

DYNAMIC RESPONSE OF STRUCTURAL SYSTEMS SUBJECTED TO
HORIZONTAL PROPAGATING SHEAR WAVES

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SUMMARY

This paper presents the numerical results of a parametric study for structures subjected to horizontal propagating shear waves. Dynamic behavior of coupled lateral-torsional systems subjected to seismic excitations is investigated analytically. Case studies are provided to show the contribution of each of the selected parameters to the rotational response of the system. Dynamic eccentricity is selected as an index to represent the level of the response. The dynamic characteristics of the systems and motions are also discussed based on the formulation of the presented approach.

INTRODUCTION

There has been evidence of serious damage in buildings due to torsional effects in earthquakes. Therefore, it is necessary that adequate torsional resistance be provided for building systems. Current design provisions in building codes or standards related to torsional effects (1-3) are mainly based on engineering judgment. These provisions are not necessarily theoretically sound. To avoid structural failures in earthquakes, research is needed to simulate the real behavior of structural systems subjected to seismic waves.

Recently, analytical models have been developed specifically for determining dynamic response of coupled lateral-torsional structural systems. For example, the effects of response due to geometric eccentricity between center of rigidity and center of mass have been investigated extensively by Kan and Chopra for linear and non-linear systems (4,5). The response of torsionally coupled elastic systems have been studied by Kung and Pecknold (6) based on a probabilistic ground motion model. Dynamic eccentricity has been estimated by Tso and Dempsey in terms of geometric eccentricity (7). However, soil-structure interaction (SSI) effects were not considered in these studies. To include SSI effects, a simple approach (8) was proposed that made use of the impedance functions and input motions as computed in references 9 and 10.

In this paper, analytical results are presented to illustrate the effects of a few parameters pertinent to the characteristics of a structural system. The approach presented in reference 8 is briefly described. The

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effects on structural response due to several significant parameters will be given in terms of dynamic eccentricity with a few case studies. The characteristics of the structural system as well as the input motions are also discussed.

APPROACH

A one-story structural system with geometric eccentricity equal to e at the first floor is shown in figure 1. The structure is subjected to earthquake excitations at the ground floor (foundation). The equations of motion may be written as:

$$m_t \ddot{U}_{yt} + e m_t \ddot{U}_{\phi t} + K_y U_{yd} + C_y \dot{U}_{yd} = 0 \quad (1)$$

$$m_t e \ddot{U}_{yt} + I_{\phi t} \ddot{U}_{\phi t} + K_{\phi} U_{\phi d} + C_{\phi} \dot{U}_{\phi d} = 0 \quad (2)$$

$$m_b \ddot{U}_{yb} + m_t \ddot{U}_{yt} + m_t e \ddot{U}_{\phi t} = f_y(t) \quad (3)$$

$$m_t e \ddot{U}_{yt} + I_{\phi t} \ddot{U}_{\phi t} + I_{\phi b} \ddot{U}_{\phi b} = f_{\phi}(t) \quad (4)$$

where m , C , K , and U are equal to the mass, damping, stiffness, and displacement, respectively. Subscript t or b denotes that the term is related to the first or the ground floor; subscript y or ϕ denotes that the term is related to the translational or rotational movements; subscript d denotes that the term is equal to the difference between the related terms of first floor and the ground floor; e.g., $U_{yd} = U_{yt} - U_{yb}$; $I_{\phi t}$ and $I_{\phi b}$ are the rotational mass moments of inertia taken with respect to the Z -axis located at the center of rigidity, for the first and ground floors, respectively; f_y and f_{ϕ} are the earthquake excitation forces at the foundation.

