LINEAR AND NONLINEAR DYNAMIC RESPONSE ANALYSIS OF COMPLEX SHELL STRUCTURES

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SUMMARY

The elastic-inelastic and geometrically nonlinear formulations and solution procedures have been developed for two types of finite elements, a 6 d.o.f. beam element and a 48 d.o.f. quadrilateral curved shell element, for the static or dynamic response analysis of beam and shell structures with one or both types of nonlinearities. Three example structures are studied: an R.C. beam, a spherical cap, and a full spherical shell. The elastic formulations are used to analyze a cooling tower shell due to earthquake and wind loads. For earthquake loads, response spectrum analysis is performed. For wind loads, frequency domain analysis is performed.

INTRODUCTION

The intent of this paper is to study the dynamic response of complex column supported thin shell structures, due to earthquake and wind loads. Finite element method is chosen and a beam and a shell element are formulated. The formulation included both geometric and material nonlinearities. The elastic formulation and the solution procedures are integrated with the response spectrum and the frequency-domain analysis methods, with specific applications to the earthquake and wind response of a cooling tower.

LINEAR ELASTIC ANALYSIS OF A COOLING TOWER SHELL

Finite Element Modeling for a Cooling Tower

Earlier analyses of cooling towers were performed for the case of fixed base (see, for example, Ref. 1). Later studies (Refs. 2, 3) revealed that the presence of discrete column supports significantly alters the seismic response behavior and the columns must be modeled accurately. Gould, Sen and Suryoutomo (Ref. 2) and Basu and Gould (Ref. 4) used an equivalent rotational-type shell element to model the columns. Gran and Yang (Ref. 3) used quadrilateral flat plate elements to model the shell and discrete column elements to model the columns. In Ref. 3, the columns were modeled more accutately, but obviously, the shell was modeled less efficiently as compared to that by using rotational-type shell elements (Refs. 2 and 4). In this study, quadrilateral shell elements with proper curvatures were employed. Because the seismic response of a cooling tower is dominated by only the eccentric modes and is characterized by membrane behavior in the middle and upper regions (Refs. 2, 3, 5), it would be advantageous to use different types of elements, ranging from

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"general" to "membrane" to model the shell optimally. A family of finite elements were adopted, modified, or extended for modeling a cooling tower. A complete description of these elements can be found in Refs. 6 and 7.

Since the base column supports and the adjoining shell region are susceptible to yielding under severe earthquakes (Ref. 3), elastic-inelastic response analyses must be performed. As a first step in that direction, a column element which can predict the elastic-inelastic dynamic behavior of reinforced concrete columns, and a quadrilateral shell element which can predict the elastic-inelastic dynamic behavior of shells were developed. A brief description of these efforts is given here, which are described in greater detail in Refs. 8 and 9.

Response Spectrum Analysis Due to Seismic Load

Method of Analysis -- The equations of motion of an elastic, viscously damped shell subjected to support motion can be written as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(t)\} \tag{1}$$

where [M], [C], and [K] are, respectively, the mass, damping, and the stiffness matrices, {q} is the vector of the relative displacements, {F(t)} is the vector of generalized nodal forces due to three components of inertial forces caused by ground motion. The three components of the inertial forces are given as

$$\begin{aligned} p_u &= \rho h \ddot{x} \cos\theta \cos\phi \\ p_v &= -\rho h \ddot{x} \sin\theta \\ p_w &= \rho h \ddot{x} \cos\theta \sin\phi \end{aligned} \tag{2}$$

in which p_u, p_v, and p_w are, respectively, the inertial forces in meridional, circumferential, and normal directions; ρ and h are, respectively, the density and thickness; θ and ϕ are, respectively, the circumferential and meridional location of a given point, and $\ddot{x}$ is the horizontal ground acceleration (parallel to $\theta = 0$ direction).

The equations of motion, Eqs. 1, can be solved using the modal superposition analysis. As the eccentric modes (modes with one circumferential wave) are known to dominate the seismic response, only these modes were considered. To avoid the tedious task of computing the Duhamel's integral for the time-history response analysis, it is a common practice in earthquake engineering to calculate only the maximum relative displacements using the response spectrum method. This approach was employed here.

