# EXTENDED APPLICATIONS OF RELATIVE ACCELERATION AND VELOCITY SPECTRA AS SEISMIC DESIGN INPUTS

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## SUMMARY

The relative acceleration and velocity response spectra have only been of academic interest so far. These, however, can be used with advantage as seismic design inputs in structural analysis for earthquake loads. The methods are presented which use these spectra as inputs in the calculation of member response as well as for generation of floor response spectra. These methods are more efficient computationally than the methods which use psuedo-acceleration spectra as design inputs. The prescription and use of relative spectra as seismic design inputs is, thus, advocated.

## INTRODUCTION

In structural analysis for earthquake loads, smoothed ground response spectra are commonly used. The response spectrum of a ground motion is the plot of the maximum response versus frequency of a single degree-of-freedom oscillator subjected to the ground motion. The plotted response quantity could be the maximum relative displacement, relative velocity, relative acceleration, absolute acceleration or any other quantity of interest (Ref. 1). In earthquake structural engineering the most commonly used plot is for the relative displacement of the mass, which directly provides the maximum deformation and thus the force in the spring. Such a plot is called the displacement spectrum.

Directly related to this spectrum are the psuedo-velocity and psuedo-acceleration spectra in which the response spectrum ordinates at a frequency are just the frequency and $(\text{frequency})^2$ times the displacement spectrum value, respectively. These two latter spectra, that is psuedo-velocity and psuedo-acceleration spectra, are more commonly used to characterize seismic design input for design of structural systems for earthquake loads. To obtain design spectra, usually the spectra of several recorded ground motions are used collectively. Smoothed out curves representing the average or other percentile values of the ensemble, have been used as seismic design input for the design of important structural facilities such as nuclear power plants, etc. See Refs. 2 & 3. Several analytical approaches have also been developed whereby these design spectra can be directly used for the calculation of design response for the design of primary and secondary structural systems.

The relative velocity and absolute acceleration spectra are often referred in the literature but, so far, have only been of academic inter-

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est. Also, the relative acceleration is a quantity which enters the equation of motion of an oscillator, its response spectrum has not been of even academic interest until very recently. The recent research by the writer and his associates (Ref. 4 & 5), and also by Hadjian (Ref. 6) has, however, shown that the relative acceleration and velocity spectra are as important as psuedo-acceleration spectra. In fact it has been found that there are some distinct analytical advantages in the use of relative acceleration and velocity spectra in lieu of psuedo-spectra in the calculation of seismic design response of primary and secondary structural systems. These extended applications are described in this paper.

## DESIGN RESPONSE AND RESPONSE SPECTRA

For a single degree-of-freedom structure, a response spectrum directly provides the design response like the maximum design force in the structural member without any special analysis. For a multi-degree-of-freedom (MDF) structure, however, the structure must be analyzed by the modal analysis approach to use response specra in the calculation of design response. For a MDF structure subjected to base acceleration, $\ddot{x}_g(t)$, the equations of motion, written in standard notations (Ref. 7),

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{r\}\ddot{x}_g(t) \tag{1}$$

are decoupled into modal equations. If [C] is a classical matrix, then a decoupled modal equation is of the following form:

$$\ddot{\nu}_j + 2\beta_j\omega_j\dot{\nu}_j + \omega_j^2\nu_j = -\gamma_j\ddot{x}_g(t) \tag{2}$$

in which $\nu_j$ = jth principal cordinate or mode displacement; and $\beta_j$, $\omega_j$ and $\gamma_j$, respectively, are the damping ratios, natural frequency and participation factor of the jth mode. If [C] is not classically damped, Eq. 2 can still be decoupled if complex-valued damped modes are used in the transformation to the principal coordinates. We will not pursue such a case here.

A response quantity S(t) of design interest can now be obtained as

$$S(t) = \sum_{j=1}^{N} \zeta_j \nu_j \tag{3}$$

in which $\zeta_j$ = modal response which is linearly related to the eigenvectors or mode shapes of Eq. 1, and N = the number of degrees-of-freedom. The modal displacement $\nu_j$ is obtained from the solution of Eq. 2 as

$$\nu_j(t) = \gamma_j \int_0^t \ddot{x}_g(\tau)h_j(t-\tau)\, d\tau \tag{4}$$

in which $h_j(t)$ is the impulse response function of Eq. 2.

