SYSTEM IDENTIFICATION OF STRUCTURES FROM AMBIENT WIND MEASUREMENTS

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SUMMARY

A procedure is outlined for estimating structural parameters based on relatively simple ambient wind measurements of story accelerations reduced to resonant frequencies, and corresponding mode shapes. Unknown parameters, corresponding to a superimposed stiffness matrix of a closely coupled shear building on to a stiffness matrix based on a finite element calculation, are systematically adjusted. The method is applied to a twelve-story apartment building located in Ottawa, Canada. It is built of reinforced concrete with primary lateral resistance furnished by shear walls in both directions.

The method developed is based on Bayesian inference with the objective function relating the errors between measurements and corresponding values minimized by a modified Newton-Raphson scheme. Not all the mode shapes and natural frequencies are required, nor do all the stories need be instrumented. Furthermore, the sensitivity of the measured quantities with respect to the parameters are based on eigenvalue and eigenvector sensitivities. These are function only of the corresponding eigenvalues and eigenvectors and take into consideration the symmetric and banded nature of the stiffness and mass matrices. An appropriate method of evaluating the effect of proposed modifications on the structural response is based on the sensitivity relationships used in the identification.

INTRODUCTION

The actual observed dynamic behavior of structures is generally different from that calculated by a finite element model. This is due to a variety of reasons related to uncertainty in material parameters, behavior at structural connections, the effect of secondary structural elements, such as infilled panels, and other simplifying assumptions. Most structural design codes relate earthquake considerations to the estimated fundamental period and its corresponding mode shape, which in all likelihood are different than what is measured.

Once a particular structure is finally built, little attention is normally given to monitor its behavior unless, of course, problems develop. For existing buildings which are scheduled for strengthening and/or modification, there is a practical need for establishing its actual behavior, and correspondingly, its behavior for the modifications proposed once complete. For instance, additional stories, weight, strengthening of a few bays and/or stories all modify the structural characteristics. The experimental procedure

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for such full-scale observation must be relatively simple, cause no additional damage, and require lightweight equipment. Parameters of a finite element model may then be adjusted in order to match more closely the observed structural behavior. Once these parameters are ascertained, prediction of the behavior due to these modifications may then be made and evaluated in a quantitative framework.

One of the easier ways of obtaining information of full-scale structures is to observe the behavior under ambient wind. No excitation equipment is needed and a good estimate of the resonant frequencies, mode shapes and even damping ratios can be determined experimentally. Of course, only a few of the frequencies and mode shapes are generally measured, the lowest ones corresponding to lateral and/or torsional motion of each story. Movement of each story must often be calibrated on a particular story, usually the roof, in calculating the mode shapes. Furthermore, measurements are often not available at all the floors.

Due to these facts and also since a mathematical model of the behavior is available from numerical procedures, identification usually proceeds within a Bayesian framework in which the mathematical model and measured quantities are weighed relative to each other (Ref. 1). The mathematical model used may be of a general nature, but it is desirable to limit the number of parameters. Within this framework, a procedure is developed in which the modified Newton-Raphson scheme is used to optimize the objective function corresponding to Bayesian inference, that is to match more closely the model and measurements. Sensitivities of the resonant frequencies and mode shapes are used which depend on the knowledge of only the eigencharacteristics under consideration. The procedure is applied to measurements of a twelve-story frame-shear wall apartment building built of reinforced concrete located in Ottawa, Canada, for lateral and torsional motions.

EQUATIONS OF STRUCTURAL DYNAMICS

Normal Modes

The equations of structural dynamics assuming viscous damping are written as follows:

$$[M]\,\{\ddot{x}\} + [C]\,\{\dot{x}\} + [K]\,\{x\} = \{f\} \tag{1}$$

in which [M], [C] and [K] are the mass, damping and stiffness matrices respectively. For no damping, there correspond the associated normal mode shapes which diagonalize both the symmetric mass and stiffness matrices, i.e.

$$[\phi]'\,[M]\,[\phi] = [I] \tag{2}$$

$$[\phi]'\,[K]\,[\phi] = [\Omega^2] \tag{3}$$

in which ' represents the transpose, [I] is the identity matrix, $[\Omega^2]$ is the diagonal matrix of squares of undamped natural frequency, and $[\phi]$ is the matrix of normal mode shapes.