The maximum value of the generalized coordinate $\eta_p(t)$ for the pth mode is given by

$$(\eta_p)_{max} = \frac{\Gamma_p}{\omega_p} S_{vp} \tag{3}$$

in which Γ_p, ω_p, and S_{vp} are, respectively, the modal participation factor, the circular natural frequency, and the psuedo-velocity corresponding to ω_p and ζ_p, with ζ_p being the damping factor for mode p. The maximum relative displacement for the given mode is then obtained by

$$\{q_p\}_{max} = \{\phi_p\} (\eta_p)_{max} \tag{4}$$

in which $\{\phi_p\}$ is the mode shape corresponding to ω_p. The total maximum response is obtained by superimposing the contribution of the considered modes. This superposition can be achieved either by the root mean square (RMS) method or by adding the absolute (ABS) values of the individual modes.

Finite Element Meshes -- Two cases were considered: one with fixed base and another with discrete column supports.

For the case of fixed base, a 5x3 mesh of general 48 d.o.f. quadrilateral shell elements was used to model a quarter of the shell. The five layers of the elements had the height ratio, starting from the base, of 1:2:3:4.02:2.825. For the case with column supports, two different meshes were used.

The first mesh (Mesh A) used to model the cooling tower had five layers of elements with the same height ratio as used for the case of fixed base. The base layer contained the "general" shell elements, the next layer contained the "general-to-membrane" quadrilateral transition elements and the remaining three layers contained the membrane quadrilateral elements.

The second mesh (Mesh B) used to model the cooling tower had six layers of elements with the height ratio, starting from the base, of 1:4:4.167:4.167:10:10. The two base layers contained "general" shell elements; the next two layers contained, respectively, the "general-to-membrane" quadrilateral transition elements and "membrane" triangular elements; and the remaining two layers contained "membrane" quadrilateral elements.

Results -- The above methods of analysis and finite element meshes were used to study the linear elastic seismic response of a cooling tower (Refs. 2, 4, and 10). The base excitation spectrum was based on an 0.12 g ground acceleration and 4% viscous damping.

The results for the natural frequencies for the first three eccentric modes for all the meshes were found to be in good agreement with the available results. The results for the maximum seismic response are presented here. Figures 1, 2, and 3 show, respectively, the variations of both RMS and ABS values of the maximum meridional, circumferential, and normal displacements along the shell height for both the fixed and flexible cases. For the column-support case, the results were obtained for both meshes A and B. But only those for Mesh A are shown. Both the meshes yielded almost identical results, with only one exception. Due to the linear assumption of the normal displacement field for triangular elements, the results in normal displacement obtained in the fourth layer of Mesh B were not in good agreement with those obtained using Mesh A. The present results agree well with those of Ref. 2 which are also shown in Figs. 1-3.

Frequency Domain Analysis Due to Wind Loads

The response of a cooling tower under dynamic wind loads was studied using probabilistic methods. A finite element formulation and Gaussian quadrature procedure were developed for studying the stationary random response of complex shells subjected to distributed random loads. The random wind loads were obtained using quasi-steady aerodynamic theory and Davenport's spectrum for wind fluctuations. Figure 4 shows the response spectral density curves for an example cooling tower subjected to three wind velocities. The material and the geometric data are also shown in Fig. 4. The details of the aforementioned quadrature procedure, wind loads and other results are given in Ref. 11.

NONLINEAR ANALYSES OF COLUMNS AND SHELLS

Nonlinear Column and Shell Element Formulation

Past experience has shown that the base columns and the adjoining shell region of a column-supported cooling tower may yield under the action of severe earthquake loads (Ref. 3). In such situations, an elastic-inelastic analysis is desirable. To perform such analysis, a 6 d.o.f. nonlinear column element and a 48 d.o.f. nonlinear doubly curved quadrilateral shell element were developed. The tangent stiffness matrix $[\bar{K}]$ of an element was obtained as

$$[\bar{K}] = \int_V [B]^T [D] [B] \, dv \qquad (5)$$

where [B] and [D] are the appropriate strain-displacement vector and the stress-strain relationships. Details of [B] and [D] for the column elements are given in Ref. 8 and those for the shell element are given in Ref. 9. For the case of a reinforced concrete column, the material nonlinearity was considered by assuming the concrete as a nonlinearly elastic-strain hardening material in compression and a linearly elastic-brittle fracture material in tension. The spread of the plastic zones for both the column and the shell elements were allowed using the concept of layered element model. The complete description of these formulations are given in Refs. 8 and 9.

