STRUCTURE IDENTIFICATION USING CLASSICAL AND NONCLASSICAL NORMAL MODES

S. T. Mau (I)
Presenting Author: S. T. Mau

SUMMARY

The nonclassical normal mode approach for dynamic response of structures is applied to vibrational test data of an arch dam using system identification techniques. Due to the effects of foundation and reservior interaction, the actual behavior of an arch dam may not be modelled truthfully by a proportional damping assumption. However, classical mode results are also included for comparison. The nonclassical mode results give a better fit to the test data. The present study indicates the feasibility of using nonclassical modes in structure identification problems. Based on the requirements of structural identification, recommendations are made on the suitable way of dynamic testing.

INTRODUCTION

The effects of nonproportional damping have been studied by several researchers before, Refs. 1-4. Bounds on damping matrix were established if the effects of nonporportional damping are to be neglected, Ref. 5. While most structure behavior can be modelled by proportional damping with subsequent benifit of using the simple classical mode solution approach, structures with pronounced foundation interaction effect may have to be modelled with a nonproportional damping matrix that necessitates the use of nonclassical normal mode formulation if the advantage of mode superposition is to be retained, Ref. 6.

In the course of studing the dynamic behavior of an arch dam, Ref. 7, it is found that the classical normal mode model may be used to obtain a general understanding of the dynamic characteristics of the dam. Closer inspection of the steady-state test data, Ref. 8, indicates that phase lag between dam crest radial displacement responses exists. This phenomenon can be explained by the effect of model interference and/or nonclassical normal mode behavior. Thus it is interesting to examine whether the test data can be explained by the classical normal modes alone or the nonclassical mode model gives better explanation.

In the following, the nonclassical normal mode formulation for dynamic problems is reviewed and basic equations rederived to fit the purpose of structure identification. The equation for nonclassical modes can also be reduced to that for the classical mode as indicated later. Thus the identification can easily be carried out for both models. The arch dam test data is used and results compared and discussed.

(I) Professor of Civil Engineering, National Taiwan University, Taipei, Taiwan, Republic of China

BASIC EQUATIONS

The conventional equations of motion for a N-DOF system,

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{q}} + \underset{\sim}{C}\,\dot{\underset{\sim}{q}} + \underset{\sim}{K}\,\underset{\sim}{q} = \underset{\sim}{F}(t) \tag{1}$$

can be put into the following first order form, (Ref. 9)

$$\underset{\sim}{R}\,\dot{z} + \underset{\sim}{G}\,z = \underset{\sim}{S}(t) \tag{2}$$

by the transformation

$$R = \begin{bmatrix} \underset{\sim}{O} & \underset{\sim}{M} \\ \underset{\sim}{M} & \underset{\sim}{C} \end{bmatrix} \qquad G = \begin{bmatrix} -\underset{\sim}{M} & \underset{\sim}{O} \\ \underset{\sim}{O} & \underset{\sim}{K} \end{bmatrix}$$
$$z = \left\{ \begin{matrix} \dot{\underset{\sim}{q}} \\ \underset{\sim}{q} \end{matrix} \right\} \qquad S = \left\{ \begin{matrix} \underset{\sim}{O} \\ \underset{\sim}{F} \end{matrix} \right\} \tag{3}$$

Assuming $\underset{\approx}{z} = e^{\alpha t}\underset{\approx}{\Phi}$ for the homogeneous solution of Eq.(2), the following eigenvalue problem of order 2N is obtained.

$$\alpha\,\underset{\approx}{R}\,\dot{\underset{\approx}{\Phi}} + \underset{\approx}{G}\,\underset{\approx}{\Phi} = \underset{\sim}{O} \tag{4}$$

There exist 2N eigenvalues α_n and 2N eigenvectors $\underset{\sim}{\Phi}_n$

$$\underset{\sim}{\Phi}_n = \left\{ \begin{matrix} \alpha_n \underset{\sim}{\phi}_n \\ \underset{\sim}{\phi}_n \end{matrix} \right\} \tag{5}$$

The eigenvectors are complex conjugate pairs if the eigenvalues are pairs.

