DYNAMICAL RESPONSE OF NON-PROPORTIONALLY DAMPED MECHANICAL SYSTEMS

K. Delinić
Kraftwerk Union AG, Erlangen, W. Germany

SUMMARY

The modal superposition method is widely used for the analysis of structural response. The standard method uses the undamped eigenvalue solution. The undamped natural modes are superposed to give the response to arbitrary excitation time histories or spectra. For small damping and for proportional (Rayleigh) damping this is a satisfactory approximation.

In the case of essentially non-proportional damping the damped eigenvalue solution leads to more accurate results. Moreover it gives the reliable possibility to optimize the effect of dashpots externally built into the structure to control its dynamical behavior. This is of practical interest not only for the design of new structures (e.g. seismis isolation) but also for the reconstruction of the old ones.

INTRODUCTION

The structural response analysis of elastic structures under different dynamical excitations such as earthquake ground motion, aircraft impact, driving vibration etc., generally is performed by the modal superposition method. There is a nice feature of the modal superposition method that by knowing the natural frequencies and modes the structure becomes dynamically evident, independently of the type of excitation applied. This enables the engineer to design the structure in such a way that the natural frequencies of the structure lie outside the essential spectral content of the excitation.

The modal superposition method allows without great loss of accuracy to reduce the number of DOF (degrees of freedom), which is originally very high in the most practical cases, to a small number of essential natural modes. This is an obvious computational advantage.

Consequently, once the eigenvalue problem for a structural system is solved the response to any type of excitation can easily be found by little computational effort using always the same few natural modes.

Consider the governing differential equation of vibration

$$M\ddot{r} + C\dot{r} + Kr = p \qquad (1)$$

At the present state of the art the modal superposition method involves the solution of the undamped eigenvalue problem

$$(\omega^2 M + K)\, x = 0 \tag{2}$$

where ω denotes the unknown circular frequency and x the corresponding natural mode. The damping is then added to this solution leading to an approximation of the damped problem.

This approximation is good enough in the following cases:

- if the damping is small or
- if the damping is nearly Rayleigh (proportional) i.e. it is of the form

$$C \approx \alpha M + \beta K \tag{3}$$

(Ref. 1).

In the design of nuclear power plants there are many cases in which the structure comprehends an essentially non-proportional damping. Such damping can either be inherit by the nature of the structural system (e.g. soil- structure interaction) or is given from outside by means of point-dashpots (examples are seismic isolation of the reactor building on helical springs and dashpots, or point-dashpots for design of pipes etc.).

The task of the design engineer in the first case is to verify if the peak values of dynamical stresses are less as permitted without changing the structure. In the second case there is the posibility to reduce the dynamic response. The effect of damping can be optimized by appropriate location of point dashpots and selection of their damping characteristics. For both tasks - verification and (or) optimization - it is necessary to solve the quadratic eigenvalue problem

$$(M\omega^2 + C\,\omega + K)\, x = 0 \tag{4}$$

subordinate to the eq. (1), where now C is neither "small" nor Rayleigh.

At the first glance it is theoretically possible to solve (4) by "direct" computation, which is exhausting already in the case with two or three DOF, Ref. 2. It is easy to see that under assumptions on C made above the solution of the quadratic eigenvalue problem is essentially complex, i.e. the eigenvalues and modes are complex. (In overdamped modes the solution is "double" real, but these modes give no contribution to the steady state solution. However they may be relevant for local design of dashpots.)

The procedures available have all some of the disadvantages like the loss of symmetry or (and) bandwidth in structural matrices and generally they include the complex arithmetic. This causes an increase in the computer time or (and) computer storage, i.e. generally an increase of the computer costs. Some of the calculations become impossible at all.

This paper presents the new method for the solution of (4) and its effect on structural response analysis for strongly non-proportional damping.

