EARTHQUAKE RESPONSE OF BUILDINGS ON WINKLER FOUNDATION ALLOWED TO UPLIFT

Solomon C. S. Yim (I)
Anil K. Chopra (II)
Presenting Author: Solomon C. S. Yim

SUMMARY

The effects of transient foundation uplift on the earthquake response of flexible structures attached to a rigid foundation mat supported on Winkler foundation with spring-damper elements distributed over the entire width of the mat are investigated. Based on the response spectra presented, the effects of foundation-mat uplift on the maximum response of structures are identified, and the accuracy of an equivalent two spring-damper-element foundation system are explored.

INTRODUCTION

The earthquake induced lateral forces on a structure, computed by dynamic analysis under the assumption that the foundation and soil are firmly bonded, will often produce a base overturning moment that exceeds the available overturning resistance due to gravity loads. The computed overload implies that a portion of the foundation mat or some of the individual column footings, as the case may be, would intermittently uplift during an earthquake. The effects of foundation uplift on the earthquake response of flexible structures have been investigated in recent analytical studies (Ref. 1-3). Whereas the flexibility and damping of the supporting soil were not incorporated in Refs. 1 and 2, they were modeled by two spring-damper elements, one at each edge of the foundation mat, in the most recent work (Ref. 3). Because the Winkler foundation model leads to considerable complication in the analysis, an equivalent two-element supporting system was developed based on the dynamics of rigid blocks (Ref. 3). Without resorting to this approximation in modeling the foundation, this paper aims to develop a better understanding of the effects of transient foundation uplift on building response. It is based on a report (Ref. 4) prepared under a research grant from the National Science Foundation.

SYSTEM CONSIDERED

The structural system considered, is a linear structure of mass m, lateral stiffness k and lateral damping c, which is supported through the

(I) Assistant Research Engineer, University of California, Berkeley, California, USA

(II) Professor of Civil Engineering, University of California, Berkeley, California, USA

foundation mat of mass m_o resting on a Winkler foundation, with spring-damper elements distributed over the entire width of the foundation mat, connected to the base which is assumed to be rigid (Fig. 1). If the dampers are absent in the Winkler foundation, the relation between the displacement and reaction force per unit width of the foundation mat is shown in Fig. 2a. The relation between the static moment M applied at the c.g and the resulting foundation-mat rotation θ, is shown in Fig. 2b where p is a static force acting in the downward direction at its center of gravity. If the mat is not bonded to the supporting elements the M-θ relation is linear, implying constant rotational stiffness, until one edge of the foundation mat uplifts from the supporting elements; thereafter the rotational stiffness decreases monotonically with increasing θ, which implies an expanding foundation-mat width over which uplift occurs. Uplift is initiated when the rotation reaches $\theta_u = p/2k_w b^2$ with the corresponding moment $M_u = pb/3$. The M-θ curve asymptotically approaches the critical moment, $M_c = pb$, corresponding to the physically unrealizable condition of uplift of the entire foundation mat from the supporting elements except for one edge. The downward force is $p = (m + m_o)g$, the combined weight of the superstructure and foundation mat, prior to any dynamic excitation, but would vary with time during vibration.

Next consider the entire structural system with a gradually increasing force f_s applied at the lumped mass m in the lateral direction. If the foundation mat is bonded to the supporting elements, which along with the structure have linear properties, the lateral force can increase without limit if the overturning effects of gravity forces are neglected. However, if the mat is not bonded to the supporting elements, one edge of the foundation mat is at incipient uplift when the lateral force reaches $f_{su} = (m + m_o)gb/3h$. Thus the corresponding base shear at incipient uplift under the action of static force is

$$V_u = (m + m_o)g \frac{b}{3h} \tag{1a}$$

The structural deformation u_u associated with this base shear and the foundation-mat rotation θ_u at incipient uplift are

$$u_u = \frac{(m + m_o)g}{k} \frac{b}{3h}; \quad \theta_u = \frac{(m + m_o)g}{2k_w b^2} \tag{1b}$$

