APPROXIMATE ANALYSIS OF EARTHQUAKE RESPONSE OF IMPACTING STRUCTURES

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SUMMARY

An approximate technique for determining the peak earthquake response of an impacting structure is presented. The method provides a reduction in required computational effort of more than two orders of magnitude over that required by direct numerical simulation. Furthermore, the peak nonlinear response is estimated directly from a linear response spectrum, without requiring the complete time history of the excitation. An evaluation of the accuracy of the approximate response is presented in a comparison of approximate and exact solutions for an ensemble of 51 earthquake excitations. A multiple linear regression model for the error is presented and used in an example of probabilistic design.

INTRODUCTION

The evaluation of the earthquake reliability of power plants, mechanical equipment, and some building structures often requires consideration of the effects of impact between adjacent structures. This vibroimpact may occur at a seismic joint between subsections of a large building, between separate but adjacent pieces of mechanical equipment in a power plant, or between piping systems in an industrial facility. Because of the strongly nonlinear nature of dynamic impact phenomena, the analysis of vibroimpact can be quite tedious and/or expensive by existing numerical procedures. Such procedures are usually based on direct integration of the nonlinear equations of motion of the system (Refs. 1 and 2), although for some simple systems an exact analytical solution is feasible (Ref. 3) for harmonic excitation.

Presented in this paper is an approximate technique for determining the peak earthquake response of an impacting structure. The method is based on an extention of a recently developed weighted equivalent linearization technique for asymmetric nonlinear structural vibrations (Ref. 4). The essence of the method is to define an equivalent linear stiffness and damping coefficient for the nonlinear system, and then solve the resulting linearized problem to obtain an approximation to the nonlinear response.

APPROXIMATE ANALYSIS PROCEDURE

Consider the base excited SDOF oscillator constrained on one side by the elastic barrier of stiffness K as shown in Fig. 1(a). The barrier is separated from the oscillator by a gap of width d when the system is at rest. Due to the one-sided nature of the barrier, the vibratory response x(t) for

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sufficiently large earthquake base excitation is typically as shown in Fig. 2, as determined by direct numerical integration of the equations of motion. Note that the negative peak values of x are much larger than the positive peak values since the system is unconstrained on the left.

Now consider a linear comparison system with equivalent linear parameters k_e and c_e as shown in Fig. 1(b). Since the barrier is nondissipative, we choose

$$c_e = c. \tag{1}$$

However, let k_e be chosen as (Ref. 5)

$$k_e = \frac{E[x_m^2 k_H(x_m)]}{E[x_m^2]} = \frac{\int_0^{x_{max}} x_m^2 k_H(x_m) f_x(x_m)\,dx_m}{\int_0^{x_{max}} x_m^2 f_x(x_m)\,dx_m} \tag{2}$$

where E [.] represents the mathematical expectation operator, x_m is the absolute value of the local peak response, $f_x(x_m)$ is the probability density function for the response peaks, x_{max} is the global peak response during the earthquake, and $k_H(x_m)$ represents the equivalent linear stiffness for the problem with harmonic base excitation. Let $k_H(x_m)$ be obtained from the weighted equivalent linearization procedure (Ref. 4) wherein the weighting function is chosen such that the approximate and exact backbone curves for free vibration coincide at all amplitudes of vibration. Then it can be shown that

$$k_H(x_m) = \frac{k}{B(x_m)} \tag{3}$$

where

$$B(x_m)' \equiv \begin{cases} \frac{1}{2}\left[\frac{1}{\sqrt{1+\left(\frac{K}{k}\right)}} + 1\right] + \frac{1}{\pi}\left\{\sin^{-1}\left(\frac{d}{x_m}\right) - \frac{1}{\sqrt{1+\left(\frac{K}{k}\right)}}\sin^{-1}\left(\frac{d}{\sqrt{x_m^2\left(1+\frac{K}{k}\right) - d^2\left(\frac{K}{k}\right)}}\right)\right\}; x_m > d \\ 1 \;;\; x_m \leq d \end{cases} \tag{4}$$

A statistical analysis of the local response peaks x_m (obtained from direct numerical simulation) for several earthquake excitations produced a histogram for x_m similar to the one shown in Fig. 3. Motivated by the shape of the histogram in the figure and by a desire for mathematical simplicity, the probability density function $f_x(x_m)$ was chosen as parabolic in x_m. The equivalent linear stiffness k_e was then obtained by substituting the parabolic $f_x(x_m)$, together with $k_H(x_m)$ from Eqs. (3) and (4), into Eq. (2) and evaluating the integrals.