By transforming the terms "U" and "f" into the frequency domain and expressing them in terms of the amplitudes, U_s or f_s , and the frequency, ω_s , equations 1 through 4 can be rewritten as:

$$\begin{bmatrix} -m_t \omega_s^2 & -e m_t \omega_s^2 & -m_b \omega_s^2 - K_{fy} & 0 \\ -e m_t \omega_s^2 & -I_{\phi t} \omega_s^2 & 0 & -I_{\phi b} \omega_s^2 \\ -m_t \omega_s^2 + k_y & -e \omega_s^2 m_t & -K_y - c_y i \omega_s^2 & 0 \\ + C_y i \omega_s & & & \\ -e m_t \omega_s^2 & -I_{\phi t} \omega_s^2 + K_{\phi} & 0 & -K_{\phi} \\ + C_{\phi} i \omega_s & & & -C_{\phi} i \omega_s \end{bmatrix} \begin{bmatrix} U_{yt} \\ U_{\phi t} \\ U_{yb} \\ U_{\phi b} \end{bmatrix} = \begin{bmatrix} -K_{fy} U_{fy}^* \\ -K_{f\phi} U_{f\phi}^* \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

where K_{fy} and $K_{f\phi}$ are impedance functions; U_{fy}^* and $U_{f\phi}^*$ are input motions for the translational and rotational directions, respectively.

ANALYTICAL STUDY

The response of the structure can be calculated once the impedance functions and the input motions of the foundation are known. With the Fast Fourier Transform technique, the dynamic response of the system may be found in the time domain (e.g., ref. 11). Dynamic eccentricity is an important variable for indicating the behavior of a coupled rotational-translational system. Therefore, it is selected as an index to represent the structural response in the investigation. For the problem under investigation, the dynamic eccentricity, $E(t)$, is considered to be equal to the torsional moment, $M(t)$, divided by the transverse shear, $V(t)$. The effects on dynamic eccentricity due to the effects of several parameters are illustrated here. These parameters are: geometric eccentricity, the aspect-ratio of the foundation mat, and the ratio of the uncoupled rotational to translational frequencies. The analytical results of a few case studies are described below. Unless otherwise specified, the general properties of the system are taken as:

$$m_t = 3.63 \times 10^7 \text{ kg}, m_{b2} = 1.85 \times 10^8 \text{ kg}, I_{\phi t} = 3.03 \times 10^9 \text{ kg-m}^2, \\ I_{\phi b} = 1.52 \times 10^{10} \text{ kg-m}^2, K_y = 1.44 \times 10^7 \text{ N/m}, K_{\phi} = 2.70 \times 10^{11} \text{ N-m/rad}, \\ C_y \text{ and } C_{\phi} = 2 \text{ percent of the critical damping ratios.}$$

The soil is assumed to have a damping ratio equal to 0.05, with Poisson's ratio equal to 0.33 and the shear modulus equal to $2.15 \times 10^8 \text{ N/m}^2$. The impedance functions and input motions are taken from references 9 and 10. The input spectrum is based on the curve shown in figure 2.

Geometric Eccentricity - Dynamic eccentricities of the first floor for cases with geometric eccentricity ratios, e/r , equal to 0.1, 0.2 and 0.32 are plotted in figure 3, where r represents the radius of gyration. These curves are obtained by solving equation 5. The foundation mat is assumed to be 25.9m x 25.9m. It is shown in the figure that the dynamic eccentricity becomes larger as the geometric eccentricity increases.

Aspect-Ratio of the Foundation Mat - To illustrate the effects due to the aspect-ratio of the foundation mat, the dynamic eccentricities are plotted in figure 4 for a case with the same structural properties as the preceding case (25.9m x 25.9m) except for the dimensions of the foundation mat. The foundation mat selected here is of rectangular shape with dimensions equal to 51.2m x 12.8m. These two cases have about the same mat area, but the input functions are different. Hence, the dynamic eccentricities are different in terms of magnitude and frequency.

Ratio of the Uncoupled Rotational Frequency to Translational Frequency - Dynamic eccentricities are shown in figure 5 as a function of the ratio between the uncoupled rotational frequency to translational frequency. The curve with a solid line in the figure is a replot of the case with $e/r = 0.2$

in figure 3. The ratio is approximately 1.5 for this case. The curve with a dotted line represents a case with the same system except with a higher $I_{\phi t}$. For this case, $I_{\phi t}$ is selected such that the rotational frequency is one-third higher than the case with a solid line. As shown in figure 5, the magnitude of the eccentricity is lower for a larger ratio of the rotational to translational frequencies, but the frequency of the dynamic eccentricity is higher if the ratio is higher. Similar results are found in references 4 and 12 although SSI effects were not included in those studies.