Method of Analysis

The incremental equations of motion for an assemblage of nonlinear finite elements are given as

$$[M] \{\ddot{q}(t+\Delta t)\} + [C] \{\dot{q}(t+\Delta t)\} + [K(t)] \{\Delta q\} = \{P(t+\Delta t)\} - \{F(t)\} \qquad (6)$$

where [M] and [C] are, respectively, the mass and damping matrices, [K(t)] is the tangent stiffness matrix at time t; $\{q(t+\Delta t)\}$, $\{\dot{q}(t+\Delta t)\}$, and $\{\ddot{q}(t+\Delta t)\}$ are the vectors of nodal displacements, velocities and accelerations at time $t+\Delta t$, respectively; $\{\Delta q\} = \{q(t+\Delta t)\} - \{q(t)\}$; $\{F(t)\}$ is the assembled nodal forces corresponding to element stresses at time t.

For the static analysis ($\{\dot{q}(t)\}$ and $\{\ddot{q}(t)\} = \{0\}$), a step by step loading combined with Newton-Raphson iteration procedure is employed for tracing the nonlinear behavior. For the dynamic analyses, numerical integration techniques as suggested by Bathe, Ramm and Wilson (12), were employed for the solution of Eqs. 6. Wilson-θ and Newmark's methods were interchangeably used. The structure was loaded in incremental steps (of load or time) and iterations were performed at every or desired step to guarantee equilibrium to any desired accuracy.

Numerical Results

Reinforced Concrete Beam -- This example was previously analyzed in Ref. 13, using ten 6-node two-dimensional elements to model the concrete and five 3-node truss elements for steel. The material properties were given in Ref. 13. The loading, geometric data and the presently used finite element modeling are shown in Fig. 5. The results, which were obtained for two complete cycles of loading-unloading, are shown in Fig. 5 for both the studies. The agreement between the present and the previous sets of results is good for first loading-unloading sequence. The unloading, however, ended at a higher deflection in this study, causing a time lag in subsequent response. It is noted that the assumption for failure surface and loading criteria was extremely simplified in this study as compared to those used in Ref. 13.

A Clamped Spherical Cap Under Uniform Pressure (Static Analysis) -- This example, having the material and geometric data as given in Fig. 6, was previously analyzed in Ref. 14 using doubly-curved rotational-type shell elements. The material nonlinearity was considered by interpreting plastic strains as initial strains. The large deflection effects were included via an incremental approach with updated coordinate system. A finite difference solution of this problem was given by Kao (Ref. 15). Material nonlinearity was again considered by treating the plastic deformation as effective plastic loads.

In this study, due to the axisymmetric nature of the problem, a 10° segment was analyzed. Four quadrilateral elements, each with eight stress layers parallel to the midsurface (needed for integration through the thickness), were used. The effect of geometric nonlinearity was accounted for by using a Lagrangian formulation. The three sets of load versus central deflection curves are plotted in Fig. 6. The present results agree well with the previous results.

A Spherical Shell Under Uniform Step Pressure (Dynamic Analysis) -- This example, having the material and geometric data as given in Fig. 7, was previously analyzed by Ishizaki and Bathe (Ref. 16) under a step pressure of 0.5 P_{cr}, ($P_{cr} = 2E(h/R)^2/[3(1-\nu^2)]^{1/2}$), using 8-node axisymmetric elements. In this study, due to the axisymmetric nature of the problem, a 10° segment of the sphere was considered. Four elements with 8 layers in the thickness direction were used. The Newmark integration scheme was employed and as in Ref. 16, a non-dimensional time step $\tau = t(E/\rho R^2)^{1/2}$ of 0.05 was used. The two sets of the time-response results are shown in Fig. 7. The present results agree well with the previous results.

