

IDENTIFICATION OF STRUCTURAL PARAMETERS AND ASEISMATIC CONTROL OF THE BUILDING

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SUMMARY

In this paper, applying the idea of model reference adaptive system (MRAS) developed in the field of automatic control engineering to aseismatic control problem of the building, implementation of the MRAS algorithm by digital computer is presented. The paper shows also a method for estimating the dynamical characteristics from noisy observations of dynamical behaviors. The equation of motion for three-dimensional multi-degree-of-freedom lumped mass system proposed in this paper are generally applicable to the vibration problems of multi-story buildings having any eccentricities of center of rigidity or center of gravity.

INTRODUCTION

For the purpose of reducing seismic response of building frame, many investigations or ideas on the earthquake isolation device for the building have been proposed. But no one gets wide practicality since almost the entire proposition has some restrictions such as input excitations to the device become allowable only when either they are within a limited frequency band or they are within a limited amplitude.

Today's electronics engineering yields the possibility of developing rather positive scheme for aseismatic control of the building than negative scheme for the earthquake isolation. The term "aseismatic control" means controlling dynamic behavior of the building under the earthquake motion to a desired state by modifying its structural parameters such as rigidity and damping of the building frame. To realize the aseismatic control system, it is necessary to develop a software that contains a procedure for how to modify the structural parameters and a hardware that contains a mechanism for how to carry out the modification actually. This paper describes the software part.

The software has to possess a high-speed performance because the seismic response of the building transits very rapidly. Therefore it can not cope with the problem by the traditional feedback control system having relatively a low-speed performance. To date, idea of the parallel model reference adaptive system (parallel MRAS) may be almost unique solution to the problem in the face. The advantage of the parallel MRAS is easy-to-implement system with a high-speed of adaptation.

Basic scheme of the parallel MRAS is given in Fig.1. The reference model indicated in Fig.1 specifies a desired performance and the adjustable system also indicated in Fig.1 is a system capable of adjusting its performance by modifying its parameters. The term disturbances represented in Fig.1 are parameter-disturbances modifying the adjustable system.

Algorithms for implementation of the parallel MRAS require, as mentioned later, a priori knowledge of initial values of the structural parameters.

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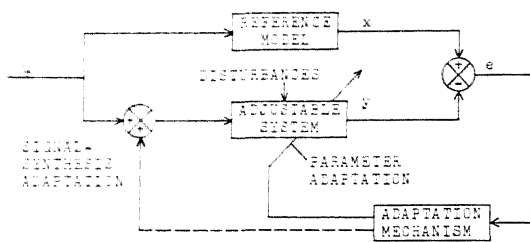


Fig.1 Basic configuration of a parallel MRAS (after Landau)

To satisfy this requirement, a method for identifying the structural parameters, i.e. stiffness matrix and damping matrix of the building frame, from noisy observations of dynamic behavior of the building is proposed. In this paper, moreover, a new description to three dimensional dynamic behaviors of the general structural system including braced frame system, framed tube system and so on is presented.

STATE-SPACE EQUATIONS OF MOTION

Consider a gauge axis set vertically through a n-story building at an arbitrary point on whose representative floor. And define the following flexibility matrix L with respect to the gauge axis on the assumption that each floor slab of the building has infinite plane rigidity. For $i=1,2,\dots,n$, and $j=1,2,\dots,n$,

$$L = \begin{bmatrix} \bar{X}X_{ij} & \bar{X}Y_{ij} & \bar{X}R_{ij} \\ \bar{Y}X_{ij} & \bar{Y}Y_{ij} & \bar{Y}R_{ij} \\ \bar{R}X_{ij} & \bar{R}Y_{ij} & \bar{R}R_{ij} \end{bmatrix} \quad (3n \times 3n)$$

where
 $\bar{X}X_{ij}$ = ith floor deflection in the direction of X when unit force in the direction of X acted on jth floor,
 $\bar{Y}X_{ij}$ = ith floor deflection in the direction of Y when unit force in the direction of X acted on jth floor,