To obtain design response, $\ddot{x}_g$ must be considered as a random process to account for all possible ground motions. We will assume $\ddot{x}_g(t)$ to be a stationary random process for analytical case. It has been verified by simulation study that this assumption is acceptable. Furthermore, considering the stationary value of the response, the maximum value of S(t), i.e. design response, $S_d$, can be written in terms of its mean square value and its peak factor, C, as

$$S_d^2 = C^2 \sum_{j=1}^{N} \sum_{k=1}^{N} \zeta_j \zeta_k \gamma_j \gamma_k \int_{-\infty}^{\infty} \Phi_g(\omega) \; H_j(\omega) H_k^*(\omega) \; dw \qquad (5)$$

where $\Phi_g(\omega)$ = the spectral density function of the ground motion. To obtain, the design response in terms of response spectra, the following relationships between design response spectra and spectral density function are used.

$$C_a^2 \int \omega_j^4 \Phi_g(\omega) \, |H_j(\omega)|^2 d\omega = R_{aj}^2 \qquad (6)$$

$$C_0^2 \int \omega^2 \Phi_g(\omega) \, |H_j(\omega)|^2 d\omega = R_{vj}^2 \qquad (7)$$

where $R_{aj}$ and $R_{vj}$, respecively, are the psuedo-acceleratin and relative velocity spectrum values at frequency $\omega_j$ and damping ratio $\beta_j$; and $C_a$ and $C_v$ are the peak factors for psuedo-acceleration and relative velocity response of the oscillator which when multiplied by the root mean square responses give their response spectrum values. The peak factors C, $C_a$ and $C_v$ will in general not be the same. However, extensive numerical simulation studies has shown that assuming them to be equal does not introduce any significant error in the calculation of design response. Thus here they are assumed to be equal. To use Eq. 6 and 7 in Eq. 5, the latter is split into terms with j=k and j≠k to give the following

$$S_d^2 = \sum_j (\zeta_j \gamma_j R_{vj}/\omega_j^2)^2 + 2 \sum_j \sum_{k=j+1} \gamma_j \gamma_k \zeta_j \zeta_k \left[A_1 R_{aj}^2 + A_2 R_{vj}^2 + A_3 R_{ak}^2 + A_4 R_{vk}^2\right] \qquad (8)$$

where $A_1$, $A_2$, $A_3$ and $A_4$ are coefficients of partial fraction (Ref. 8). The terms with single summation represent the popular SRSS mode response combination rule. The double summation terms improve the results and in some cases they must be considered to obtain the correct design response.

It is seen that in the double summation terms we need relative velocity spectra. Often the relative velocity spectrum value is assumed equal to the psuedo velocity spectrum value, especially when the former is not available. This assumption is reasonable if the dominant structural frequencies are within the range of the excitation frequencies. For the dominant structural frequencies higher than the excitation frequencies, this assumption can lead to significant errors in the calculated response (Ref. 4).

It is precisely in these cases that the relative acceleration spectra also play an important role. When a response quantity has a significant contribution from the high frequency modes, the terms associated with such modes in Eq. 7 must be retained, otherwise sizeable errors are likely to be introduced in the calculation of response. This situation can be alleviated if $S_d$ is expressed in an alternative form, employing relative acceleration spectra rather than psuedo-acceleration spectra.

For this, with the help of Eq. 2, Eq. 3 is re-written in terms of modal velocity, $\dot{\nu}_j$, and acceleration, $\ddot{\nu}_j$, as

$$S(t) = - \sum \frac{\xi_j}{\omega_j^2} \{\gamma_j \ddot{x}_g(t) + 2\beta_j \omega_j \dot{\nu}_j + \ddot{\nu}_j\} \tag{9}$$

The root mean square value of this quantity when multiplied by the peak factor gives the design response. This value can be expressed in terms of C, spectral density function and modal frequency response function. After some simplification, the design response can be written as (Ref. 4)

$$S_d^2 = S_s^2 + \sum (\zeta_j \gamma_j / \omega_j^2)^2 [2\omega_j^2(1-2\beta_j^2) R_{vj}^2 - R_{rj}^2]$$

$$+ 2 \sum \sum \frac{\zeta_j \zeta_k \gamma_j \gamma_k}{\omega_j^2 \omega_k^2} [\omega_j^2(1-4\beta_j^2) R_{vj}^2 + \omega_k^2(1-4\beta_k^2) R_{vk}^2$$

$$+ B_1 R_{vj}^2 + B_3 R_{vk}^2 + (B_2-1) R_{vj}^2 + (B_4-1) R_{rk}^2] \tag{10}$$

where $B_1$, $B_2$, etc., are the coefficients of partial fractions (Ref. 4), $R_{rj}$ is the relative acceleration response spectrum defned as

$$C^2 \int_{-\infty}^{\infty} \omega^4 \phi_g(\omega) |H_j(\omega)|^2 d\omega = R_{rj}^2 \tag{11}$$

and $S_s$ = the response calculated for the inertia forces corresponding to the maximum ground accleration applied statically on the structure.