The approximate solution for the peak response is then obtained by requiring the maximum potential energy stored in the system at the global peak displacement x_{max}, to equal the maximum kinetic energy achieved by the mass during the earthquake. This may be expressed as (Ref. 5)

$$\frac{1}{2}\, k\, x_{max}^{2} = \frac{1}{2}\, m\, [SV(\sqrt{k_e/m})]^2. \tag{5}$$

where $SV(\omega_n)$ is the spectral velocity determined from the linear response spectrum for a system with natural frequency ω_n and damping coefficient c. This equation is implicitly a transcendental equation since k_e is a function of x_{max} through Eqn. (2), and it may be rewritten as

$$x_{max} = SV(\sqrt{k_e/m})/\sqrt{k/m}. \tag{6}$$

The solution of Eqn. (6) generally requires an iterative numerical approach.

Scaling of Earthquake Accelerograms

Since the transient impact behavior of the system is highly nonlinear, it is necessary to scale the earthquake accelerograms in order to minimize the statistical variations in the accuracy of the response caused by differences in the amplitude and frequency content of the various earthquake accelerograms. In this study each earthquake accelerogram was characterized by a characteristic frequency ω^* and a characteristic amplitude a^* determined as follows:

$$(\omega^*)^2 \equiv \frac{\int_0^{\omega_{max}} \omega^2 SV(\omega)\, d\omega}{\int_0^{\omega_{max}} SV(\omega)\, d\omega} \; ; \; a^* \equiv \frac{\int_0^{\omega_{max}} \omega SV(\omega)\, d\omega}{\omega_{max}} \tag{7}$$

where ω_{max} represents a preselected upperbound on the natural frequency of structures under consideration. Note that these accelerogram scale factors are dependent upon the first and second moments of the response spectrum $SV(\omega)$ vs. ω. Typical values of the scale factors determined from Eqs. (7) for a subset of the ensemble of earthquakes used in this study are shown in Table 1.

Parameter Studies

Comparisons between the exact (determined by direct numerical simulation of the equations of motion) and approximate (determined by the method outlined above) peak earthquake response x_{max} for an ensemble of 51 earthquake accelerograms were obtained for a set of parameters which included three different values for the nondimensional parameters (K/k), (ma*/kd), and $(\omega^*/\sqrt{k/m})$. Presented in Table 2 are typical numerical results obtained in the parameter study for the case when (K/k) = 100. Due to space limitations, only a small subset of these studies can be presented here.

A primary objective of the parameter studies is to evaluate the accuracy of the approximate solution for engineering purposes. Thus, a statistical analysis of the errors in the approximate solutions was performed.

Statistical Evaluation of Errors

Shown in Fig. 4 is a typical histogram of the error in the approximate response x_{max} for an ensemble of 51 earthquakes for a single set of parameters, where the error ε is defined as

$$\varepsilon \equiv \frac{x^e_{max} - x_{max}}{x^e_{max}} \tag{8}$$

where x^e_{max} is the exact peak response and x_{max} is the approximate response. As shown in the figure, the approximate response is sometimes conservative and sometimes nonconservative, but with an average μ_ε which is much smaller than the standard deviation σ_ε.

Presented in Table 3 are the means and standard deviations of the error ε and the absolute error $|\varepsilon|$ over the ensemble of all 51 earthquakes and all nine combinations of the parameters $(\omega^*/\sqrt{k/m})$ and (ma^*/kd). Thus, each row in the Table 3 represents the results of a total of 459 simulations.

Significant statistical variation in the error is associated with the accelerogram scale factors a^* and ω^*. To facilitate the construction of quantitative probabilistic estimates of the error in the response prediction x_{max}, a multiple linear regression of the error ε was performed. The factors considered in the regression model were (ma^*/kd) and $(\omega^*/\sqrt{k/m})$. The expected value of the error was modeled as

$$E[\varepsilon] = C_o + C_1(\omega^*/\sqrt{k/m}) + C_2(ma^*/kd) \tag{9}$$

where C_o, C_1, and C_2 are regression coefficients. Shown in Table 4 are the values for these coefficients for three different stiffness ratios obtained from an analysis of the ensemble of responses to 51 scaled earthquakes. The regression analysis was based on the assumption that the variance σ^2_ε is independent of a^* and ω^*. Also presented in Table 4 are the values for s^2_ε, the statistical variance about the constructed regression line.