CONCLUSIONS

Some developments pertinent to the linear and nonlinear dynamic response analysis of complex shell structures were presented. Linear elastic seismic and wind response analyses were performed for a cooling tower, with and without column supports. For the case of a column-supported cooling tower, the base columns were modeled using discrete column elements. A variety of doubly curved shell elements capable of modeling the shell according to the distribution of dominating membrane and bending behaviors were used. Response spectrum analysis was performed for the seismic loads and frequency domain analysis was performed for the wind loads.

Materially and geometrically nonlinear column and shell finite elements were developed to perform nonlinear response analysis of complex shells with column supports. Examples for an R.C. column and two spherical shells were given to evaluate the present developments. Efforts are underway to study the nonlinear dynamic response of column-supported shells, such as a cooling tower.

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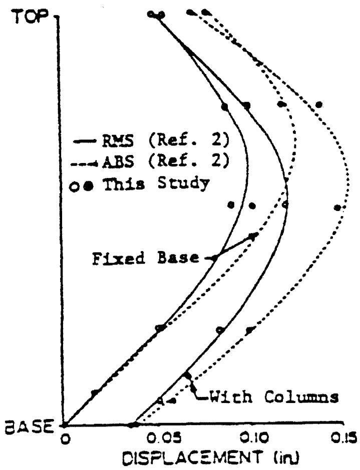

Fig. 1 Variation of the Meridional Displacement.

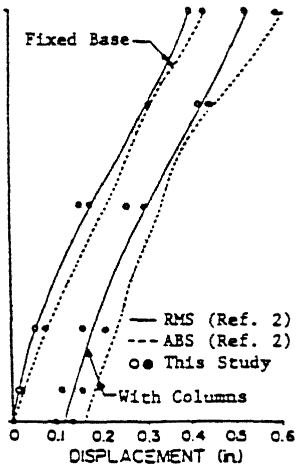

Fig. 2 Variation of the Circumferential Displacement.

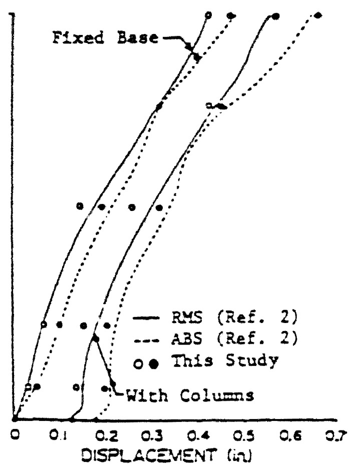

Fig. 3 Variation of the Normal Displacement.

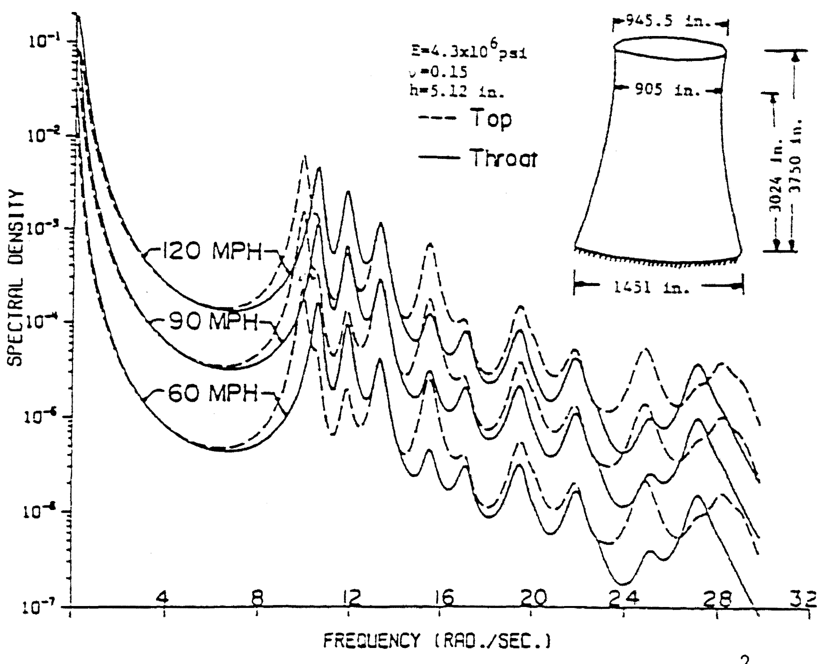

Fig. 4 Auto Spectral Density of the Normal Displacement ($In.^2$- Sec./Rad.) at the Top and Throat for Three Different Wind Velocities at the Throat.