Several other papers are concerned with the treatment of (non-proportional) damping in the computation of structural response, such as Clough, Mojtahedi /3/, Duncan, Taylor /4/, Novak, El-Hifnawi /5/, Traill-Nash /6/, Tsai /7/ and Warburton /8/.

DESCRIPTION OF THE METHOD

Let us modify the problem (1) by the use of substitution

$$\left.\begin{aligned} & y = L_2 r \\ & L_2^{-T}\ L_1^{\ T}\, z = y + L_2^{-T}\ C\, L_2^{-T}\, y \\ & \left.\begin{aligned} & M = L_2^T L_2 \\ & K = L_1^T L_1 \end{aligned}\right\} \text{(Cholesky)} \end{aligned}\right\} \tag{5}$$

The new system is now

$$\begin{pmatrix} \dot{y} \\ \dot{z} \end{pmatrix} = A \begin{pmatrix} y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ L_1^{-T} p \end{pmatrix} \tag{6}$$

where

$$A = \begin{pmatrix} -L_2^{-T}\ C\ L_2^{-1} & L_2^{-T}\ L_1^{\ T} \\ -L_1\ \ L_2^{-1} & 0 \end{pmatrix} \tag{7}$$

It has the double size and the symmetry is lost. Nevertheless there is another type of symmetry of (7) called "J-symmetry", Ref. 9. The matrix A in (7) is of type

$$A = \begin{pmatrix} A & B \\ -B^T & D \end{pmatrix} \tag{8}$$

where A and D are symmetric. It has the property that

$$JAJ = A^T \tag{9}$$

with

$$J = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{10}$$

I being the unit matrix of order n.

There is an obvious storage advantage of J-symmetric matrices in comparisson to general (2n) x (2n) matrices: it needs roughly $2n^2$ storage which is about half as for a non J-symmetric case.

In addition the use of the real arithmetic reduces the storage for another half.

Let us shortly review some of the properties of the J-symmetric matrices, Ref. 10. There is an analogous of orthogonal matrix in terms of the J-orthogonal defined by the property

$$U^{-1} = J\,U^T J \tag{11}$$

J being the matrix in eq. (10). It is easy to verify that if U is J-orthogonal the similarity transformation

$$A \longmapsto U^{-1}\,A\,U \tag{12}$$

preserves J-symmetry. This is an important fact for the diagonalization procedure.

The most important result is concerned with the decoupling of the system (6). A "brutal" diagonalization of the matrix A would cause introducing complex arithmetic. The corresponding complex natural modes would have no kind of orthogonality property which would itself destroy J-orthogonal structure of the iterates of A.

Therefore the new method uses a sort of "block-diagonalization" the big matrix A, which is enough to evaluate the damped eigenfrequencies and corresponding real modal dampings.

DAMPED MODAL ANALYSIS

Without going into technical details let us now assume the big matrix A to be block-diagonalized by the J-orthogonal T and that all modal parameters are known.

Transforming the coordinates in (6) by

$$\begin{pmatrix} y \\ z \end{pmatrix} = T \begin{pmatrix} u \\ v \end{pmatrix} \tag{13}$$

where u and v are time dependend u-vectors the equation (6) changes to

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\beta & \gamma \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} g \\ h \end{pmatrix} \tag{14}$$

where

$$\begin{pmatrix} g \\ h \end{pmatrix} = T^{-1} \begin{pmatrix} 0 \\ L_1^{-T} p \end{pmatrix} . \tag{15}$$

For each k there is an explicite solution of (14)

$$\begin{pmatrix} u_k\ (t) \\ v_k\ (t) \end{pmatrix} = \exp\ (A_k t) \left[\begin{pmatrix} c_{1k} \\ c_{2k} \end{pmatrix} + \int_0^t \exp\ (-A_k \tau) \begin{pmatrix} g_k(\tau) \\ h_k(\tau) \end{pmatrix} d\tau \right] \quad (16)$$

where

$$A_k = \begin{pmatrix} \alpha_k & \beta_k \\ -\beta_k & \gamma_k \end{pmatrix}$$

and c_{1k}, c_{2k} are constants depending on initial conditions. It is easy to construct the explicite formula for exp $(A_k t)$, which then inherits all the characteristics comming from the overdamped, underdamped or critically damped natural modes.