$\bar{R}X_{ij}$ = ith floor rotation when unit force in the direction of X acted on jth floor, $\bar{X}Y_{ij}$ = ith floor deflection in the direction of X when unit force in the direction of Y acted on jth floor, $\bar{Y}Y_{ij}$ = ith floor deflection in the direction of Y when unit force in the direction of Y acted on jth floor, $\bar{R}Y_{ij}$ = ith floor rotation when unit force in the direction of Y acted on jth floor, $\bar{X}R_{ij}$ = ith floor deflection in the direction of X when unit moment acted on jth floor, $\bar{Y}R_{ij}$ = ith floor deflection in the direction of Y when unit moment acted on jth floor, $\bar{R}R_{ij}$ = ith floor rotation when unit moment acted on jth floor, in which the positive directions of the X,Y and the gauge axis are specified by a cartesian reference frame.

Since stiffness matrix K equals inversion of the flexibility matrix L , the equation of motion for three dimensional elastic vibration of multistory building frame subjected to earthquake ground motion may be written as:

$$M\ddot{\Xi} + C\dot{\Xi} + K\Xi = -M\ddot{\Xi}_0(t) \quad (1)$$

where

$$M = \begin{bmatrix} m_1 & & & 0 & & 0 \\ & m_n & & & & \\ 0 & & m_1 & & & 0 \\ & & & m_n & & \\ 0 & & & & J_1 & \\ & & & & & J_n \end{bmatrix}, \quad \Xi(t) = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_n \\ \eta_1 \\ \vdots \\ \eta_n \\ \zeta_1 \\ \vdots \\ \zeta_n \end{bmatrix}, \quad \Xi_0(t) = \begin{bmatrix} \xi_0 \\ \vdots \\ \xi_0 \\ \eta_0 \\ \vdots \\ \eta_0 \\ \zeta_0 \\ \vdots \\ \zeta_0 \end{bmatrix},$$

$K=L^{-1}$ and $C = \gamma_0 M + \gamma_1 K$ in which m and J are floor concentrated mass and mass moment of inertia with respect to the gauge axis respectively, ξ and η are floor displacements relative to the base of the building in the direction of X and Y respectively, ζ is relative floor rotation to the base with respect to the gauge axis, ξ_0 , η_0 and ζ_0 are motions of the base due to earthquake, γ_0 and γ_1 are Rayleigh's coefficients of damping.

Defining further the state vector

$$x(t) = [\xi_1 \dots \xi_n \eta_1 \dots \eta_n \zeta_1 \dots \zeta_n \dot{\xi}_1 \dots \dot{\xi}_n \dot{\eta}_1 \dots \dot{\eta}_n \dot{\zeta}_1 \dots \dot{\zeta}_n]^T = [x_1 \dots x_{6n}]^T, \quad (2)$$

the continuous-time state-space equation equivalent to the Eq.(1) can be obtained as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3)$$

where the system matrices

$$A = \begin{bmatrix} 0 & I \\ \frac{(3n \times 3n)}{-(M^{-1}K)} & \frac{(3n \times 3n)}{-(M^{-1}C)} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ \frac{(3n \times 3)}{-1} \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \text{in which} \quad l = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad (n \times 1)$$

and the system disturbance $u(t) = [\ddot{\xi}_0 \ddot{\eta}_0 \ddot{\zeta}_0]^T = [u_x u_y u_r]^T$.

For convenience of the implementation on digital computer, the following discrete-time state-space equation which is the discrete-time analog of Eq. (3) is introduced (Ref.2). That is

$$x(k+1) = \Psi x(k) + \Gamma u(k) \quad (4)$$

where $\Psi = \exp[A\Delta t]$, $\Gamma = A^{-1}[\Psi - I]B$, $x(k) = x(t)$, $u(k) = u(t)$ in which $k \cdot \Delta t \leq t < (k+1)\Delta t$ for some $k=0,1,2,\dots$, and Δt is sampling time interval.

ALGORITHMS FOR PARALLEL M.R.A.S.