Frequency Response

For sinusoidal excitation and steady response at a frequency $\bar{\omega}$

$$\{f\} = \{F\}\, e^{i\bar{\omega}t} + \{F\}^*\, e^{-i\bar{\omega}t} \tag{4}$$

$$\{x\} = \{X\}\, e^{i\bar{\omega}t} + \{X\}^*\, e^{-i\bar{\omega}t} \tag{5}$$

in which {F} and {X} are in general complex vectors and * representes its complex conjugate, there results for the case under consideration:

$$\{X\} = [H]\, \{F\} \tag{6}$$

in which

$$[H] = [\phi] \left[[\Omega^2] - \bar{\omega}^2\, [I] + i\bar{\omega}\, 2[\zeta]\, [\Omega]\right]^{-1} [\phi]' \tag{7}$$

where it has been assumed that the normal modes also diagonalize the damping matrix, i.e.

$$[\phi]'\, [C]\, [\phi] = 2[\zeta]\, [\Omega] \tag{8}$$

for $[\zeta]$ a diagonal matrix of modal damping ratios.

These expressions have been generalized to the case of arbitrary viscous damping resulting in complex modes and to a slightly more general expression for the frequency response expression, Eqn. 6 (Ref. 2). Suffice it to say that for excitations near one of the undamped natural frequencies, the response is dominated in the corresponding mode shape when damping is small as is normally the case. Such sinusoidal testing has been performed in a number of structures but requires rather cumbersome equipment and deals with fitting the frequency response function (Ref. 3).

Random Response

A much more pleasing approach is to monitor movement of the building, say its horizontal acceleration at various stories in an ambient wind environment. On line fast Fourier transform techniques convert these time signals to cross-spectral densities. Knowing that the matrix of response cross-spectral densities for the system under consideration is given by (Ref. 4)

$$[S_{xx}] = [H]\, [S_{ff}]\, [H]^{\dagger} \tag{9}$$

in which [H] is given by Eqn. 7, † represents the complex conjugate transpose and $[S_{ff}]$ is the matrix of cross-spectral densities of the loads at each story. Under the assumption that these are uncorrolated and have identical variance, σ^2, this reduces to the expression that these are proportional to the square of the amplitude of the frequency response matrix

$$[S_{xx}] = \sigma^2\, [H]\, [H]^{\dagger} \tag{10}$$

Since these loads are not measured, only the building motion is monitored; all that is required is that a reference signal be recorded for each time interval, for instance the motion of the top story as a calibration point. This permits the use of only a few channels, some staying at the roof, while the others are moved from story to story, but not necessarily at all stories.

SYSTEM IDENTIFICATION

In order to weigh the model and measurements, the objective function, s, corresponding to Bayesian inference, is optimized

$$
\begin{aligned}
s = &\{d - D\}' \, [w_d] \, \{d - D\} \\
&+ \{p - P\}' \, [w_p] \, \{p - P\}
\end{aligned}
\tag{11}
$$

in which D are the measured quantities, d, are the calculated values based on the parameters, p, and P, are estimates to the parameters. $[w_d]$ and $[w_p]$ are weighting matrices, usually assumed to be diagonal, suggesting that the parameters and measurements are statistically independent. Normally, the diagonal elements are taken as the inverse of the variance of the quantities being weighed.

Modified Newton-Raphson

A modified Newton-Raphson scheme, based on a first order Taylor series expansion of the measurement quantities, d, deals with the following approximation to the objective function at the ℓ-th iteration.

$$
\begin{aligned}
s \approx s(p^{\ell}) + &\left\{d^{\ell} + [\nabla] \, \{\delta\} - D\right\}' \, [w_d] \left\{d^{\ell} + [\nabla] \, \{\delta\} - D\right\} \\
&+ \{p^{\ell} + \delta - P\}' \, [w_p] \, \{p^{\ell} + \delta - P\}
\end{aligned}
\tag{12}
$$

in which δ is the change in parameter and

$$\nabla_i = \frac{\partial s}{\partial p_i} \tag{13}$$

When optimized with respect to δ, this yields the relation

$$[A] \, \{\delta\} = a \tag{14}$$

in which

$$\{a\} = [\nabla]'[w_d] \, \{d^{\ell} - D\} + [w_p] \, \{p^{\ell} - P\} \tag{15}$$

$$[A] = [\nabla]'[w_d] \, [\nabla] + [w_p] \tag{16}$$

Only first order derivatives of d are used in this approach compared to the classical Newton-Raphson which requires second order derivatives as well.