As the lateral force continues to increase beyond f_{su}, the foundation mat separates over increasing width from its supporting elements. The lateral force $f_{sc} = (m + m_o)gb/h$ corresponds to the physically unrealizable condition of uplift of the entire foundation mat from the supporting elements except for one edge. Thus the limiting value for base shear under the action of static forces is

$$V_c = (m + m_o)g\frac{b}{h} \tag{1c}$$

The structural deformation due to this base shear is

$$u_c = \frac{(m + m_o)g}{k} \frac{b}{h} \tag{1d}$$

EQUATIONS OF MOTION

The differential equations governing the small-amplitude motion of the system of Fig. 1 due to horizontal ground acceleration $\ddot{u}_g(t)$ can be expressed as

$$m\ddot{u} + m(h\ddot{\theta}) + c\dot{u} + ku = - m\ddot{u}_g(t) + \frac{m(u + h\theta)}{h}(\ddot{v} + g) \tag{2a}$$

$$\frac{m_o b^2}{3h^2}(h\ddot{\theta}) - c\dot{u} + (1 + \varepsilon_1^3)c_w \frac{b^3}{3h^2} (h\dot{\theta}) + (1 - \varepsilon_1^2)\varepsilon_2 c_w \frac{b^2}{2h} \dot{v}$$

$$- ku + (1 + \varepsilon_1^3)k_w \frac{b^3}{3h^2} (h\theta) + (1 - \varepsilon_1^2)\varepsilon_2 k_w \frac{b^2}{2h} v = 0 \tag{2b}$$

$$(m + m_o)\ddot{v} + (1 + \varepsilon_1)c_w b\dot{v} + (1 - \varepsilon_1^2)\varepsilon_2 c_w \frac{b^2}{2h} (h\dot{\theta})$$

$$+ (1 + \varepsilon_1)k_w bv + (1 - \varepsilon_1^2)\varepsilon_2 k_w \frac{b^2}{2h} (h\theta) = - (m + m_o)g \tag{2c}$$

where contact coefficient ε_1 is equal to unity during full contact but depends continuously on foundation-mat rotation θ and vertical displacement v during partial uplift as follows:

$$\varepsilon_1 = \begin{cases} 1 & \text{contact at both edges} \\ \varepsilon_2 v/b\theta & \text{left or right edge uplifted} \end{cases} \tag{3a}$$

and contact coefficient ε_2 depends on whether one or both edges of the foundation mat are in contact with the supporting elements:

$$\varepsilon_1 = \begin{cases} -1 & \text{left edge uplifted} \\ 0 & \text{contact at both edges} \\ 1 & \text{right edge uplifted} \end{cases} \tag{3b}$$

The vertical displacements at the edges of the foundation mat, measured from the initial unstressed position, are

$$v_i = v \pm b\theta(t) \qquad i = 1,r \tag{4a}$$

Because the Winkler foundation cannot extend above its initial unstressed position an edge of the foundation mat would uplift at the time instant when

$$v_i(t) > 0 \qquad i = 1,r \tag{4b}$$

The equations of motion for the system of Fig. 1 are nonlinear as indicated by the dependence of the coefficients ε_1 and ε_2 on whether the foundation mat is in full or partial contact with the supporting system; and on the contact width.

EQUIVALENT TWO-ELEMENT FOUNDATION SYSTEM

The solution of the nonlinear equations of motion is complicated by the fact that after lift off these equations depend continuously on the varying degree of contact between the mat and its supporting elements. In contrast, the equations of motion are relatively simple for a system with foundation mat resting on two spring-damper elements, one at each edge of the foundation mat. In the latter case, the nonlinear equations of motion depend on three discrete contact conditions -- both edges of foundation mat are in contact with supporting elements, the left edge is uplifted, or the right edge is uplifted -- but for each contact condition the governing equations are linear. Because of the relative simplicity of the two-element supporting system, it is of interest to define its properties in such a way that it can model the more complicated Winkler foundation.

The equations of motion for the idealized one-story structure supported through a foundation mat resting on a Winkler foundation were presented in the preceding section and those for a two-element foundation in Chapter 2 of Ref. 4. If the foundation mat is bonded to the supporting elements the equations of motion for the structure supported on Winkler foundation are identical to those for the same structure on a two-element foundation with the following properties: $k_f = bk_w$, $c_f = bc_w$ and half base-width = $b/\sqrt{3}$. This two-element foundation is exactly equivalent to the Winkler foundation if uplift is not permitted.