Let the following three discrete-time state-space equations express the reference model, the adjustable system and uncontrollable system.

$$\text{Reference model} : x(k+1) = \Psi_M x(k) + \Gamma_M u(k) \quad (5)$$

$$\text{Adjustable system} : y(k+1) = \Psi_S(k+1)y(k) + \Gamma_S(k+1)u(k) \quad (6)$$

$$\text{Incontrollable system} : z(k+1) = \Psi_S(0)z(k) + \Gamma_S(0)u(k) \quad (7)$$

In the reference model, the transition matrix Ψ_M is calculated from a system matrix A_M which consists of mass matrix M_M , stiffness matrix K_M and damping matrix C_M . The matrices M_M , K_M and C_M may be assumed imaginatively for the purpose of realizing a desired fashion in which the reference model will behave under the system disturbance $u(k)$. For example, when stress distribution of a building frame having some imaginary dynamical characteristics stays within elastic range even if the building is subjected to any strong earthquake ground motion, such a building frame becomes one of the reference models. To know the time varying transition matrix $\Psi_S(k)$ and the time varying disturbance transition matrix $\Gamma_S(k)$ in the adjustable system is the objective of the computation for minimizing the difference $[x(k)-y(k)]$ through the algorithms mentioned later. The reason of presenting the uncontrollable system having a time-invariant transition matrices $\Psi_S(0)$ and $\Gamma_S(0)$ which specify initial state of the adjustable system is for showing effectiveness of the adaptive control by comparing output of the adjustable system with that of the uncontrollable system.

The adaptation algorithms, after Landau (Ref.3), for the parallel MRAS are summarized as follows:

for $k=0,1,2,\dots$,

$$\left. \begin{aligned} Q(k) &= [I + D\{y^T(k)(P + \bar{P})y(k) + u^T(k)(R + \bar{R})u(k)\}]^{-1} \\ e(k) &= x(k) - y(k) \\ v(k) &= De(k) \\ \Psi_s(k+1) &= \sum_{l=0}^{k-1} Q(l)v(l)[Py(l)]^T + Q(k)v(k)[(P + \bar{P})y(k)]^T + \Psi_s(0) \\ \Gamma_s(k+1) &= \sum_{l=0}^{k-1} Q(l)v(l)[Ru(l)]^T + Q(k)v(k)[(R + \bar{R})u(k)]^T + \Gamma_s(0) \end{aligned} \right\} \quad (8)$$

where $6n \times 6n$ matrix D becomes a solution of the Lyapunov equation $\Psi_M^T D \Psi_M - D = -N$ for an arbitrary positive definite matrix N if the eigenvalues of Ψ_M are less than one in absolute values, $6n \times 6n$ matrices P and \bar{P} and 3×3 matrices R and \bar{R} are positive definite constant matrices to be assumed, $6n \times 6n$ matrix $\Psi_0(0)$ and $6n \times 3$ matrix $\Gamma_0(0)$ are assumed to be known and I is $6n \times 6n$ identity matrix.

Once the matrices $\Psi_s(k)$ and $\Gamma_s(k)$ are obtained through the above algorithms, the time-varying system matrices $A_s(k)$ and $B_s(k)$ which are necessitated to control the seismic behavior of the building can be calculated. The $3n \times 3n$ matrix $A_s(k)$ and the $6n \times 3$ matrix $B_s(k)$ may be express formally as $A_s(k) = (1/\Delta t) \ln \Psi_s(k)$ and $B_s(k) = [\Psi_s(k) - I]^{-1} A_s(k) \Gamma_s(k)$. But to execute these calculations rapidly and accurately, some device on computation procedure becomes necessary.

IDENTIFICATION OF STRUCTURAL PARAMETERS

To carry out the adaptive control of seismic behaviors of the building by virtue of the algorithms (8), a priori knowledge of the values of the matrices $\Psi_s(0)$ and $\Gamma_s(0)$ are required. The problem how to know the matrices $\Psi_s(0)$ and $\Gamma_s(0)$ is the problem how to identify the structural parameters.

