by the transformation of L. This indicates that, based on the idea shown in Fig.2, it is quite easy to conduct the eigenvalue problems of multistory buildings. Namely, by executing a set of small size eigenvalue problems, the large size eigenvalue problems of the buildings can easily be conducted. Furthermore, in the process, information on story-mode coupling relations can be obtained.

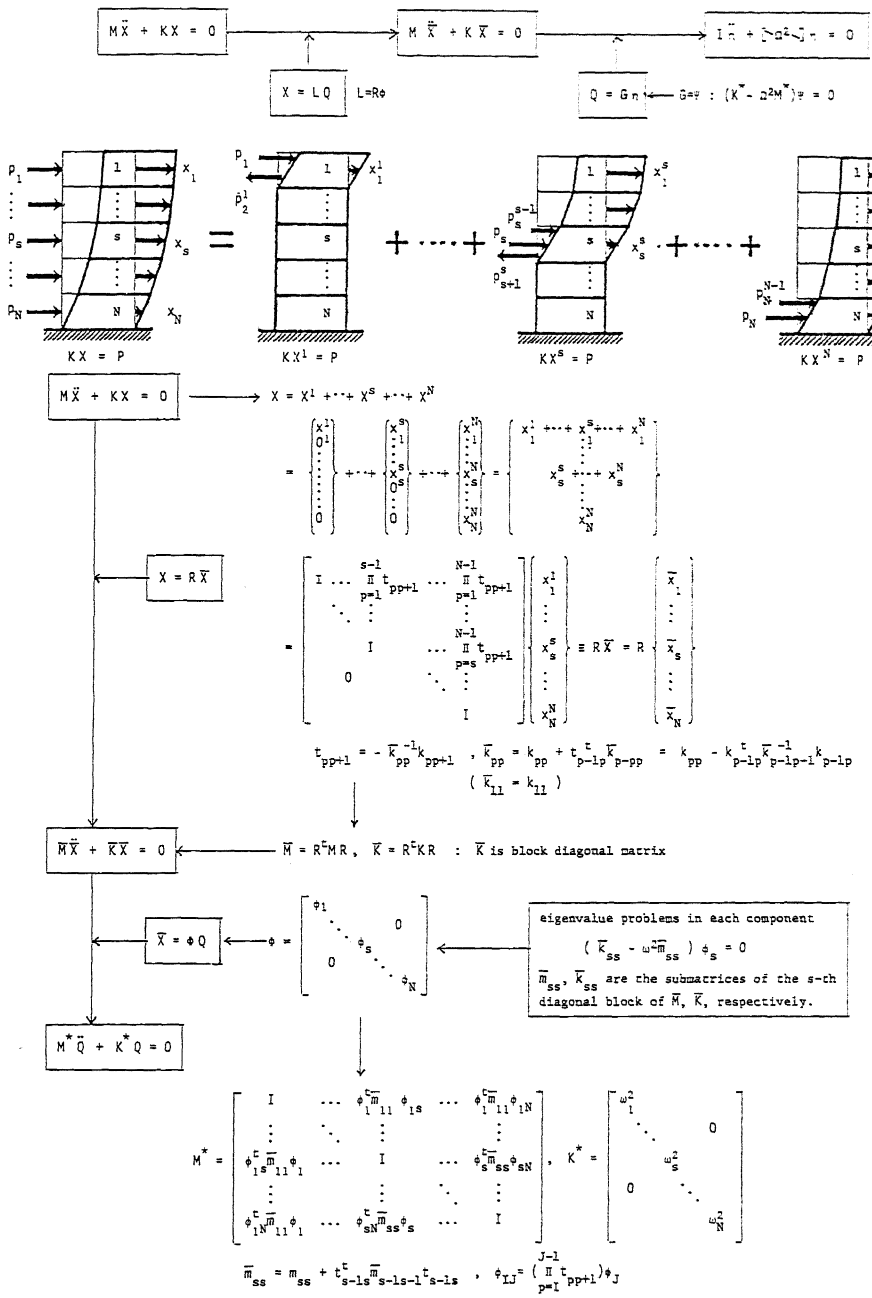


Fig.2 Story-Mode Coupling Analysis

A NUMERICAL EXAMPLE

A numerical example, using the story-mode coupling analysis, is shown in Figures 3 and 4, and Table 1. In the lower part of Fig.3, the model of a 5-story building and its substructures are shown. The building model, with 3174 degrees of freedom, is devided into 9 substructures. The first four mode shapes are illustrated in the upper part of the figure. The component modes, that is story modes, calculated in the process of the analysis are also shown in Fig.4. The values of M^*, story-mode coupled mass, are shown in Table 1.