For nonhomogeneous solutions, a modal superposition approach can be used,

$$\underset{\approx}{z} = \Sigma\,\underset{\sim}{\Phi}_n\; y_n(t) \tag{6}$$

The modal equations, using orthogonality of the eigenvectors, is then,

$$R_n\,\dot{y}_n - \alpha_n R_n y_n = \underset{\sim}{\phi}_n^T\,\underset{\sim}{F}(t), \quad n = 1,2,\ldots 2N \tag{7}$$

where

$$R_n = 2\alpha_n\,\underset{\sim}{\phi}_n^T\,\underset{\sim}{M}\,\underset{\sim}{\phi}_n + \underset{\sim}{\phi}_n^T\underset{\sim}{C}\underset{\sim}{\phi}_n$$

For the present formulation, the forcing function is periodic, i.e.,

$$\underset{\sim}{F}(t) = \underset{\sim}{f}\,e^{i\omega t}$$

and the solution for $\underset{\sim}{q}$, obtained from Eqs.3,6 and 7, is

$$\underset{\sim}{q} = \Sigma\,\underset{\sim}{\phi}_n\,\frac{\underset{\sim}{\phi}_n^T\,\underset{\sim}{f}}{(i\omega-\alpha_n)R_n}\,e^{i\omega t} \tag{8}$$

The eigenvectors $\underset{\sim}{\phi}_n$ consist of real and imaginary parts,

$$\underset{\sim}{\phi}_n = \underset{\sim}{\phi}_{nr} + i\underset{\sim}{\phi}_{ni}$$

The complex eigenvalues can be expressed in terms of equivalent natural frequency ω_n and equivalent damping ratio ξ_n.

$$\alpha_n = -\omega_n\xi_n + i\omega_n \sqrt{1 - \xi_n^2}$$

The complex coefficient R_n can also be represented by its real and imaginary parts R_{nr} and R_{ni}. After lengthy but straightforword substitution, the real part of the displacement response, due to the nth mode, can be put into the following form

$$\begin{aligned} q_n &= A_n \{ (P_1 W_1 + P_2 W_2) \cos\omega t + (P_1W_3 + P_2W_4) \sin\omega t\} \phi_{nr} + \\ & A_n \{ (P_1 W_2 - P_2 W_1) \cos\omega t + (P_1W_4 - P_2W_3) \sin\omega t\} \phi_{ni} \end{aligned} \tag{9}$$

where

$$P_1 = 2(\phi_{ni}^T f + d_n\phi_{nr}^T f)$$

$$P_2 = 2(d_n\phi_{ni}^T f - \phi_{nr}^T f)$$

$$W_1 = \omega_n\xi_n(\omega^2 + \omega_n^2)$$

$$W_2 = \omega_n (\omega^2- \omega_n^2) \sqrt{1 - \omega_n^2}$$

$$W_3 = \omega \quad \omega^2 - \omega_n^2 (1-2\xi_n^2)$$

$$W_4 = -2\omega\omega_n^2\xi_n \sqrt{1 - \xi_n^2}$$

$$A_n = R_{ni}\{ (1 + d_n^2) \{ (\omega^2+ \omega_n^2)^2- 4\omega^2\omega_n^2(1 - \xi_n^2)\}\}^{-1}$$

and

$$d_n = R_{nr} / R_{ni}$$

Eq.(9) is the nonclassical normal mode representation of the displacement. The classical normal mode representation can be easily obtained from Eq.(9) by letting $\phi_{ni}=0$, $R_{nr}=0$ and $P_1=0$. After some manipulation, the familiar modal solution is reproduced.

$$q_n = (\frac{-\phi_{nr}^T f}{m_n}) \frac{(\omega^2-\omega_n^2) \cos\omega t - 2\omega_n\xi_n\sin\omega t}{(\omega^2- \omega_n^2)^2+ (2\omega_n\xi_n)^2} \phi_{nr} \tag{10}$$

SYSTEM IDENTIFICATION

Eqs. (9) and (10) can be used to identify natural frequency and damping ratio from frequency response curves obtained in steady-state forced vibration tests of a structure. They can also be used to identify mode shapes if response amplitudes and phase angles of enough points of a structure are obtained from steady-state tests.

In the present application, the test data of Techi arch dam, (Ref.8) is used. The dam is 180m high with a crest length of about 290m. A thorough dynamic test, consisting of ambient and steady-state forced vibration tests was carried out in 1969 by a Berkeley team with the assistance of the personnel from the National Taiwan University. Details of the dam and the test results were summarized in a report, Ref.8. It suffices for the present purpose to indicate that fifteen measuring stations on the crest was used to record responses from forced vibrational tests. Symmetric and antisymmetric periodic forces were applied at stations 7 and 10, Fig.1. Frequency response data points were obtained and system identification was carried out assuming classical normal modes, Ref.7. Reasonably good fit was obtained, although there were certain discrepencies left unexplained. It was determined that the first five modal frequencies are 2.69, 3.29, 4.62, 4.97, and 5.61 Hz, and the corresponding modal damping ratios are 0.035, 0.041, 0.095, 0.041 and 0.052, respectively.