Consider the following set of equations, called the system equation, for $k=1,2,3,\dots$,

$$x(k+1) = \Psi x(k) + \Gamma u(k) \quad (9a)$$

$$f(k) = Ix(k) + n(k) \quad (9b)$$

where $6n \times 1$ vector $f(k)$ denotes the measurement vector and $6n \times 1$ vector $n(k)$ is the measurement error vector and is assumed to be Gaussian white noise vector with zero mean. Eliminating the state vector $x(k)$ from Eqs.(9), one may obtain the expression given below:

$$f(k+1) = \Theta_k[\omega(k) - \nu(k)] + n(k+1) \quad (10)$$

where $6n \times (6n+3)$ matrix $\Theta_k = [\Psi \Gamma]_k$ and

$(6n+3) \times 1$ vectors $\omega(k) = \begin{bmatrix} f(k) \\ u(k) \end{bmatrix}$ and $\nu(k) = \begin{bmatrix} n(k) \\ 0 \end{bmatrix}$. To estimate the parameter matrix Θ_k from a finite number of observations by means of the least-squares estimation procedures (Ref.4), let an error function be the following form.

$$J = \sum_{i=1}^k \|f(i+1) - \Theta_k[\omega(i) - \nu(i)]\|_{\mathbb{H}}^2 \quad (11)$$

With taking care of the differential calculus, one may obtain the following result.

$$\frac{\partial J}{\partial \Theta_k} = 2 \sum_{i=1}^k [-f(i+1)\{\omega^T(i) - \nu^T(i)\} + \Theta_k\{\omega(i) - \nu(i)\}\{\omega^T(i) - \nu^T(i)\}] \quad (12)$$

Since the vector $n(k)$ is composed of White noise entries, Eq.(12) can be written as the reduced form given below for sufficient large number of k .

$$\frac{\partial J}{\partial \Theta_k} \triangleq -2 \sum_{i=1}^k f(i+1) \omega^T(i) + 2 \Theta_k \left[\sum_{i=1}^k \omega(i) \omega^T(i) + k V_k \right] \quad (13)$$

or in matrix forms,

$$\frac{\partial J}{\partial \Theta_k} \triangleq -2 F_k \Omega_k^T + 2 \Theta_k [\Omega_k \Omega_k^T + k V_k] \quad (14)$$

where $F_k = [f(2)f(3)\dots f(k+1)]$, $\Omega_k = [\omega(1)\omega(2)\dots\omega(k)]$ and

$$V_k = E[V(k) V^T(k)] = \begin{bmatrix} \sigma_{(1)}^2 & & & & & \\ & \sigma_{(2)}^2 & & & & \\ & & \ddots & & & \\ & & & \sigma_{(6n)}^2 & & \\ \hline & & & & 0 & \\ & & & & & 0 \end{bmatrix}$$

(6n x 3) (3 x 3)

in which the notation $E[\cdot]$ shows the mathematical expectation. For some matrix $\hat{\Theta}_k$ Eq.(14) can be equated to zero, leading to an expression for $\hat{\Theta}_k$ at which the error function J becomes an extremum. Therefore

$$\hat{\Theta}_k = F_k \Omega_k^T [\Omega_k \Omega_k^T + k V_k]^{-1}. \quad (15)$$

If the covariance matrix V_k of the measurement error vector is known, employing input and output observations Eq.(15) provides the estimates for the matrices Ψ and Γ . Then the estimate for the system matrix A can be calculated accurately by the following formula (Ref.2) when the estimate $\hat{\Psi}$ has distinct eigen-values.

$$\hat{A} = [p_1 \ p_2 \ \dots \ p_{6n}] \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{6n} \end{bmatrix} [p_1 \ p_2 \ \dots \ p_{6n}]^{-1} \quad (16)$$

where $\lambda_i = (1/\Delta t) \ln \mu_i$ and $p_i = q_i$ for $i=1,2,\dots,6n$, in which μ_i are the distinct eigen-values of $\hat{\Psi}$ and q_i are its corresponding eigen-vectors. The estimates for the structural parameters K and C looking for, then, are given as follows:

$$\begin{bmatrix} 0 & I \\ \hat{K} & \hat{C} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & -M \end{bmatrix} \cdot \hat{A} \quad (17)$$