If the mat is not bonded to the supporting elements, the relation between the static moment M applied at the c.g and the resulting foundation-mat rotation is shown in Fig. 3 for the two systems. The M-θ relation is linear with the same rotational stiffness for the two systems until θ reaches $\theta_u = p/2k_w b^2$ when one edge of the foundation mat on Winkler foundation is at incipient uplift. For larger rotation angles the M-θ relation for the two systems are different. The constant rotational stiffness implied by the linear M-θ relation continues to apply for the equivalent two-element supporting system until θ reaches

$\theta_u\sqrt{3}$ when one edge of the foundation mat uplifts from one of the supporting elements; thereafter no additional moment can be developed. On the other hand the M-θ relation for the Winkler supporting system is nonlinear for $\theta > \theta_u$ with the rotational stiffness decreasing monotonically with increasing θ. The M-θ curve asymptotically approaches the critical moment M_c = pb. Whereas the equivalent two-element supporting system is an exact representation of the Winkler foundation system for rotation angles $\theta < \theta_u$, it is only an approximation if the ground motion is intense enough to cause significant uplift. The accuracy of this approximation is explored by comparing the earthquake response of structures supported on the two foundations.

PRESENTATION AND DISCUSSION OF RESULTS

The response of a structural system to the north-south component of the El Centro, 1940 ground motion, computed by the special numerical procedures developed for the solution of the equations of motion (Ref. 4), is presented in Fig. 4. During the initial phase of the ground shaking, the foundation mat remains in contact with the supporting elements over its entire width. As the ground motion intensity builds up, the two edges of the foundation mat alternately uplift in a vibration cycle, including partial separation of the mat from the supporting elements. In this example the foundation mat uplifts over a significant portion of its width in every vibration cycle during the strong phase of ground shaking, with the duration of uplift depending on the amplitudes of foundation-mat rotation. As the intensity of ground motion decays toward the later phase of the earthquake, the foundation-mat uplift becomes negligible and full contact is maintained for long durations.

The base shear coefficient $\mathcal{V}_{max} = V_{max}/w$, where V_{max} is the maximum base shear, and w is the weight of the superstructure, is plotted as a function of the natural vibration period of the corresponding rigidly supported structure. Also presented are the results for the corresponding rigidly supported structure not allowed to uplift, which is simply the standard pseudo-acceleration response spectrum, normalized with respect to gravitational acceleration. Included in the response spectra plots is $\mathcal{V}_u$, the base shear coefficient associated with the value of base shear, V_u, at which uplift of an edge of the foundation mat is initiated (Equation 1a): $\mathcal{V}_u = V_u/w$. Also included is $\mathcal{V}_c$, the critical base shear coefficient associated with the static asymptotic base shear, V_c (Equation 1d) which corresponds to the uplift of the foundation mat from its supporting springs over the entire width, i.e., the foundation mat is standing on its edge: $\mathcal{V}_c = V_c/w$. These base shear coefficients depend on the mass ratio m_o/m and slenderness-ratio parameter h/b, but are independent of the vibration period T. The differences between the response spectra for the two linear systems, the structure with foundation mat bonded to the supporting elements and the corresponding rigidly supported structure, are due to the change in period and damping resulting from support flexibility (Ref. 4). The base shear developed in structures with relatively long vibration periods is below the static value at incipient uplift and throughout the earthquake the foundation mat remains in contact over its entire width with the supporting elements. If foundation mat uplift is prevented, the maximum base shear at some vibration periods may exceed the incipient-uplift value. For the selected system parameters and ground motion, Fig. 5 indicates that this occurs for all vibration periods shorter than the period where the linear spectrum first attains the incipient-uplift value. If the foundation mat of such a structure rests on the Winkler spring-damper elements only through gravity and is not bonded to these elements, partial separation occurs and this has the effect of reducing the base shear. However, the base shear exceeds the value at incipient-uplift because even under static forces the base moment, and hence base shear, continue to increase considerably beyond this value (Fig. 2b). Furthermore the base shear is not reduced to as low as the critical value based on static considerations. Although this asymptotic value can never be exceeded under static forces, depending on the state -- displacement, velocity and acceleration -- of the system, the deformation and base shear may exceed the critical values during dynamic response, as seen in Fig. 5. Because the base shear developed in linear structures (foundation-mat uplift prevented) tends to exceed the incipient-uplift value by increasing margins as the vibration period decreases, the foundation mat of a shorter-period structure has a greater tendency to uplift over a greater portion of its width, which in turn results in the incipient-uplift base shear being exceeded by a greater margin although it remains well below the linear response.