In trying to determine the crest mode shapes, it is assumed in the present study that these frequencies and damping ratios are approximately accurate, to make things easier. Only the radial crest displacements are to be determined using measured radial displacement amplitude and phase data. To reduce the number of parameters to a manageable degree, it is further decided that only a representative three-point data from first three tests, with forcing frequencies at 2.65, 3.23 and 3.26 Hz, is used for fitting, (see Table). Since relative phase exists between responses of different points, the error is defined as the square of the vector difference bdtween the measured and calculated displacements summed over the three points and three tests.

Both classical and nonclassical normal mode models are used based on Eqs. 10 and 9 respectively. For classical normal mode model, each mode assumed involves three unknown parameters, the m_n and the two modal displacements normalized with respect to a unit first modal displacement. Also, a common factor can be eliminated from Eq.10. Thus the number of unknown parameters is three times the number of modes less one. On the other hand, the available testing data as displayed in the Table consists of a total of fifteen numbers, i.e., five from each test. Thus up to five modes with forteen parameters can be included for data fitting. A gradient search method is used for the minimization of the error. It is found that not until the five modes are used, the errors are rather too large. Only the last two, i.e., the four-mode and the five-mode model results are listed in teh Table.

For the nonclassical mode model based on Eq.9. each mode consists of seven parameters, three imaginary displacements and two real displacements normalized with respect to a unit real displacement plus d_n and R_{ni}. Again a common factor can be eliminated. Thus a total of thirteen parameters is associated with a two-mode model. The results for the two-mode model are listed in the last line of the Table.

The five classical modes and the two nonclassical modes obtained are shown in Fig.2. The calculated response amplitude and relative phase are shown in Fig.3 together with the measured ones.

DISCUSSIONS

It is seen from the above results that a two-mode model in the nonclassical approach gives better explanation to the test data than a five-mode model in the classical approach, in the sense of better curve fitting. Also the testing frequencies are clustered near the first two natural frequencies determined from previous studies, Thus a two-mode model seems to be more resonable than a five-mode approach. However, since an additional imaninary part is included in each nonclassical mode, a two-mode model is really equivalent in number to a four-mode model of the classical approach in terms of mode shape determination. Thus, it can not be concluded, from the numerical evidence alone, that the nonclassical mode approach is the better one for the present problem. However, consideration of foundation-dam interaction has lead to the conclusion before, Ref.3, that nonproportional damping effect can not be neglected. Therefore it is reasonable to conclude that for the present test case, where foundation-reservior-dam interaction compounds the damping problem, the nonclassical mode approach offers a more reasonable explanation to the test data.

Concentrating on the nonclassical results now, Fig.3 and the Table, it is obvious that the remaining error may be attributed to the influence of the third and/or higher modes. The inclusion of higher modes in the identification process would require the inclusion of tests at other forcing frequencies. In the present application, only higher frequency testing data are available, which in turn would require additional modes. Thus, it becomes clear that more tests at lower frequencies are important to the identification process. This is quite a departure from the common practice in testing, when only tests at frequencies close to the estimated natural frequencies are performed. Also, the phase lag between the exciting force and response is a valuable information and should be recorded. It is felt that the present study has indicated the possibility of utilizing both amplitude and phase data in the process of structure identification. Such a procedure may be the only way of identifing mode shapes for structure with closely spaced vibrational frequencies.

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ACKNOWLEDGEMENT

The work reported herein was supported by National Science Council, Republic of China, through Grant No. NSC-68E-0410-02(03) and NSC70-0210-E002-04, under the US-ROC cooperative research program.