To implement the above estimation procedure, it is necessary to evaluate fundamentally the covariance matrix V_k . By a similar way that is described in Ref.5, one may obtain the required expression for estimating the l th diagonal element of the matrix V_k . For an abbreviated suffix-notation $r(=6n)$,

$$\hat{\sigma}_{(l)}^2 = \frac{1}{k-r} \sum_{j=r+1}^k p_l^2(j) (1 + \sum_{i=1}^r a_i^2)^{-1} \quad (18)$$

where,

$$p_l(k) = f_l(k) + \sum_{i=1}^r a_i f_l(k-i) - \sum_{i=1}^r \beta_i u(k-i) \quad (19)$$

in which $f_l(k)$ is the l th element of the measurement vector $f(k)$, a_1, a_2, \dots, a_r are coefficients of the characteristic polynomial of $\hat{\Psi}$ which is determined from the approximated equation $[\hat{\Psi}^r \hat{\Gamma}]_k = F_k \Omega_k^T [\Omega_k \Omega_k^T]^{-1}$, and $\beta_1, \beta_2, \dots, \beta_r$ are defined as given below:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{r-1} \\ \beta_r \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ a_1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{r-2} & a_{r-3} & \dots & 1 \\ a_{r-1} & a_{r-2} & \dots & a_1 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{r-1} \\ g_r \end{bmatrix}, \quad \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{r-1} \\ g_r \end{bmatrix} = T \hat{\Gamma}^T,$$

$$T = [J_1^T (J_1 \hat{\Psi}^T)^T (J_1 (\hat{\Psi}^T)^2)^T \dots (J_1 (\hat{\Psi}^T)^{r-1})^T]^T,$$

$$J_1 = \begin{bmatrix} 1 & \dots & 1 & 1 & 1 & \dots & 1 \\ 0 & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix}$$

NUMERICAL EXAMPLE FOR IDENTIFICATION

As a basis for numerical verification to the theory of identification, three-story, braced steel-building-frame as shown in Fig.2 was chosen. The gauge axis is set vertically through the points A. Each floor has a uniformly distributed mass whose total value and the moment of inertia with respect to the gauge axis are 0.05 ton·sec /cm and 2333.3 ton·cm·sec respectively. Each bracing bar is assumed to be available for tensile force but to be unavailable for compressive force. The stiffness matrix of this structure shown in Table 1 is evaluated in consideration of axial deformation of

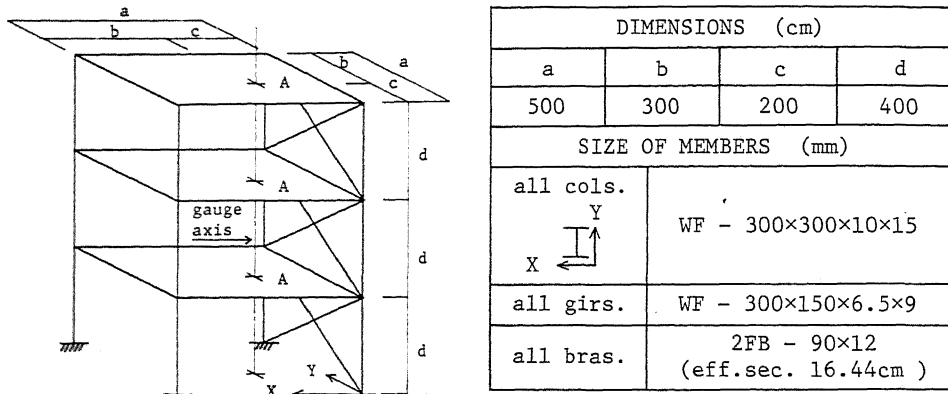


Fig.2 Building frame model for numerical tests

columns and shear deformation of column-girder joint panels.