Eigensensitivity

The measured quantities are the eigenvalues and eigenvectors of the generalized eigenvalue problem which for no damping is

$$\Big[[K] - \Omega_i^2 \, [M]\Big]\{\phi_i\} = \{0\} \qquad i=1, 2, \ldots, m \tag{17}$$

for m the number of measured natural frequencies and/or modes, Ω_i^2 and $\{\phi_i\}$ are the eigenvalues and associated eigenvectors respectively. The sensitivity of the eigenvalue is readily obtained by a premultiplication by $\{\phi_i\}'$ and subsequent chain rule differentiation

$$\frac{\partial\ \Omega_i^2}{\partial\ p} = \frac{\{\phi_i\}'\left[\left[\frac{\partial K}{\partial p}\right] - \Omega_i^2\ \left[\frac{\partial M}{\partial p}\right]\right]\{\phi_i\}}{\{\phi_i\}'[M]\ \{\phi_i\}} \tag{18}$$

The eigenvector $\{\phi_i\}$ satisfies the relation (Ref. 5)

$$\left[[K] - \Omega_i^2\ [M]\right]\left\{\frac{\partial\ \phi_i}{\partial\ p}\right\} = -\left[\left[\frac{\partial K}{\partial p}\right] - \Omega_i^2\ \left[\frac{\partial M}{\partial p}\right] - \frac{\partial\ \Omega_i^2}{\partial\ p}\ [M]\right]\{\phi_i\} \tag{19}$$

which is necessarily singular. The additional constraint that one of the elements of the modes is constant, for instance the one at the roof equal to one, yields an additional equation which for n, the number of stories, is

$$\phi_n = 1 \tag{20}$$

$$\frac{\partial\ \phi_n}{\partial p} = 0 \tag{21}$$

There results when Equation (21) is added to Equation (19)

$$[B]\ \left\{\frac{\partial\ \phi_i}{\partial\ p}\right\} = \{b\} \tag{22}$$

in which $\{b\}$ is the right-hand side of Equation (19) and

$$[A] = \left[[K] - \Omega_i^2\ [M]\right] + \beta\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & & \\ 0 & & 0 & & \\ 0 & & & & 1 \end{bmatrix} \tag{23}$$

in which β is a scalar multiplier to assure the relative importance of the constraint such that [A] is not singular. The only nonzero element added corresponds to the floor about which the modes are normalized. The particular nature of the matrices such as symmetry, sparsity and banded structure are thus maintained.

EXAMPLE

Building

The building studied is a twelve-story frame-shear wall apartment building of reinforced concrete located in Ottawa, Canada. The masses are assumed known and concentrated at each story and there are thirteen unknown parameters in both of the horizontal and in the torsional direction of movement

$$[K] = p_{13}\ [K_a] + [K_s] \tag{24}$$

in which $[K_a]$ is the analytical stiffness matrix, p_{13} is a scalar factor and $[K_s]$ is an assumed shear-building stiffness matrix having twelve interstory springs.

$$
[K_s] = \begin{bmatrix} k_1 + k_2 & -k_2 & & & & \\ -k_2 & k_2 + k_3 & -k_3 & & & \\ & -k_3 & k_3 + k_4 & -k_4 & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & \cdot & k_{11} + k_{12} & -k_{12} \\ & & & & -k_{12} & k_{12} \end{bmatrix} \tag{25}
$$

Results

The mass quantities used for the twelve story building are shown in Table 1. It was assumed that the two lateral and torsional motions were uncoupled, and estimates to parameters of a shear building model were determined in each of the three motions separately. Relative weights for the stiffness parameters, k, the measured natural frequencies, Ω^2, and modes ϕ, are given in Table 2, together with that of a constant multiplier, p_{13}, of the analytically determined stiffness matrix. These correspond to errors of about 1% except for stiffness at 10%.

Measurements were taken during construction of the building of the first three mode shapes and frequencies in both lateral motions X, Y, and in torsion θ. The measured, initial and final mode shapes are given in Fig.1-3. No measurements were available for the third mode in Y motion. The results on the frequencies are given in Table 3. Convergence was attained in less than twelve iterations in all three instances, with the objective function reduced by a factor of a hundred.

Table 1 - Elements of the Diagonal Mass Matrix

Story	Mass (kip-sec^2/ft)	Mass Moment of Inertia (kip-ft-sec^2)
1-11	66.1	300 000.
12	85.9	390 000.

Table 2 - Weights

Variable	Lateral $1/\sigma^2$		Torsional $1/\sigma^2$
	X	Y	θ
k	10^{-10}	10^{-12}	10^{-18}
p_{13}	10^4	10^4	10^4
Ω	1	1	1
ϕ	10^4	10^4	10^4

The interstory stiffness parameters were initially set equal to zero and p_{13} was set equal to 2/3 in order to match the first frequency more closely. The final values of the thirteen parameters are given in Table 4, where a negative sign merely indicates that there was too much stiffness at that particular level in the analytical model.