Table. Relative amplitude and phase of displacement responses

	Test 1			Test 2			Test 3			Err.
	S7	S10	S11	S7	S10	S11	S7	S10	S11	
Mes.	0.343	1.000	0.737	1.000	0.733	0.396	1.347	1.000	2.209	
	163.9	0.0	5.0	0.0	-72.6	-126.7	-152.6	0.0	43.6	-----
4-M-C	0.461	0.983	0.669	0.806	0.722	0.622	1.437	1.035	2.126	
	164.7	0.0	15.5	0.0	-55.5	-124.5	-155.3	0.0	39.9	0.208
5-M-C	0.296	1.006	0.736	0.819	0.722	0.628	1.436	1.034	2.132	
	166.6	0.0	15.4	0.0	-61.1	-127.6	-152.8	0.0	42.3	0.146
2-M-N	0.407	0.904	0.789	0.914	0.799	0.449	1.393	1.013	2.173	
	142.8	0.0	15.7	0.0	-69.9	-133.3	-151.1	0.0	43.1	0.078

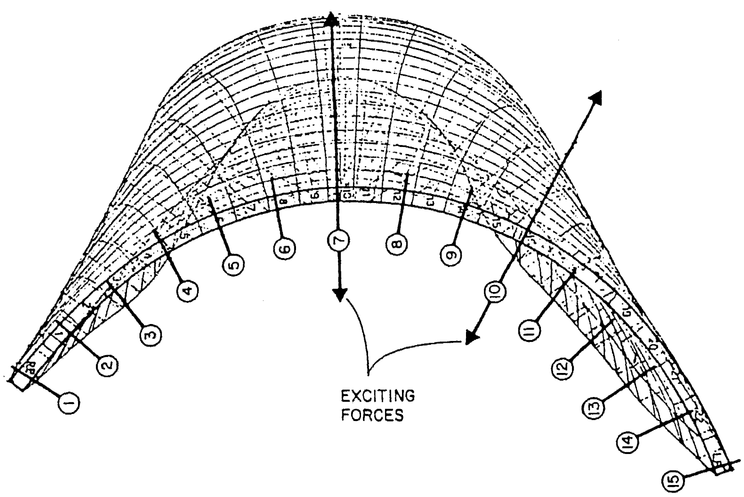

Fig.1 Plan of Techi dam showing recording and exciting stations (Ref.8)