Table 1 stiffness matrix
(true values, ton/cm)

0.229396E+02	-0.129292E+02	0.259643E+01
-0.129292E+02	0.185065E+02	-0.871747E+01
0.259643E+01	-0.871747E+01	0.655170E+01
.....	abbreviated.

Table 2 Rayleigh's damping matrix
(true values, ton·sec/cm)

0.725587E-01	-0.374846E-01	0.752763E-02
-0.374846E-01	0.597061E-01	-0.252739E-01
0.752763E-02	-0.252739E-01	0.250465E-01
.....	abbreviated.

Assuming the damping factors for both first and second natural circular frequencies as 0.02, the Rayleigh's damping matrix may be calculated as shown in Table 2. From a point of view of the aseismatic control scheme, it is desirable to estimate these matrices without any ground motions since the matrices are required as a priori knowledge. The afore-mentioned theory of identification is also applicable to the case of free vibration i.e. $u(k) = 0$ with a given $x(0)$.

Regarding the maximum realization of the measurement noise at so-called 3σ level as 0.05 percent of the maximum response with reference to Table 3,

Table 3 Performances of observation devices

Maximum error for measurable range				Root mean square
	SENSOR	AMPLIFIER	A/D CONVERTER	
For DISPLACEMENT	0.00003 %	0.0001 %	0.0488 %	0.049 %
For VELOCITY	0.01 %			0.050 %

the Gaussian white noise sequence for simulating measurement error whose standard deviation is 0.016 percent of the maximum response can reasonably be assumed. Making thus the white noise sequences corresponding to the elements of the state vector and then adding them to the response sequences