Knowing vectors u and v the final result for structural response is given by the expression

$$r(t) = L_2^{-1} \sum_{k=1}^{s} (u_k(t) \cdot t_k + v_k(t) \cdot t_{k+n}) \quad (17)$$

where L_2 is given by (5) and t_i denotes the first n components of the i-th column in T. The summation bound s takes care of the fact that normally only a few frequencies (smallest s frequencies) are essential for the dynamical response, $s \leq n$. The rest of them can then be neglected for calculating r(t), cf. also Ref. 6.

EXAMPLE

The example illustrates the new method. A simple mechanical system consists of a bending beam supported by a pair of spring-dashpots, Fig. 1. The point-dashpots give rise to strongly non-proportional damping in the system.

In Fig. 2 the new method is compared with the classical solution by the use of (undamped) modal analysis. The uniform material damping is set 0.02. In the classical approach the dashpots having the damping characteristics c = 20 kNs/m are replaced by "equivalent" springs having stiffness k = 200 kN/m. As a comparisson a 2/3 weaker dashpots (c = 13 kNs/m) are considered additionally. The vertical response at the point A of the beam is calculated due to the vertical support excitation by pure sine time function at the eigenfrequencies.

In Fig. 3 the modal parameters of the first five natural modes vs. the variation of dashpot characteristics are presented.

CONCLUSION

The important fact is that the new quadratic solver is of the same order of effort as the classical procedures. It can therefore be seen as a possible branch-point in the classical approach to the natural mode analysis.

REFERENCES

/1/ CLOUGH, R. W., J. PENZIEN, "Dynamics of Structures", Mc Graw-Hill, New York,, 1975.

/2/ HURTY, W. C., M. F. RUBINSTEIN, "Dynamics of Structures", Prentice Hall, Clifton, 1964.

/3/ CLOUGH, R. W., MOJTAHEDI, S., "Earthquake Response Analysis Considering Non-Proportional Damping", Earthquake Engineering and Structural Dynamics 4, 1976, 489-496.

/4/ DUNCAN, P. E., EATOCK TAYLOR, R., "A Note on the Dynamic Analysis of Non-Proportionally Damped Systems", Earthquake Engineering and Structural Dynamics, 7, 1979, 99-105.

/5/ NOVAK, M., EL-HIFNAWI, L., "Damping of Structures due to Soil- Structure Interaction", Proc. Fifth. Collog. on Industr. Aerodyn. Aachen 1982.

/6/ TRAILL-NASH, R. W., "Modal Methods in the Dynamics of Systems with Non-Classical Damping", Earthquake Engineering and Structural Dynamics, 9, 1981, 153-169.

/7/ TSAI, N. C., "Modal Damping for Soil Structure Interaction", J. Eng. Mech. Div. ASCE, 100, 1974, 323-341.

/8/ WARBURTON, G. B., "State-of-the-Art in Structural Dynamics", preprint, Cambridge 1982.

/9/ MAL'CEV, "Osnovy Lineinoi Algebry", Nauka, Moskva, 1970.