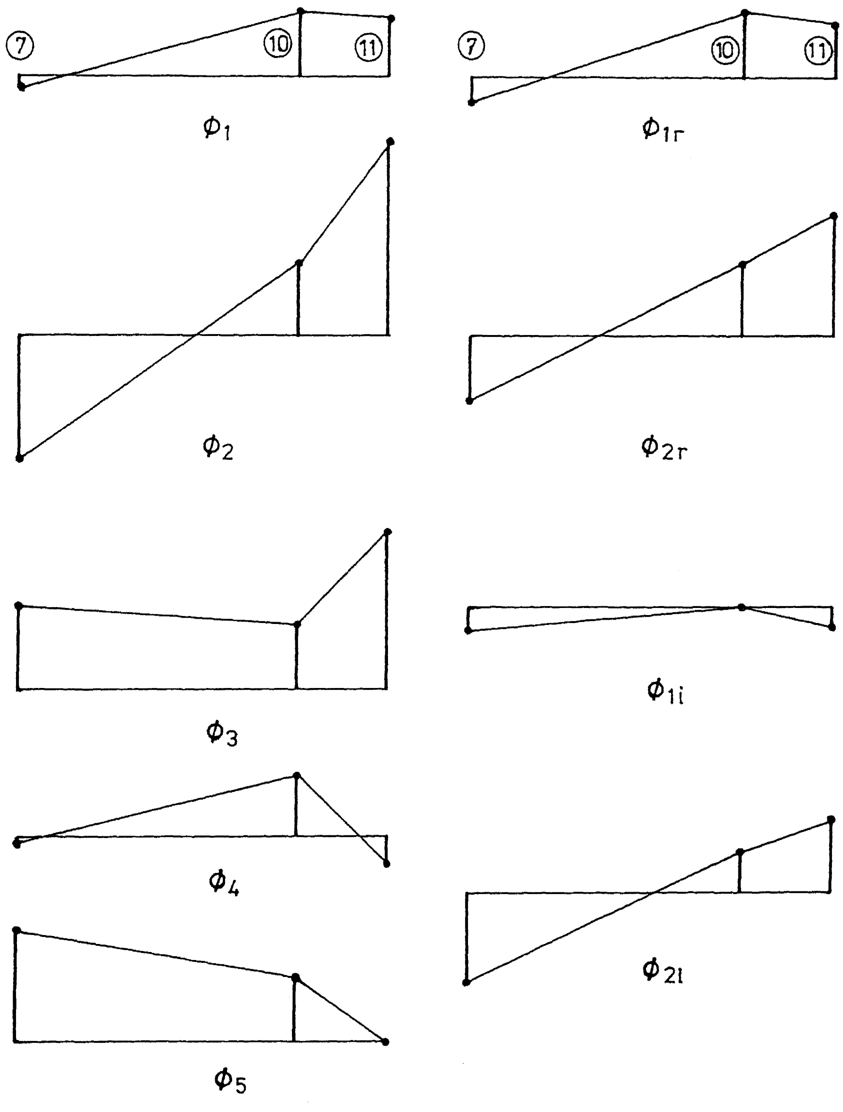

Fig.2 Calculated classical (L) and nonclassical (R) modes

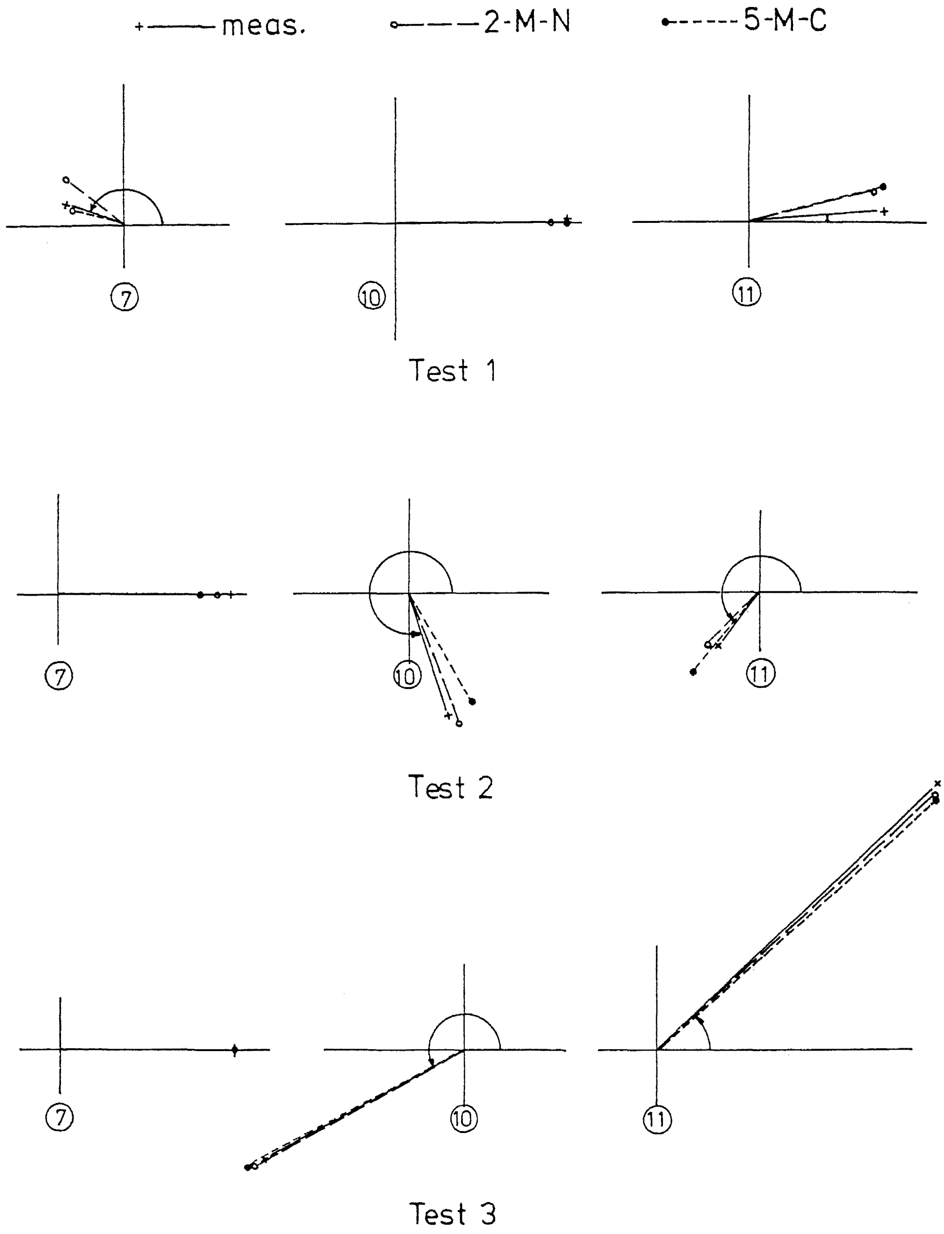

Fig.3 Relative amplitude and phase of radial displacements