ROCKING, TIPPING AND UPTHROW OF SIMPLE STRUCTURES BY HORIZONTAL MOTION

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SUMMARY

This paper presents results from the analysis of simple structures, supported on foundation systems which permit uplift, to horizontal earthquake, impulse and harmonic excitation. The structures considered include rigid blocks and linear oscillators with one or more degrees of freedom. The foundation studied is a simplified two-spring model. The results can be extended to apply to the more realistic Winkler foundation, through an equivalence between these two foundation models. The overall response of the system is nonlinear, but the small amplitude response, even with partial uplift, does consist of a sequence of linear responses, a fact that aids the solution. For a sufficiently strong horizontal impulse, complete separation (upthrow) occurs, even in the absence of vertical excitation.

INTRODUCTION

The phenomenon of partial separation (lift-off) of the base of a structure or object from its foundation during strong ground shaking has been observed in many earthquakes. To analyze this phenomenon, the simple two-spring foundation, shown in Figure 1(a), is used. The more realistic Winkler foundation is shown in Figure 1(b). Unfortunately, the Winkler foundation leads to highly nonlinear equations of motion, even for the simple case of a rigid superstructure, and does not seem to be amenable to analysis. However, an equivalence between these two foundation models can be determined, so the compicated behavior of the Winkler foundation can be approximated by the more readily analyzed two-spring system. In the analysis, the horizontal translation of the base of the structure relative to the soil is neglected, except in the last section, where the upthrow of objects is examined.

DYNAMICS OF ROCKING SLENDER RIGID BODIES

In this two-dimensional configuration (see Figure 1a), the system possesses two degrees of freedom, namely, vertical motion and rocking in the plane of motion. Two different sets of equations of motion can be derived for the two regimes of the response: during full contact, when both springs are in contact with the base, and after lift-off, when only one spring is in contact. Since it is assumed that the foundation springs cannot take tension, uplift occurs when the upward displacement of the point of contact of either spring is greater than the static deflection, δ. The equations of motion, which can be linearized and solved analytically, are given in detail in Reference 1.

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For free oscillations of the block by a horizontal impulse, the response of the system during full contact is harmonic and uncoupled, and the vertical motion is not excited initially. After uplift, however, both degrees of freedom are excited and the response consists of harmonic and hyperbolic terms. For many applications the harmonic term is small and the hyperbolic term can be approximated by an inverted parabola. This approach permits the estimation of the apparent rocking period, T, and the apparent ratio of critical damping. The word "apparent" is used here because the overall response is nonlinear and the system does not possess a natural period in the classical sense. To estimate the apparent period, one can calculate the time when the parabolic term of the rocking response attains its maximum value and assume that this time corresponds to a quarter of the period. This analysis gives

$$T = \frac{4}{P_1}\left[\sin^{-1}\left(\frac{1}{\beta}\right) + \sqrt{\beta^2-1}\right] \tag{1}$$

where P_1 is the frequency of rocking during full contact and β is a measure of the strength of the excitation. The "normalized impulse", β is defined as the ratio of the maximum angle of rotation which would occur for a given impulse if uplift were not allowed (i.e., if the springs could take tension) divided by the angle at which lift-off first happens in the absence of vertical oscillations. As expected, lift-off results in a softer vibrating system. A similar analysis show that the apparent damping has a general tendency to decrease with the amount of lift-off.

Because of the elongation of the rocking period, the behavior of a structure allowed to uplift may be very different from the response without uplift. It cannot be concluded, however, that the effects of uplift are generally beneficial; this seems to depend on the parameters of the system and the characteristics of the excitation. This is illustrated in Figures 2 and 3; in the example of Figure 2, the excitation was a horizontal impulse with β equal to 8, while in Figure 3, a 0.5g harmonic base excitation was applied. Comparing these undamped responses with and without uplift, it is seen that lift-off increases the amplitude of rocking vibrations in the case of the impulse excitation and decreases it for the harmonic ground motion. The relative roof acceleration, however, was decreased in both cases.