In Fig.4, locally complicated story-modes appear due to the existences of wall elements and additional masses. In each story-mode, the span-direction displacements near the hard-wall-sections 1 and 2 are small. The second and third story-modes of component numbers from 5 to 8 are torsional modes whose centers are the hard-wall-sections 1 and 2 respectively. By the effects of additional masses which are evaluated as the weight of the penthouse of the building, the second and third modes of component number 4 are locally excitated around the hard-wall-section 1.

The coupling relation of story-modes are shown in Table 1. From the table, the rough magnitudes of interactions between stories can be deduced. For example, it is shown that the interaction between the two kinds of tortional modes is comparatively great. Other information on story-mode interactions can be extracted from table.

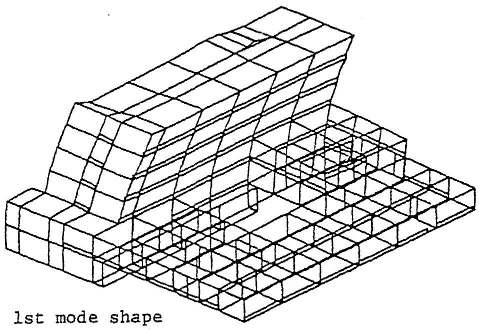
Since the mode shapes of the total building are synthesized by the story-mode, similar modes as in the stories appear. The second and third modes in the total building are evidently torsional modes, appearing also in the stories.

CONCLUSIONS

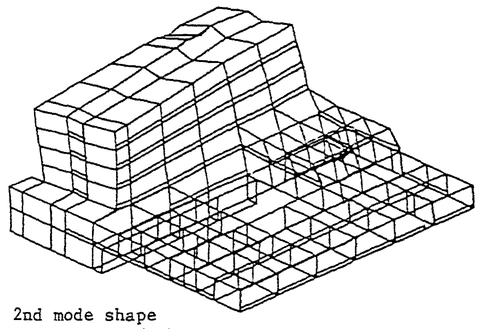
In this paper, the conception of the two-stage eigenvalue analysis based on the component mode method is developed in order not only to perform efficient analyses but also to evaluate the coupling relations between substructures. As an application of the method, the idea of story-mode coupling analysis for multistory buildings is also developed, and information on story-mode coupling relations is examined both theoretically and numerically.

REFERENCES

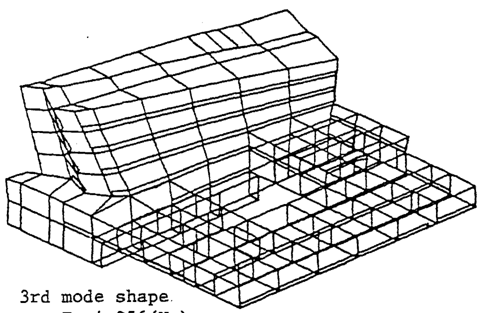
1) Yee-Tak Leung,"AN ACCURATE METHOD OF DYNAMIC SUBSTRUCTURING WITH SIMPLED COMPUTATION",International Journal for Numerical Methods in Engineering, Vol.14, 1979, pp.1241-1256
2) S.Rubin,"Improved Component-Mode Representation for Structural Dynamic Analysis", AIAA Journal, Vol.13, No.8, August, 1975, pp.995-1006
3) W.A.Benfield and R.F.Hruda,"Vibration Analysis of Structures by Component Mode Substitution", AIAA Journal, Vol.9, No.7, July, 1971, pp.1255-1261
4) E.L.Wilson,Ming-Wu Yuan, and J.M.Dickens,"DYNAMIC ANALYSIS BY DIRECT SUPPERPOSITION OF RITZ VECTORS", Earthquake Engineering and Structural Dynamics, Vol.10, 1982, pp.813-821