It is seen that Eq. 10 uses relative acceleration and relative velocity spectra as seismic design inputs. The main advantage of Eq. 10 over Eq. 8 is that the modes with frequencies higher than the input frequency can be excluded from the summation without affecting the accuracy of the results. That is, higher modes need not be obtained explicitly in this procedure and only a first few modes are adequate. Thus, the method is more efficient computationally especially for stiff-structures or where a response quantity receives significant contribution from the higher modes.

For usual cases also, i.e., where the dominant modes are within the range of input motion frequencies, this method provides a more accurate response than Eq. 8 for the same number of modes used in the analysis. This is due to a very desirable characteristic of relative acceleration spectra. Fig. 1 shows averaged psuedo-acceleration and relative acceleration spectra obtained for an ensemble of synthetically generated accelerograms. For higher frequencies, the psuedo acceleration spectra approach a constant value equal to the maximum ground acceleration. On the other hand, the relative acceleration spectra diminish very fast for frequencies higher than the highest frequency in the input. Thus omitting the modes with frequency higher than the highest frequency wave in the input will cause a smaller error in the approach which employs relative acceleration spectra as input, i.e. Eq. 10, than the one which uses psuedo-spectra as input, i.e., Eq. 8.

## RELATIVE SPECTRA AS INPUT FOR GENERATION OF FLOOR RESPONSE SPECTRA

Similar advantages are realized when relative acceleration and relative velocity spectra are used as input in generation of floor spectra. The expression of floor response spectrum value which uses relative spectra as inputs is obtained when the absolute acceleration of the floor, $x_a(u)$ is defined as

$$\ddot{x}_a(u) = \ddot{x}_u + \ddot{x}_g = \ddot{x}_g + \sum_{j=1}^{N} \phi_j(u)\,\ddot{\nu}_j \qquad (12)$$

where $\ddot{x}_u$ = relative acceleration of floor and $\phi_j(u)$ modal displacement of the floor. However, if Eq. 12 is defined in terms of modal displacement and velocity as

$$\ddot{x}_A(u) = -\sum_j \phi_j(u)\,(\omega_j^2 \nu_j + 2\beta_j \omega_j \dot{\nu}_j) \qquad (13)$$

the floor spectrum expression is obtained in terms of psuedo-acceleration and relative velocity spectra. Eq. 13 has formed the basis for the development of floor response spectra generation techniques developed by various researchers so far. Using the acceleration defined by Eq. 12 as the input to an oscillator on the floor the maximum value of the oscillator response, that is, the response spectrum value is obtained as:

$$\begin{aligned} R_a^2(\omega_0,\beta_0) &= r_m^2[A_g^2 - R_r^2(\omega_o) + 2\,\omega_0^2\,R_v^2(\omega_o)] \\ &+ \sum_{j=1}^{n} \gamma_j\,\phi_j\,[\{2\,r_m\,A_1 + \gamma_j\,\phi_j\,A_2\}\,\omega_o^2\,R_v^2(\omega_o) \\ &+ \{2\,r_m\,B_1 + \gamma_j\,\phi_j\,B_2\}\,R_j^2(\omega_o) \\ &+ \{2\,r_m\,C_1 + \gamma_j\,\phi_j\,C_2\}\,\omega_o^2\,R_v^2(\omega_j) \end{aligned}$$

$$+ \{2\, r_m\, D_1 + \gamma_j\, \phi_j\, D_2\}\, R_r^2(\omega_j)\, \}]$$

$$+ 2 \sum_{j=1}^{n} \sum_{k=j+1}^{n} \gamma_j\, \gamma_k\, \phi_j\, \phi_k\, [\omega_o^2\, (A_j + A_k)\, R_v^2(\omega_o)$$

$$+ (B_j + B_k)\, R_r^2(\omega_o) + \omega_o^2\, \{C_j\, R_v^2(\omega_j) + C_k\, R_v^2(\omega_k)\}$$