Required Computational Effort

The CPU time required to obtain the response for each member of the ensemble was evaluated for both the exact and approximate analysis. These times are a rough measure of the computational effort required to obtain the response by each technique. The CPU times were then statistically analyzed, with the results that for the exact analysis the mean time required was 1230 sec with a standard deviation of 690 sec, while the mean time required for the approximate analysis was 6.5 sec with a standard deviation of about 1 sec. Thus, the approximate method was found to reduce the computational effort by nearly a factor of 200.

EXAMPLE APPLICATION INVOLVING PROBABILISTIC DESIGN

The approximate response x_m and regression results for the associated error ε may be used to construct an iterative design procedure based on the probability of exceedence of a predetermined upperbound X. A flow chart for the procedure is given in Fig. 5. The design problem consists of selecting the necessary barrier stiffness K in order that the probability P that $x_{max} > X$ does not exceed an acceptable level P_a.

For example consider a system with known mass m, stiffness k, damping coefficient c, and gap width d, subjected to an earthquake with known response spectrum, and hence known a* and ω*. For simplicity let $\omega^*/\sqrt{k/m} = 1$ and $(ma^*/kd) = 5$ and choose the earthquake to be the accelerogram from the Kern County earthquake of 1952, S69E component. Furthermore let $P_a = 0.2$ and $(X/d) = 8$. Then, choosing an initial guess of $(K/k) = 50$ one finds that $(x_m/d) = 5.44$ and the estimated exact response (x^e_{max}/d) has mean 6.85 and standard deviation 1.70. Assuming a Gaussian distribution, the estimated probability of exceedence is $P(x^e_{max} > 8) = 0.25$, which is not acceptable. So, on the second iteration, a stiffness ratio of $(K/k) = 100$ is selected, and the corresponding probability of exceedence is then found to be 0.22, which is still not acceptable. Finally, on the third iteration, a stiffness ratio of $(K/k) = 150$ is selected, with the result that the probability of exceedence is 0.20, which meets the required criteria, and the design is complete. An independent evaluation of the exact response $(x^e_{max}/d) = 7.27$, which indeed is smaller than $X = 8$, as required.

ACKNOWLEDGEMENT

This research was supported in part by a grant from the National Science Foundation.

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TABLE 1

Accelerogram Scale Factors

Earthquake	a^* (g)	ω^* (rad/sec)
Imperial Valley 1940 (S00E)	0.334	44.5
Kern County 1952 (S69E)	0.203	32.6
Eureka 1954 (N11W)	0.183	30.6
Borrego Mountain 1968 (S00W)	0.0272	28.6
Long Beach 1933 (N82W)	0.163	39.5

TABLE 2

Parameter Studies for Exact and Approximate Response Peaks

		PEAK RESPONSE (X_{max}/d) K/k = 100					
EARTHQUAKE	$\frac{\omega^*}{\sqrt{k/m}}$	$\frac{ma^*}{kd} = 5$		$\frac{ma^*}{kd} = 6$		$\frac{ma^*}{kd} = 7$	
		EXACT	APPROX.	EXACT	APPROX.	EXACT	APPROX.
Imperial Valley 1940 (S00E)	1	5.5787	4.1563	4.9892	4.8380	5.8731	5.1523
	2	5.2921	5.2189	6.4713	6.0359	8.2839	6.9049
	3	6.9717	5.3760	9.4887	6.3370	11.0355	7.9013
Kern County 1952 (S69E)	1	6.4709	5.2076	9.3198	5.8978	10.0489	6.4856
	2	7.9989	8.2113	9.2244	9.8013	10.4854	11.7313
	3	9.1093	9.0717	10.3745	11.6686	11.3055	13.7193
Eureka 1954 (N11W)	1	6.7320	5.7792	7.9029	6.4266	10.3115	7.0406
	2	10.6570	8.2515	11.5625	8.9521	12.4640	9.8982
	3	11.2716	11.7889	13.7926	14.1745	14.9787	16.5599
Borrego Mountain 1968 (S00W)	1	8.1361	5.1862	9.0637	5.8249	9.4433	6.7250
	2	8.1439	6.2826	11.6371	6.8049	12.1793	7.2755
	3	7.4293	8.5874	10.3588	9.6497	12.6322	10.6648
Long Beach 1933 (N82W)	1	6.6574	4.8628	9.4073	5.5243	10.4181	6.0099
	2	9.4613	6.8502	10.0027	7.7577	9.3451	8.6377
	3	9.6117	8.6377	10.9573	11.8081	10.5629	14.7345