CHARACTERISTICS OF THE SYSTEM AND MOTIONS

The problem solved here has included the SSI effects. To illustrate this, the horizontal displacements on the first floor are computed as a function of the frequency as shown in figure 6. The curve with the solid-line is the displacement amplitude for the case with $e/r = 0.2$. The curve with the dotted line represents the same case computed by not considering the soil-structure interaction. The vertical axis represents the amplitude ratio between the response and the input amplitude. The horizontal axis represents the ratio between the uncoupled rotational and the translational frequencies of the first floor. By considering the soil-structure interaction, the peak amplitude shifts toward the left of the frequency axis. For the present case, the translational frequency of the system shifts to about 85 percent of the original uncoupled translational frequency.

Accidental eccentricity due to seismic waves is caused by the effect of ground motion about a vertical axis (1). Based on the approach presented, accidental eccentricity is only a special case of the general solution. Namely, to solve equation 5 by setting $e = 0$. The accidental eccentricities corresponding to the cases as given in figures 3 and 4 are shown in figure 7. It is shown that the eccentricity is larger for a structure with a larger aspect ratio of the mat. On the other hand, the response of the symmetric building cannot be ignored. The solutions obtained here verify the concepts presented in reference 3.

SUMMARY

Analytical results of a study are given in this paper for structures subjected to shear waves. Dynamic eccentricity is selected as an index. It is found that the rotational responses of a structural system depend greatly on the geometric eccentricity, the dimensions of the foundation mat, and the ratio of rotational to translational frequencies. Therefore, it is important that all of these parameters shall be considered in determining the design eccentricity. The related design provision in current building codes that consider only the effects of the geometric eccentricity should be updated. To develop consistent seismic provisions based on this approach, further study is needed for structures under various conditions.

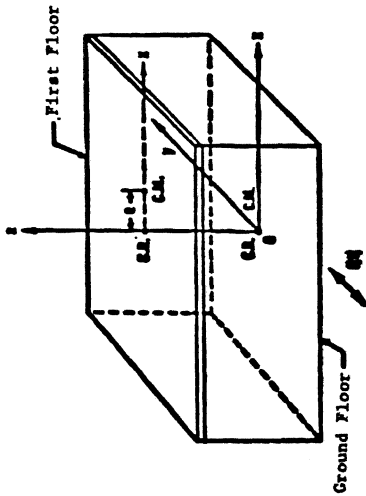


Figure 1. One-story building system subjected to earthquake excitations at the foundation mat

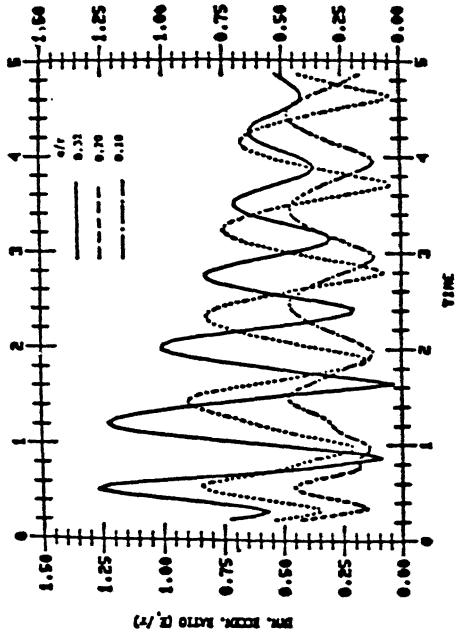


Figure 3. Dynamic eccentricities for a case with foundation mat equal to 25.9 m x 25.9 m for $a/r = 0.1, 0.2$ and 0.32 , respectively

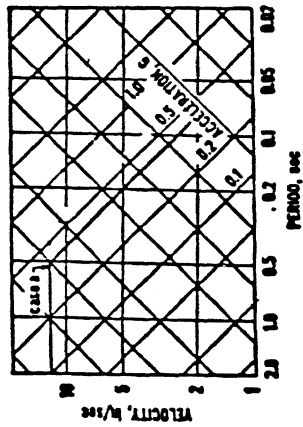


Figure 2. Spectrum assumed as the free field motions.