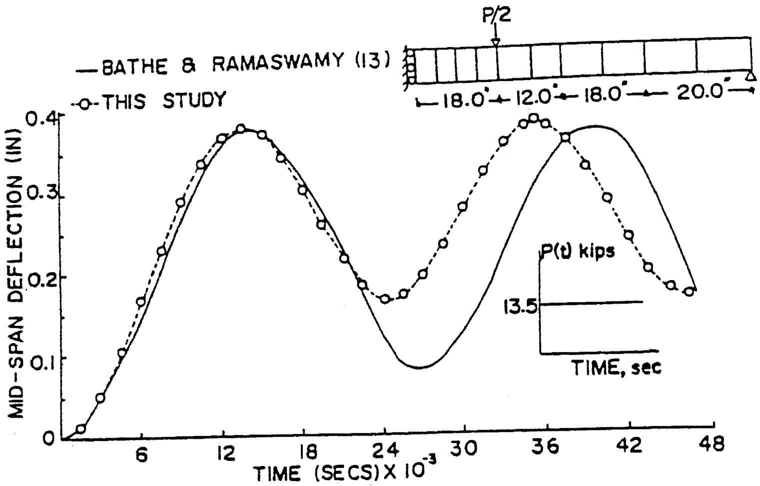

Fig. 5 Nonlinear Dynamic Response of a Reinforced Concrete Beam.

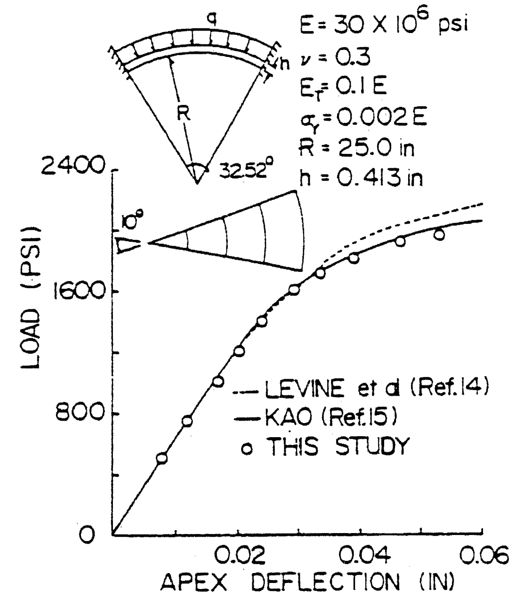

Fig. 6 Elastic-inelastic, Large Deflection Response of a Clamped Spherical Cap.

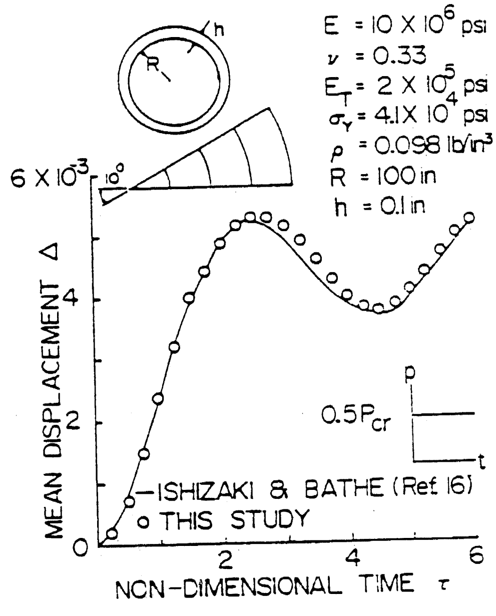

Fig. 7 Dynamic Elastic-inelastic Response of a Complete Spherical Shell.