In contrast, the more realistic Winkler foundation leads to complicated equations, which after lift-off become highly nonlinear due to the varying length of contact. An equivalence between these two models can be defined, however, which permits the use of the convenient two-spring model. This equivalence can be established by equating the vertical forces and the moments about the center of mass for the two models. An approximate equivalence, valid in both regimes of response, is given by the following equations

$$k = \frac{1}{\beta^2}\left(\tfrac{1}{2}k_o a\right) + \left(1 - \frac{1}{\beta^2}\right)\left(\tfrac{3}{4}k_o \tilde{S}\right) \tag{2a}$$

$$c = \frac{1}{\beta^2}\left(\tfrac{1}{2}c_o a\right) + \left(1 - \frac{1}{\beta^2}\right)\left(\tfrac{3}{4}c_o \tilde{S}\right) \tag{2b}$$

$$\xi = \frac{1}{\beta^2}\left(\frac{\sqrt{3}\cdot a}{6}\right) + \left(1 - \frac{1}{\beta^2}\right)\left(\frac{a}{2} - \frac{\tilde{S}}{3}\right) \tag{2c}$$

in which $\tilde{S}$ is an average value of the length of contact during lift-off for the Winkler foundation. It is given by the approximate relation

$$\tilde{S} \cong \frac{a}{\sqrt{\beta}} \tag{3}$$

The other symbols in Equations (2) are explained in Figure 1. The first terms in parentheses in these equations correspond to equivalence during full contact and the second terms to equivalence after lift-off. For the overall equivalence, the two values are combined using β^2 as a weighting factor.

The effectiveness of this equivalence is demonstrated in Figure 4 for $\beta = 8$. The results of an equivalence based only on the response during full contact, are also shown. Although Equations (2) were derived for impulse response, they may hold also for forced vibrations, as is shown in Figure 5, where a harmonic base excitation was used.

DYNAMICS OF ROCKING FLEXIBLE STRUCTURES

The general case of an N-DOF superstructure (Figure 6) is examined next. The two-spring model is again used to model the foundation and it is assumed that the superstructure possesses classical normal modes.

The equations of motion indicate that the two additional degrees of freedom introduced by the deformability of the foundation are, in general, coupled with those of the superstructure. As a result, analytical solutions are quite complicated. However, some conclusions can be drawn by working in the frequency domain (Ref. 2). The effects of the soil-structure interaction, and the uplift, are mainly concentrated in the apparent fundamental period of the system, which increases with the amount of lift-off. This period is always larger than either the fundamental period without lift-off or the fundamental period of the fixed-base superstructure. In contrast to the first mode, the second and higher modes of the structural model appear not to be affected significantly by either the soil-structure interaction or the uplift. Approximations for the natural frequencies of the system and the corresponding values of damping are given in Reference 2.

Like the case of rocking rigid blocks, the solution of the equations of motion after lift-off for a flexible superstructure includes hyperbolic terms upon which harmonic terms are superimposed. The angle of rotation increases nonlinearly with the excitation, but the effect of uplift on the amplitude of the relative story displacements and the resulting stresses is not clear, although the appearance of the response is greatly affected. This behavior is illustrated in Figure 7 for an example based on Caltech's Millikan Library, subjected to the S16E component of the Pacoima Dam accelerogram (San Fernando earthquake of 1971). A simple approximate solution, based on the response of the first mode only, is also shown.

UPTHROW OF OBJECTS

As noted above, one of the effects of lift-off is that vertical vibrations are excited even for purely horizontal base motions. For strong ground excitation, the amplitude of the vertical oscillations can be large enough to cause complete separation of the block from the foundation. Upthrow of objects has been inferred from observations after intense earthquake shaking and has been interpreted by many as evidence of vertical ground accelerations greater than gravity. This may be, but other mechanisms are possible. For example, very short blocks supported by two springs experience complete separation for any horizontal excitation strong enough to cause partial uplift (Ref. 1). However, this analysis presumed a constraint against horizontal movement of the base. For this reason, horizontal springs and dashpots have been added to the two spring foundation of Figure 1(a) and additional analyses performed. For simplicity, only the case of a horizontal impulse excitation, which imparts an initial horizontal velocity, v_o, was investigated.

The minimum required value of the horizontal impulse for upthrow of a block can be determined numerically. Such analyses are reported in Reference 3. The excitation is measured by the dimensionless quantity $\omega_y v_o/g$, wherein ω_y is the natural frequency of vertical vibrations during full contact and g is the acceleration of gravity. In Figure 8, this quantity is plotted versus the aspect ratio, b/a, for three different values of ω_x/ω_y, ω_x being the natural frequency of horizontal vibrations during full contact. In Figure 9, the dependence of this quantity on ω_x/ω_y is shown for b/a = 2, for four damping coefficients. The irregularities of the results make it difficult to draw general conclusions. It can be said that for short blocks, the required value of the initial impulse for complete separation increases as the stiffness of the horizontal springs decreases and the aspect ratio decreases. For tall blocks, the dependence of upthrow on these two parameters is not clear; the results are mixed. Foundation damping is also very important and high values of effective damping may prevent complete separation in practical cases.