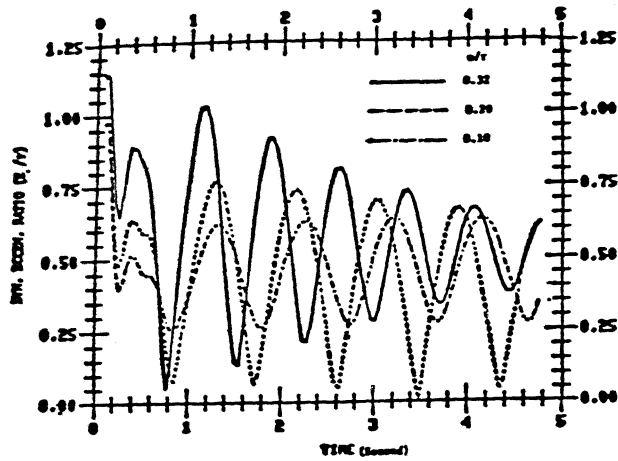


Figure 4. Dynamic eccentricities for a case with foundation mat equal to 51.2 m x 12.8 m for $e/r = 0.1, 0.2$ and 0.32 , respectively

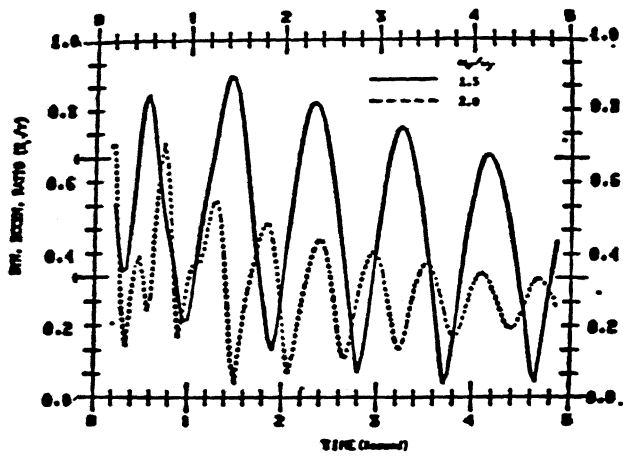


Figure 5. Dynamic eccentricities for cases with different ratios of rotational to translational frequencies.

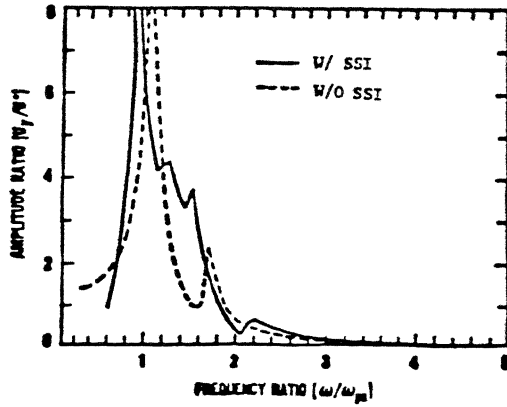


Figure 6. Soil-structure interaction (SSI) effects on the amplitude of the structural response, $e/x = 0.2$

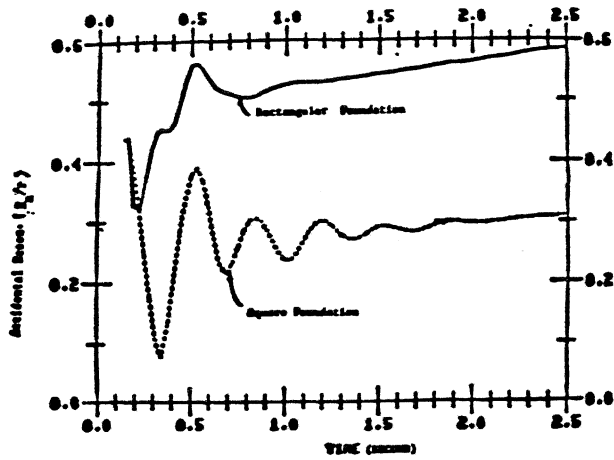


Figure 7. Accidental eccentricities due to seismic waves for two cases.

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