$$+ D_j\, R_r^2(\omega_j) + D_k\, R_r^2(\omega_k)\,] \qquad (14)$$

where $A_1$, $B_1$ ..., $A_2$, $B_2$ ..., $A_j$, $B_j$ ..., $A_k$, $B_k$ , etc. are the coefficients of partial fraction (Ref. 5), $R_a(\omega_o, \beta_o)$ = absolute floor acceleration response spectrum value for an oscillator of frequency $\omega_o$, and damping ratio $\beta_o$. For stiff structures and generation of floor spectra for floors close to base, Eq. 14 is much more efficient than the approaches which use psuedo acceleration spectra as inputs in as much only a first few modes need to be included in the analysis with this approach. The floor spectra expressions based on Eq. 13 will require a large number of modes to achieve the same accuracy of the calculated spectra. For other situations where high frequency modes are not necessarily predominant, both approaches, i.e. Eq. 14, and the approaches which employ psuedo-acceleration spectra will be equally good. Still, however, for the same number of modes to be used in an analysis, Eq. 14 will provide a more accurate value of response.

In Fig. 2 are shown the floor spectra generated for the floor close to the base of a 30 degree-of-freedom structure by the two approaches. Only first four out of 30 modes were used in the analysis. The upper curve is obtained from Eq. 14. This curve did not change much when all 30 modes were used. The lower curve was also obtained with 4 modes but employed an approach where psuedo acceleration spectra were used as the inputs. When all 30 modes were used in the latter approach, the floor spectrum curve coincided with the upper curve. This clearly shows the efficiency of the approach which employs relative acceleration and velocity spectra as inputs, such as Eq. 14.

## CONCLUSIONS

The relative acceleration and velocity spectra, which have not been of much structural design use so far, however, have been shown to be better alternatives to the commonly used psuedo-acceleration spectra as seismic design inputs. These relative spectra can be used as design inputs for the calculation of design response of primary structures as well as for development of seismic design inputs for secondary systems in terms of floor response spectra.

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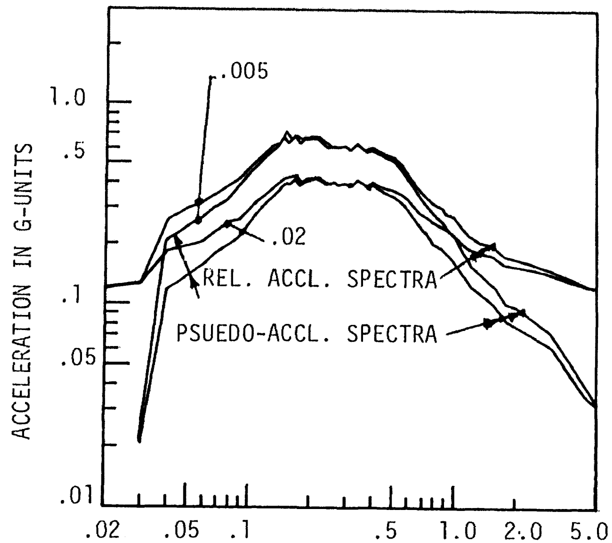


FIG. 1: PSUEDO- AND RELATIVE GROUND ACCELERATION SPECTRA FOR .005 and .02 DAMPING RATIOS - MEAN OF 33 TIME HISTORY SPECTRA.

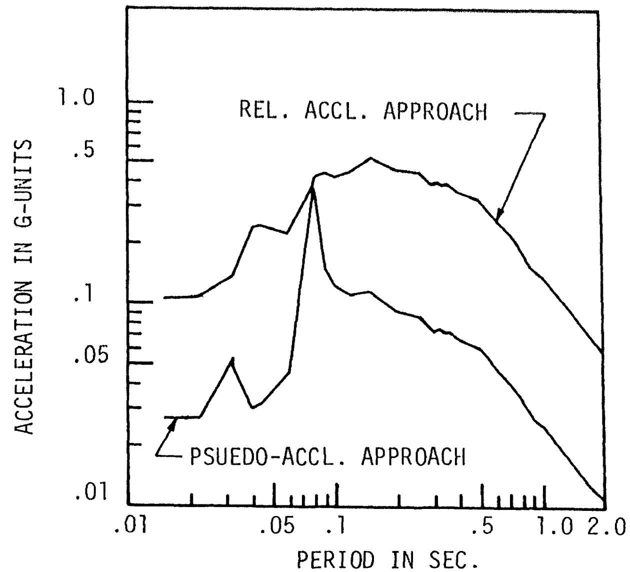


FIG. 2: FLOOR RESPONSE SPECTRA OBTAINED BY APPROACHES EMPLOYING RELATIVE AND PSUEDO-ACCELERATION SPECTRA AS INPUTS ( DAMPING RATIO = .005.)