TABLE 3

Global Mean and Standard Deviation of Error and Absolute Error for Different Stiffness Ratios

$\frac{K}{k}$	μ_ε	σ_ε	$\mu_{\lvert\varepsilon\rvert}$	$\sigma_{\lvert\varepsilon\rvert}$
10	0.052	0.222	0.186	0.132
100	0.077	0.244	0.204	0.154
10^8	0.046	0.286	0.233	0.172

TABLE 4

Error Regression Coefficients and Conditional Variance

$\frac{K}{k}$	c_0	c_1	c_2	s_ε^2
10	0.08306	−0.09136	0.02533	0.04360
100	0.2429	−0.0849	0.002494	0.05429
10^8	0.2905	−0.1562	0.01138	0.06554

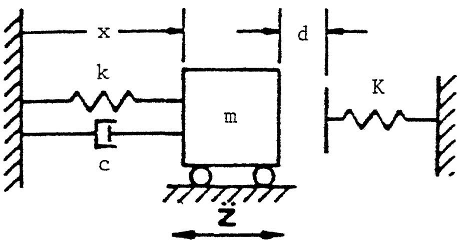

Fig. 1(a) - Base Excited SDOF Oscillator with single Elastic Barrier

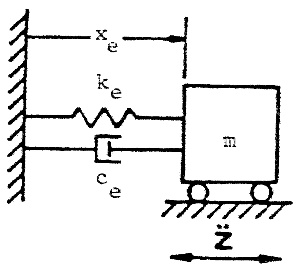

Fig. 1(b) - Equivalent linear Base Excited SDOF Oscillator

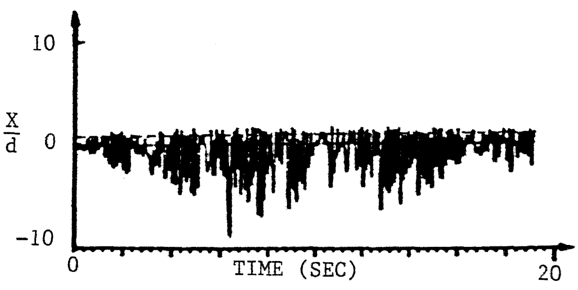

Fig. 2 - Typical Vibroimpactive Response Time History

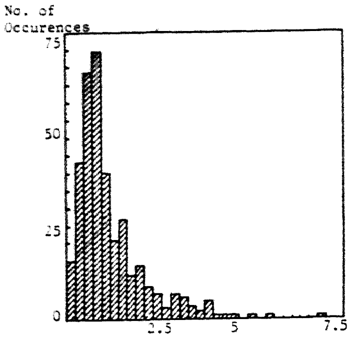

Fig. 3 - Histogram of local peaks of Vibroimpactive Response in Fig. 2

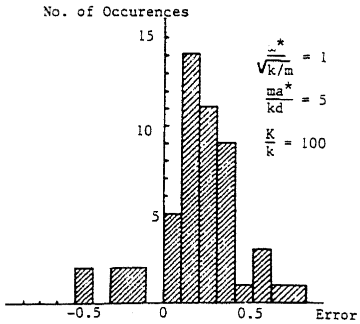

Fig. 4 - Histogram of Error ε for 51 Earthquake Responses

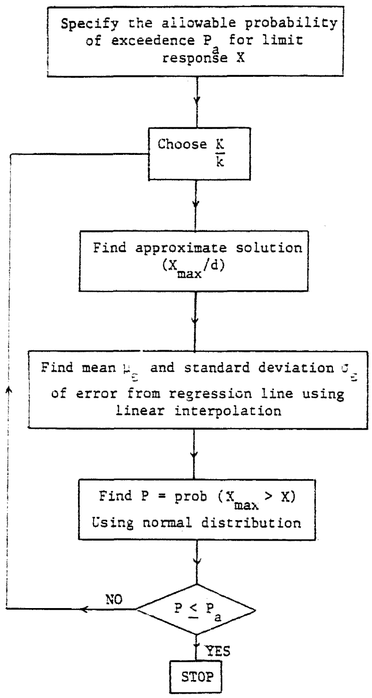

Fig. 5 - Flow Chart of Probabilistic Design Example