In most instances, complete separation happened during the second half period of rocking. For larger excitations, upthrow occurs during the first half period. Upthrow may or may not be followed by complete overturning.

Although these results cannot be applied directly to earthquake excitation, they indicate that the horizontal component of some strong earthquakes may be able to cause upthrow of some objects. Because complete separation of a body is also clearly possible under vertical excitation only, the possibility of upthrow is thought to be higher when vertical excitation is present.

ACKNOWLEDGEMENT

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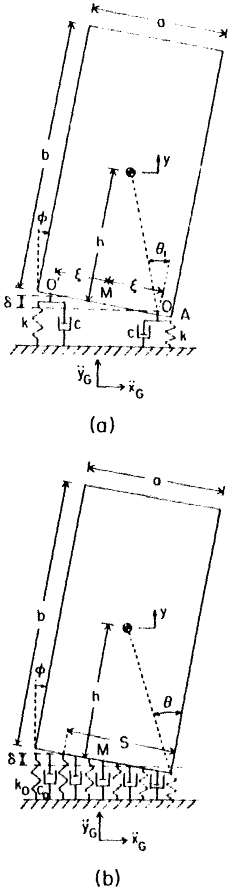

Fig. 1. Rocking block on (a) two-spring foundation and (b) Winkler foundation.

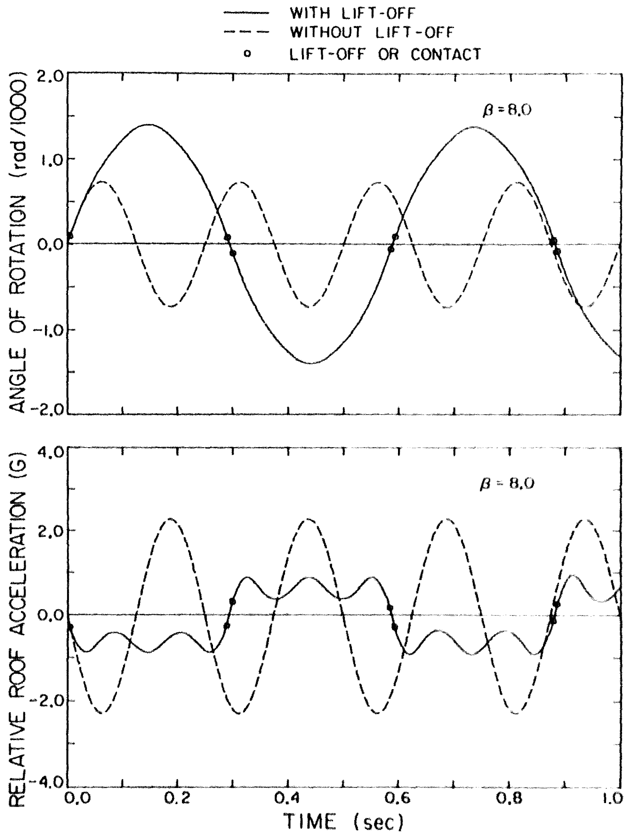

Fig. 2. Effect of lift-off on the angle of rotation and the horizontal, relative roof acceleration for free vibrations ($\beta=8$).

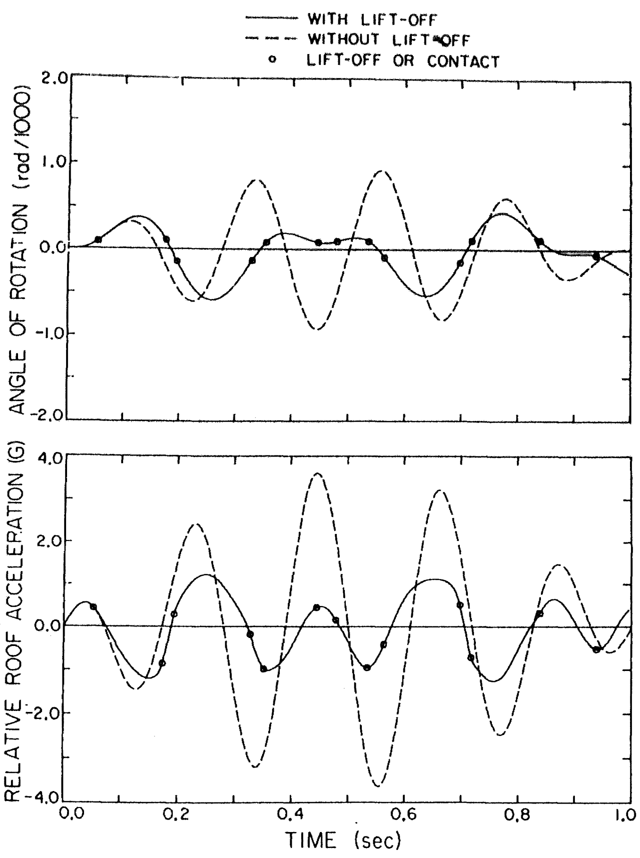

Fig. 3. Effect of lift-off on the angle of rotation and the horizontal, relative roof acceleration for harmonic excitation.