Also shown in Fig. 5 is the response spectrum for the equivalent two-element system defined earlier. This response spectrum is identical to that for the Winkler system for the relatively long periods because the base shear developed is below the incipient-uplift value and the foundation mat does not uplift from its supporting elements and for this condition the two-element supporting system is exactly equivalent to the Winkler supporting system. Uplift occurs for any structure if the corresponding ordinate of the linear response spectrum exceeds the static base shear coefficient at incipient uplift, which is $1/3\alpha$ for a Winkler system and $1/\sqrt{3}\alpha$ for the equivalent two-element system. Because the base shear developed in linear structures tends to increase as the vibration period decreases, uplift in a Winkler system is initiated at a longer period compared to the two-element system. However, because the uplift is limited in extent and duration in the range of periods bounded on the low side by the period at which uplift is initiated in a two-element system and on the high side by the period at which uplift is initiated in a Winkler system, the difference between the response spectra for the two systems is small in this period range. For shorter vibration periods outside this range the equivalent two-element system consistently underestimates the maximum response because, as shown in Fig. 3, the equivalent two-element system does not adequately represent the moment-rotation relation for larger rotation angles. Overall, the equivalent two-element system is successful in displaying the main effects of foundation uplift on maximum structural response, and it provides results that are reasonably close to those for a Winkler system over a wide range of vibration periods, excluding the very short periods.

REFERENCES

1. Meek, J.W., J. Struct. Div., ASCE,101, ST7, pp. 1297-1311, 1975
2. Meek, J.W., Earthq. Eng. Struct. Dyn., 6, pp 437-454, 1978.
3. Psycharis, Report No. EERL 81-02, Calif. Inst. Tech., Pasadena, 1981; Earthq. Eng. Struct. Dyn. 11, pp. 57-76 and 501-521 (1983).
4. Yim, C.S., and Chopra, A.K., Report No. UCB/EERC 83-09, Univ. of Calif., Berkeley, 1983.

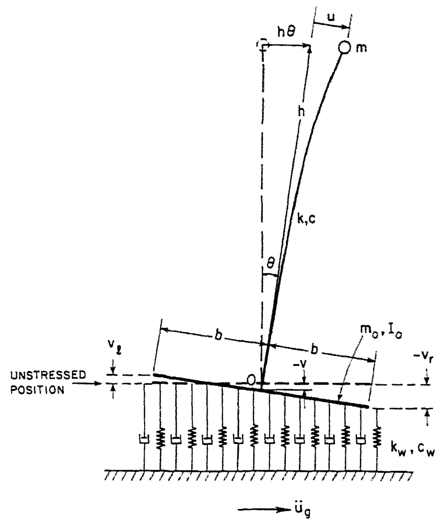
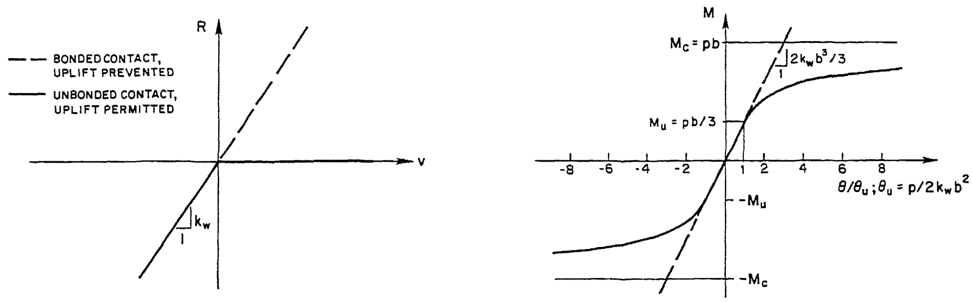


Fig. 1 Flexible Structure on Winkler Foundation.