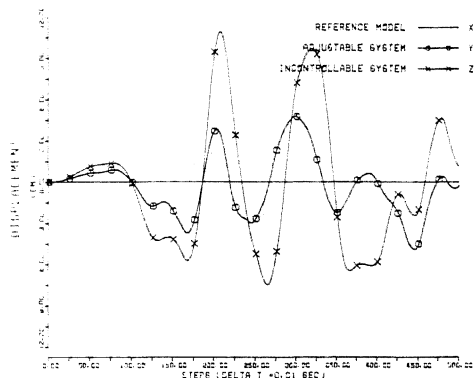


Fig.3 Displacement responses in the direction of X

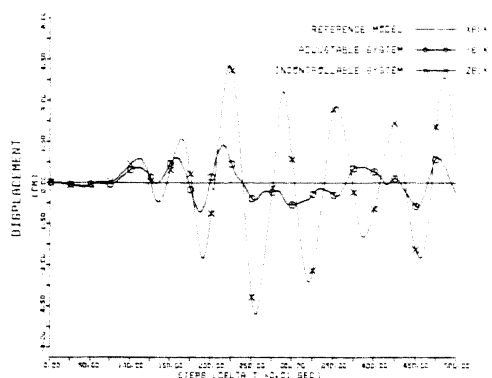


Fig.4 Displacement responses in the direction of Y

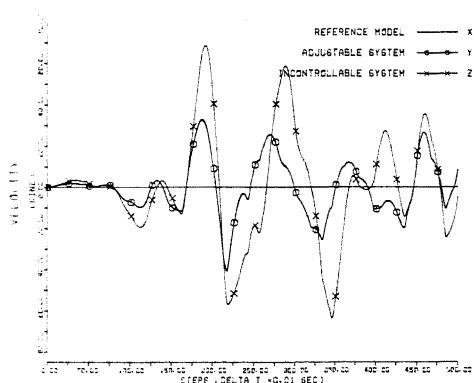


Fig.5 Velocity responses in the direction of X

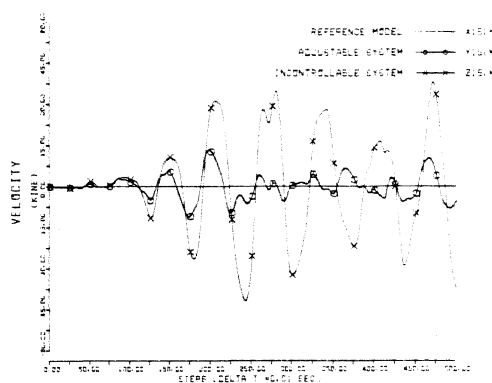


Fig.6 Velocity responses in the direction of Y

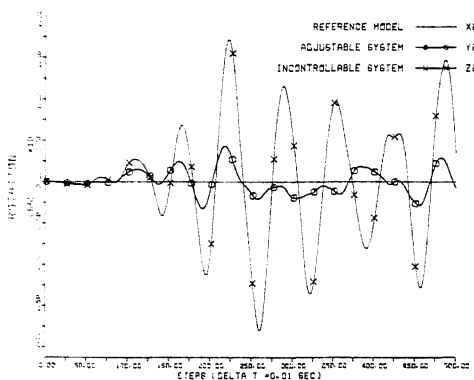


Fig.7 Rotation responses

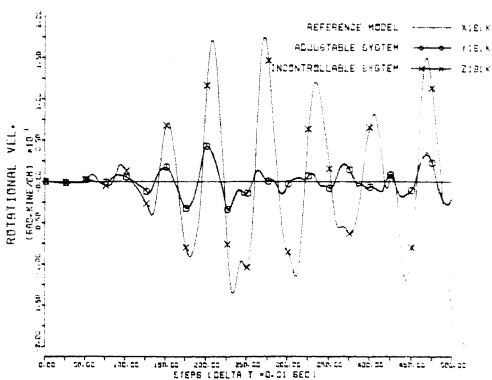


Fig.8 Rotational velocity responses

yielded by the free vibration gives the observation data to be used for the computer simulation.

By virtue of theory of identification one may obtain the estimates shown in Table 4, Table 5 and Table 6 using 300 samples with 0.01 sec. interval.

Table 4 Estimated stiffness matrix
(ton/cm)

0.229409E+02	-0.129304E+02	0.239693E+01
-0.129311E+02	0.185083E+02	-0.871819E+01
0.239740E+01	-0.871840E+01	0.655208E+01
.....	abbreviated.

Table 5 Estimated damping matrix
(ton*sec/cm)

0.725540E-01	-0.374817E-01	0.752688E-02
-0.374764E-01	0.597004E-01	-0.252722E-01
0.752341E-02	-0.252709E-01	0.250456E-01
.....	abbreviated.

Table 6 Representative estimated variance of measurement noise

Number of samples	$\sigma_{(1)}^2$ (true value)	$\hat{\sigma}_{(1)}^2$ (estimated)	Degree of estimation
300	0.24926E-06	0.25105E-06	99.3 %

NUMERICAL EXAMPLE FOR ASEISMATIC CONTROL

Suppose a system composed of the model that is shown in Fig.2 and an appropriate equipment by which lateral rigidity and damping characteristics of the model can be adaptively modified. And further suppose a case where the model shown in Fig.2 has such imaginative damping factors as the values 0.5 for both first and second natural circular frequencies. Let the system and the model like these be the adjustable system and the reference model respectively. The employed system disturbance vector $U(k)$ consists of the components S00E and S90W of El Centro 1940 earthquake record without the ground rotation. The employed system matrix $A_s(0)$ is determined by the true-value stiffness and damping matrices as shown in Table 1 and Table 2 respectively not by those estimates, in order to examine the accuracy that the adaptation algorithms may have. Carrying out the MRAS algorithms by digital computer gives the following results. From Fig.3 to Fig.8 show the time history responses of top floor of the models during five second. These responses are measured with reference to the gauge axis that is specified in Fig.2. There were assumed as: $z(0) = y(0) = 10x(0)$, $P = 0.2\bar{P}$ and $R = 0.2\bar{R}$.

One may ascertain by examining these figures that there exists a marvelous agreement between the response of the reference model and that of the adjustable system. These two responses are so agreeable that one can not distinguish between them in the scale of the presenting figures. One may also CONCLUDE that the scheme of aseismatic control is successful by comparing the response of the adjustable system with that of the uncontrollable system.

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