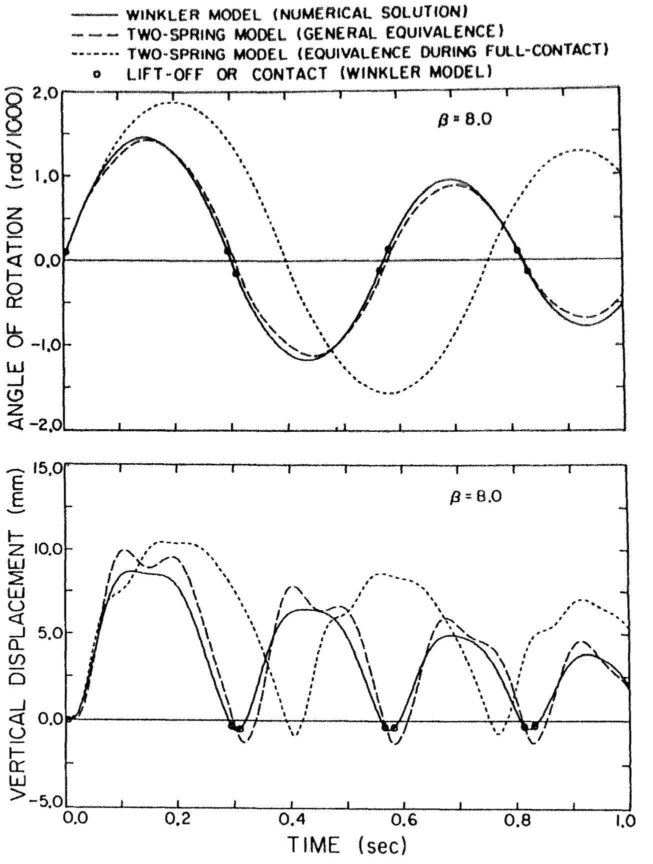

Fig. 4. Free oscillations of a simplified model of Caltech's Millikan Library (rigid block assumption) for β=8.

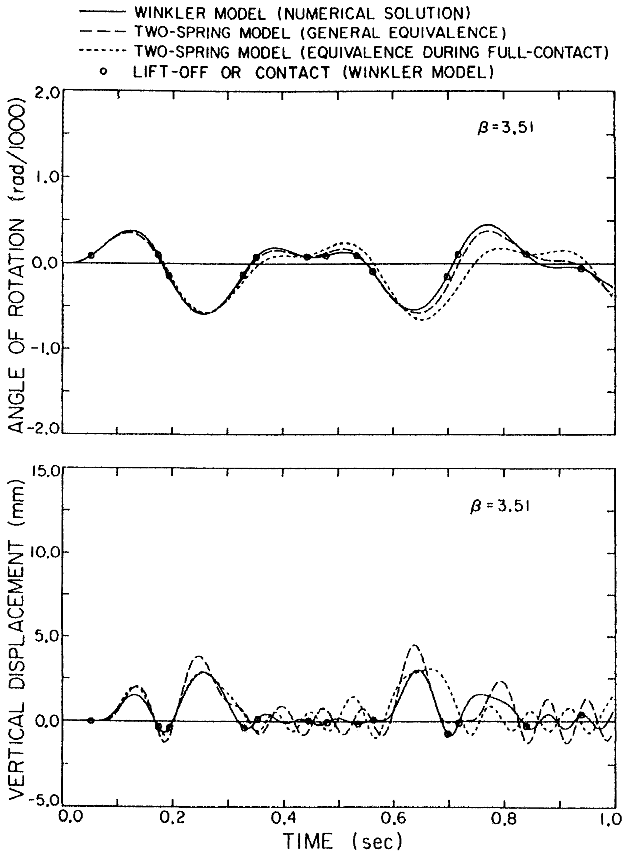

Fig. 5. Forced, undamped vibrations of a simplified model of Millikan Library (rigid block assumption) for horizontal harmonic excitation.