Table 3 - Frequencies (Hertz)

	X			Y			θ		
	1	2	3	1	2	3	1	2	3
Initial	1.40	5.44	10.99	1.95	7.38	15.69	2.14	8.15	17.13
Measured	1.36	5.17	9.84	1.60	5.91	12.70	1.81	6.90	14.10
Final	1.35	5.17	9.84	1.60	5.91	12.90	1.82	6.90	14.10

Table 4 - Final Parameters

	X	Y	θ
Elements	Parameters (2)	Parameters (2)	Parameters (2)
k_1	0.28E+06	-0.47E+06	-0.15E+09
k_2	0.27E+06	-0.33E+06	-0.37E+09
k_3	0.17E+06	0.83E+05	-0.75E+08
k_4	-0.98E+05	0.17E+06	-0.33E+08
k_5	0.40E+06	-0.12E+05	-0.73E+09
k_6	0.43E+05	-0.11E+06	-0.16E+10
k_7	0.22E+05	0.23E+05	0.26E+10
k_8	-0.14E+06	0.17E+06	0.18E+10
k_9	0.35E+06	0.21E+06	0.77E+09
k_{10}	-0.36E+04	-0.56E+06	-0.12E+10
k_{11}	0.42E+05	0.14E+06	-0.18E+10
k_{12}	0.31E+06	0.68E+06	0.47E+10
p_{13}	0.6365	0.6660	0.6759

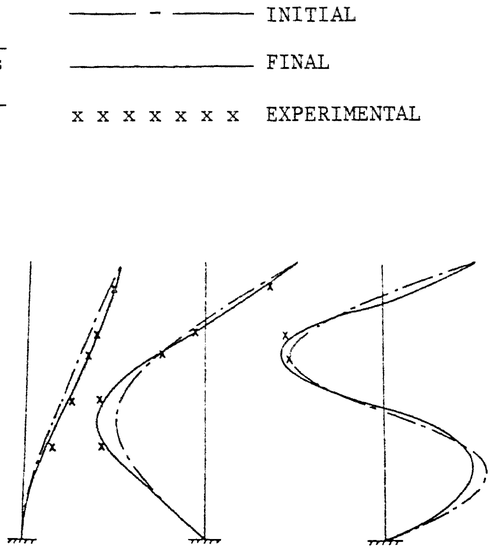

Fig. 1 - Lateral X

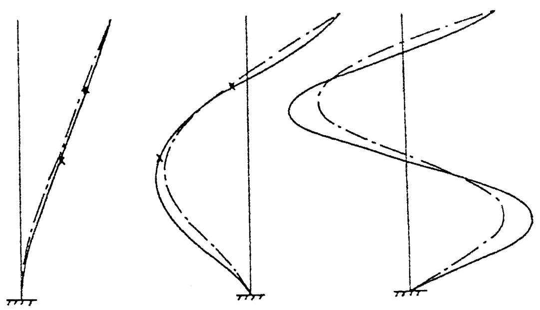

Fig. 2 - Lateral Y

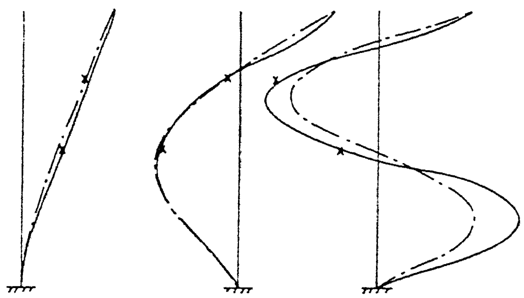

Fig. 3 - Lateral θ

Similar results for the completed building are reported elsewhere (Ref. 7). It should be emphasized that not all stories were instrumented and that only a few of the mode shapes and natural frequencies were used in the identification procedure. Once an appropriate finite element model has been adjusted to fit more closely observed dynamic behavior, in this case resonant frequencies and mode shapes, it is relatively easy to anticipate the effect of small design changes on the dynamic response. For instance, a change in either a story mass and/or stiffness affects the resonant frequency and mode shape according to Eqn. 18-23. With regards to the frequency response, its sensitivity is also readily determined (Ref. 3).

CONCLUSION

The application of a system identification technique to obtain actual physical parameters is not an easy one. Usually, little data is available; the data acquisition procedure should not be too cumbersome and yet reliable. Nevertheless, it is useful to modify parameters of a model, in this case a finite element model, in order to match these data more closely, particularly when design changes are planned. The procedure outlined here is useful in this regard and appears to converge quite nicely, provided of course some realistic structural model is postulated. It is based on relative weighing of measurements and initial parameters and utilizes efficient numerical techniques for its sensitivity calculations.

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