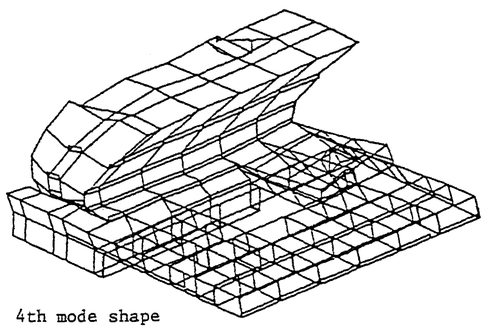
1st mode shape
F= 2.252(Hz)
T= 0.444(Sec)



2nd mode shape
F= 3.233(Hz)
T= 0.309(Sec)



3rd mode shape
F= 4.256(Hz)
T= 0.235(Sec)



4th mode shape
F= 6.122(Hz)
T= 0.163(Sec)

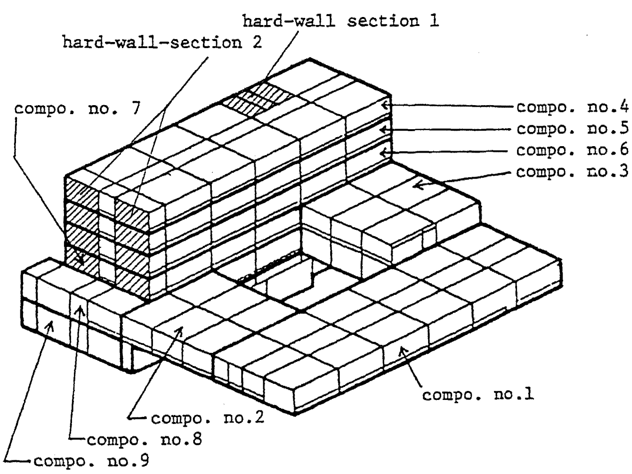


Fig.3 Model and Modal Shapes

SUBSTRUCTURE DATA

compo.no.	nodes	D.O.F
1	66	396
2	35	210
3	50	300
4	52	312
5	52	312
6	54	324
7	55	330
8	68	408
9	97	582
total	529	3174

MODEL DATA

904	beam,column elements
455	wall elements
529	nodes
3174	degree-of-freedom

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1st	1.00 0. 0.	-0.40-0.15-0.34	-0.33 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* 0.02 0.02	-0.01-0.02 0.04	0.01-0.01 0.02
	0. 1.00 0.	0.03 0.14-0.23	0.03 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	* -0.02 *	0.01 * *
	0. 0. 1.00	-0.15-0.31-0.05	0.09 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	* 0.03-0.01	-0.02 * *
2nd		1.00 0. 0.	0.20 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* -0.01-0.01	0.03 0.01-0.02	* 0.04-0.02
		0. 1.00 0.	0.01 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	* -0.02-0.05	0.02-0.02-0.04
		0. 0. 1.00	0.13 * *	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	-0.02 * *	* -0.03 *
3rd			1.00 0. 0.	0. 0. 0.	0. 0. 0.	0. 0. 0.	0.06-0.10-0.09	0.08 0.04-0.12	-0.01 0.07-0.08
			0. 1.00 0.	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	* * *	* * *
			0. 0. 1.00	0. 0. 0.	0. 0. 0.	0. 0. 0.	* * *	* * *	* * *
4th				1.00 0. 0.	-0.81-0.02 0.30	0.74-0.01-0.04	-0.64-0.07-0.05	-0.57 0.02 0.04	-0.05-0.58 0.14
				0. 1.00 0.	* 0.20 0.22	* 0.14-0.14	0.01 0.06-0.14	-0.02-0.09-0.11	0.11-0.02-0.09
				0. 0. 1.00	-0.04 0.65 0.15	0.04 0.58 *	-0.06 0.43-0.16	-0.07-0.44-0.01	0.44-0.03 0.06
5th					1.00 0. 0.	-0.92 * 0.03	0.82 0.08 0.06	0.74-0.03-0.08	0.03 0.73-0.19
					0. 1.00 0.	* 0.91 0.12	-0.05 0.74-0.16	-0.06-0.72 0.05	0.70 * 0.14
					0. 0. 1.00	-0.01 0.07-0.70	0.07-0.11-0.59	-0.04-0.03-0.55	0.11-0.12-0.52
6th						1.00 0. 0.	-0.93-0.07-0.05	-0.84 0.03 0.11	-0.03-0.81 0.23
						0. 1.00 0.	-0.04 0.83-0.30	-0.07-0.83-0.07	0.83-0.02 0.05
						0. 0. 1.00	-0.07 0.22 0.35	0.10-0.03 0.84	-0.10 0.21 0.81
7th							1.00 0. 0.	0.92 * -0.19	* 0.87-0.31
							0. 1.00 0.	0.03-0.91 0.17	0.87 0.11 0.28
							0. 0. 1.00	0.19 0.22 0.89	-0.35 0.28 0.80
8th								1.00 0. 0.	-0.04 0.96-0.12
								0. 1.00 0.	-0.97-0.05-0.13
								0. 0. 1.00	-0.15 0.12 0.94
9th									1.00 0. 0.
									0. 1.00 0.
									0. 0. 1.00