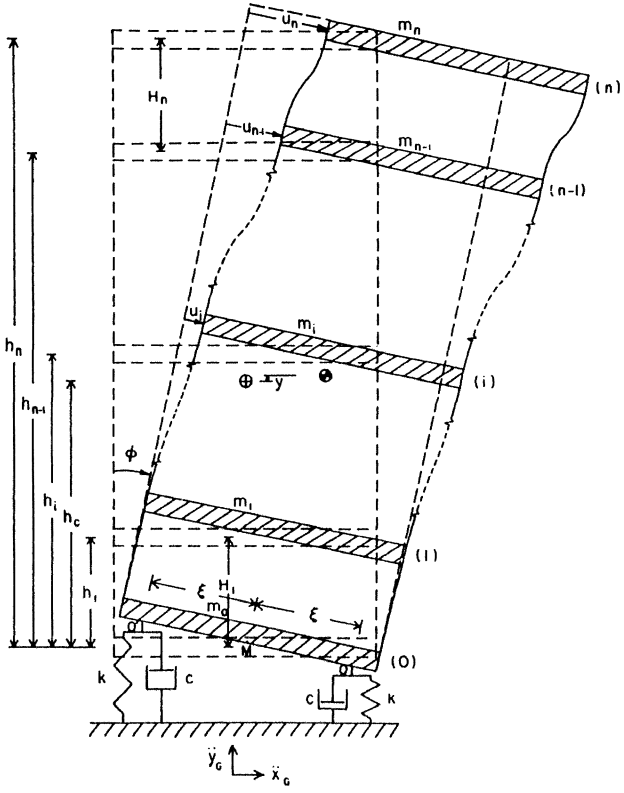

Fig. 6. Rocking n-story structure on two-spring foundation.

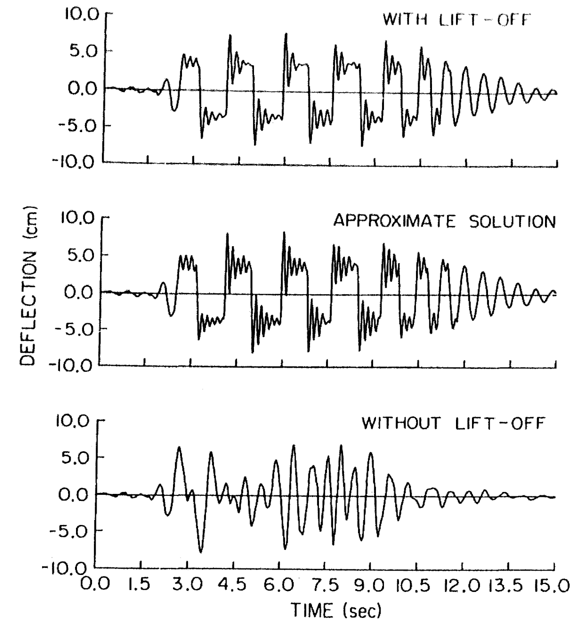

Fig. 7. Roof deflection with respect to the base, excluding rotations, of a model of Millikan Library subjected to the S16E component of the Pacoima Dam ground acceleration.

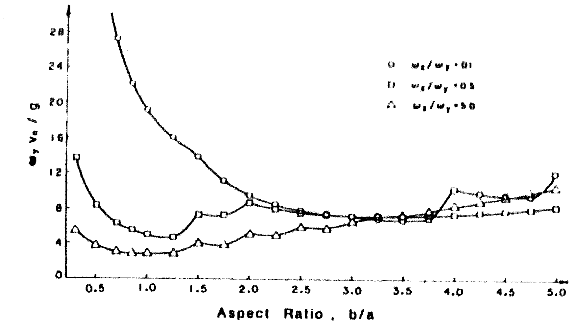

Fig. 8. Relation of the minimum required value of $\omega_y v_o/g$ for upthrow to the aspect ratio b/a ($\zeta_x=\zeta_y=10\%$)

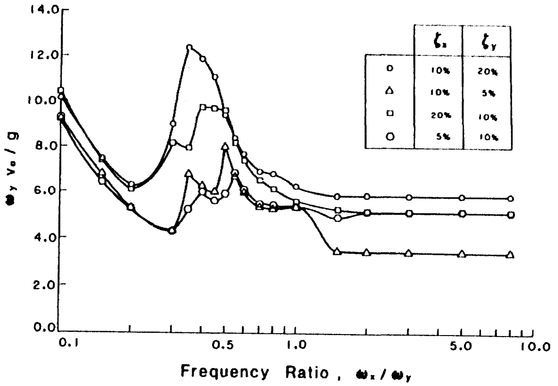

Fig. 9. Dependence of the minimum required value of $\omega_y v_o/g$ for upthrow upon the frequency ratio ω_x/ω_y and the damping values (b/a=2).