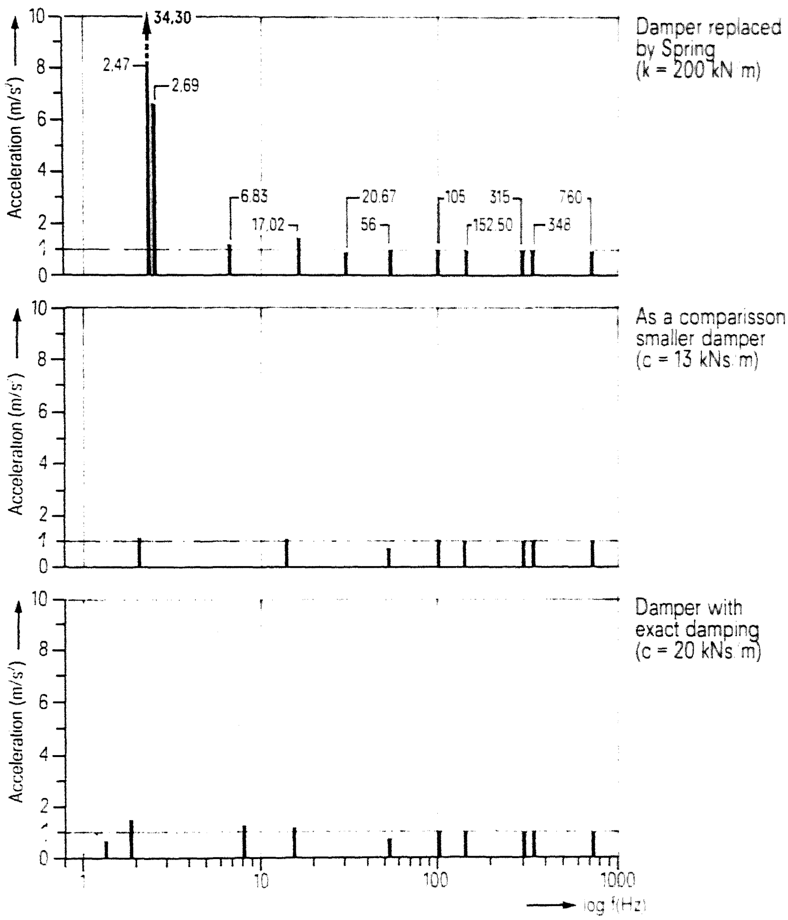

Fig. 2. Transfer for vertical acceleration in point A

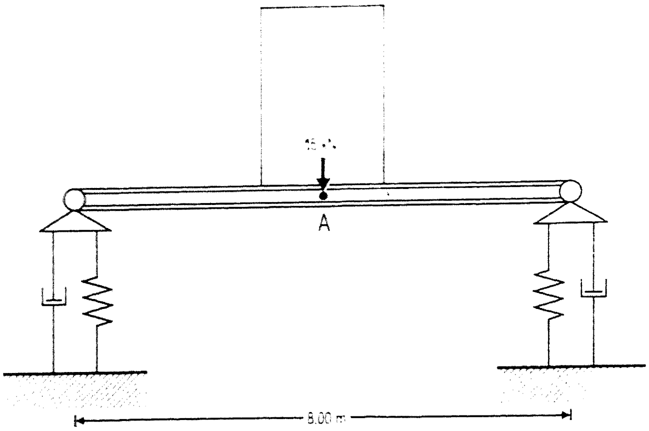

Fig. 1. Mechanical model of a bending beam

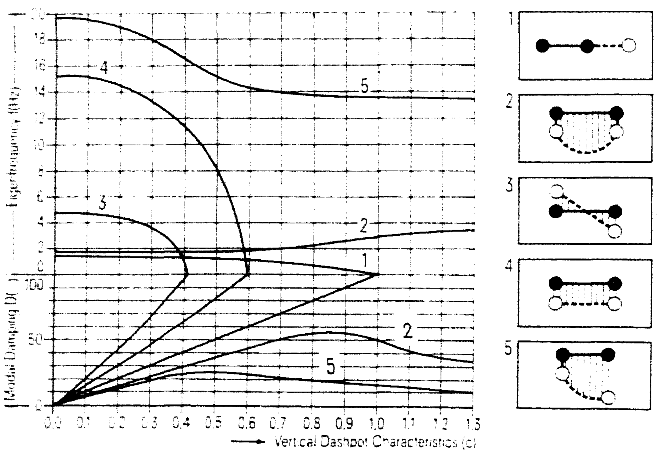

Fig. 3. K2 - diagram of the beam