(a) Reaction force-displacement relation for unit width of the supporting elements

(b) Moment-rotation relation for foundation mat

Fig. 2 Properties of Winkler Foundation.

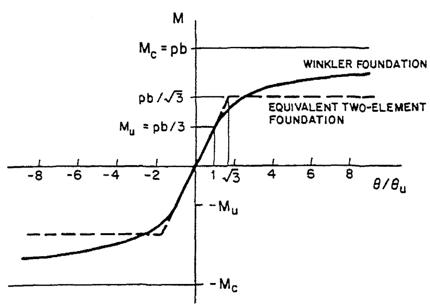


Fig. 3 Moment-rotation Relations for Foundation Mat on Winkler Foundation and on Equivalent Two-element Foundation.

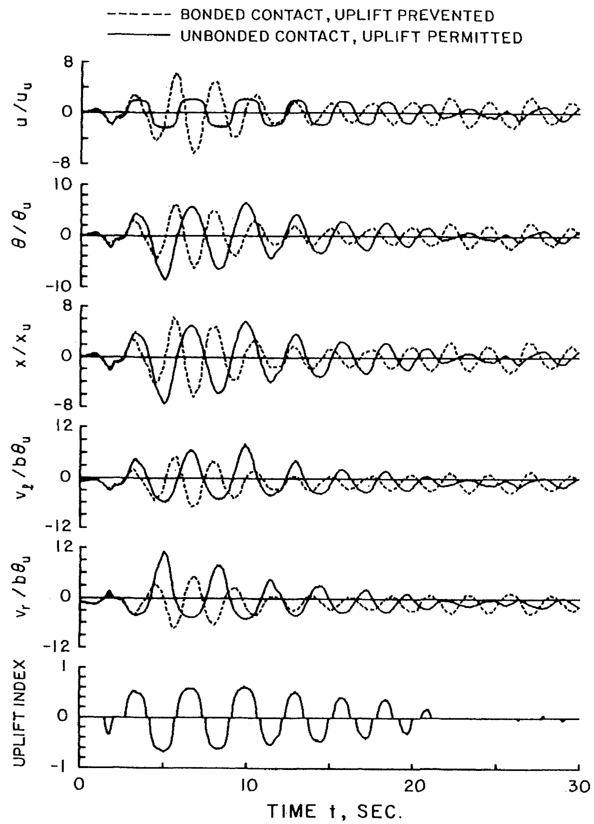


Fig. 4 Response of a Structure to El Centro Ground Motion for Two Conditions of Contact Between the Foundation Mat and the Supporting Elements.

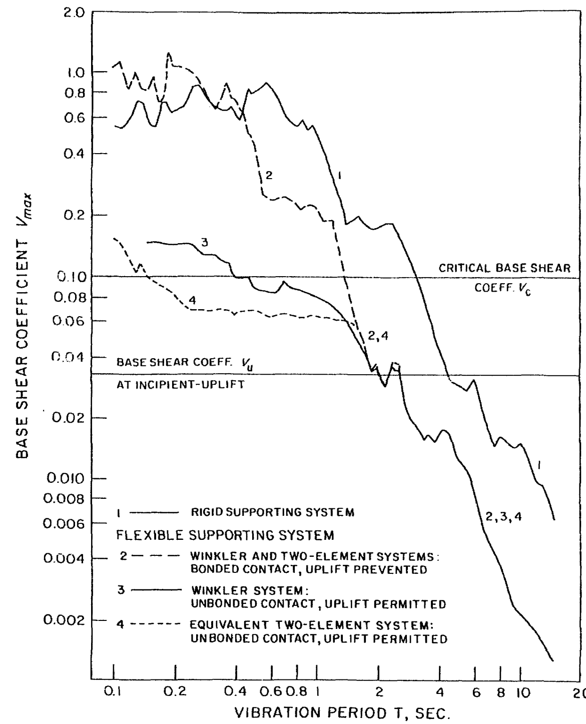


Fig. 5 Response Spectra for Structures Subjected to El Centro Ground Motion for Four Support Conditions.