'*' is a small value less than 0.01

Table 1 Modally Coupled Mass M^*

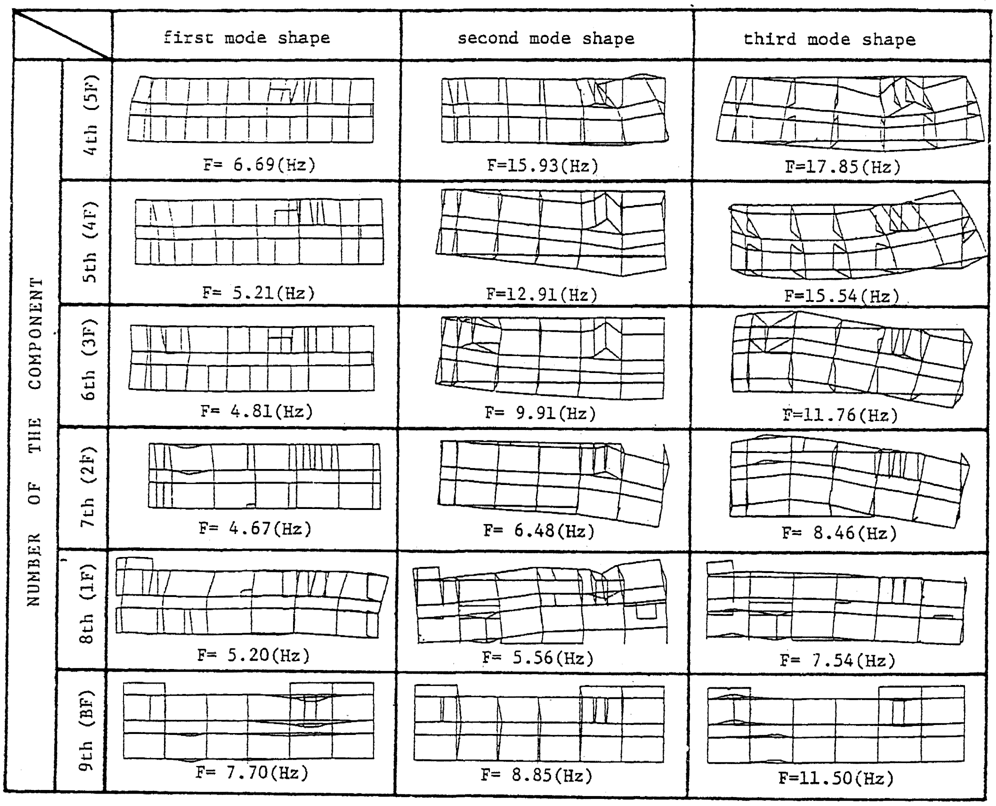


Fig.4 Modal Shapes of the Components