CRITERIA FOR OVERTURNING OF RIGID BODIES BY SINUSOIDAL AND EARTHQUAKE EXCITATIONS

Y. ISHIYAMA (I)

SUMMARY

This investigation deals with overturning of rigid bodies on a rigid floor subjected to sinusoidal and earthquake excitations. Experiments and simulations of frequency sweep tests were conducted, and it is concluded that the horizontal velocity as well as the acceleration must be taken into account as criteria for overturning. Simulations by earthquake excitations show that the criteria are also applicable to the earthquake excitations. Therefore it is possible to estimate the lower limits of the maximum horizontal acceleration and velocity of the input excitations, from the overturning of bodies.

INTRODUCTION

Despite their familiarity and apparent simplicity, the motions and overturning of rigid bodies in response to earthquake excitations pose extremely difficult problems when exact solutions are sought. Since the dawn of seismology and earthquake engineering, the behavior of rigid bodies during earthquakes has intrigued a number of seismologists and earthquake engineers for over a hundred years (Ref. 1). These interests have been mainly motivated by the possibility of estimating the peak acceleration of earthquake excitations at sites for which no seismographic records are available. Typical examples are the contributions of Japanese researchers to the estimation of seismic intensities by observing the overturning of tombstones in the graveyards. Another promising aspect of this type of study, which has attracted the attention of many researchers in recent years, is its applicability to preventing building furniture and equipment from overturning when exposed to shaking during earthquakes.

In this paper, motions and overturning of a rigid body in response to sinusoidal and earthquake excitations are experimentally and theoretically studied. Criteria for the overturning of bodies are proposed after experiments and simulations of frequency sweep tests. Simulations by earthquake excitations show that the criteria are also applicable to earthquake excitations. Therefore we can estimate the lower limits of the maximum horizontal acceleration and velocity of the input excitations from the overturning of bodies.

COMPUTER SIMULATION PROGRAM

When a body on a floor is subjected to earthquake excitations, the body may remain at rest while the excitations are not large. When the excitations become large enough the body may rock, slide, jump or may undergo some combination of these motions. The computer program, which has been developed by the present author (Ref. 2), can deal with any plane motions of a rigid body on a rigid floor subjected to the horizontal and vertical excitations. In the

(I) Head, Building Engineering Division, International Institute of Seismology and Earthquake Engineering, Building Research Institute, Ministry of Construction, Government of Japan

program, the motions are classified into six categories; i.e. (1) rest, (2) slide, (3) rotation, (4) slide rotation, (5) translation jump, and (6) rotation jump (See Fig. 1); and the equations of motion are solved numerically.

One of the applications of the program is as follows: If we tilt a body slightly and release it, the body starts rotating and it may continue rotating or it may transit into a slide rotation. Simulations on this kind of motion have been carried out, changing the size of the body, coefficients of friction and restitution coefficients. In the case that breadth-height-ratio (b/h, See Fig. 2) is not greater than the coefficient of friction, the body continues the rocking motion, repeating the impact between the body and the floor (See Fig. 3a). If b/h becomes greater than the coefficient of friction, the body slides to the reverse side of tilting after impact (See Figs. 3b, c). If b/h becomes even greater still, the body undergoes slide rotation to the tilting side after rotation before impact and slides to the reverse side of tilting after impact (See Figs. 3d - g).

OVERTURNING BY SINUSOIDAL EXCITATIONS

Experiments on Frequency Sweep Tests Using a Shaking Table (Ref. 3)

In order to study the conditions for the overturning of bodies, experiments on frequency sweep tests were conducted, using a shaking table. In the experiments on frequency sweep tests the acceleration, velocity or displacement amplitude was kept constant, gradually shifting the frequency, and the conditions when the body overturned were measured. Test specimens were rectangular columns of various sizes and materials. They were placed on a 5 cm thick granite board which was mounted on the shaking table. In some cases, the granite board was covered by a 0.3 cm thick vinyl resin sheet, to change the coefficients of friction and restitution coefficients.

Typical experimental results of the overturning of bodies by the frequency sweep tests are shown in Fig. 4. In the figure the ordinate is the normalized acceleration which is defined as the acceleration amplitude devided by the horizontal acceleration "a" necessary to initiate rocking motion of the body, i.e.

$$a = \frac{b}{h} g \quad : \text{West's formula} \qquad (1)$$

where g is the gravitational acceleration; and the abscissa is the normalized velocity which is defined as the velocity amplitude divided by the velocity v necessary to overturn the body by a single horizontal shock, i.e.

$$v = \sqrt{\frac{2g}{r} (i^2 + r^2) \frac{1 - \cos \alpha}{\cos^2 \alpha}} \qquad (2)$$

where α is the angle between the vertical line and the line from O to G of the body at rest, i is the radius of gyration of the body about G and r is the distance from O to G (See Fig. 2). It is found that the results in the figure are not affected much by the size and materials of the test specimens and by the materials of the floor.

Simulations of Frequency Sweep Tests

Simulations of frequency sweep tests were conducted with or without vertical excitations, using the computer simulation program explained in the previous section. Carrying our series of frequency sweep simulation tests, it is found that the results are not affected much by the parameters involved; i.e. b/h, size of the body, coefficient of friction, restitution coefficient, the magnitude of vertical excitations, and the phase difference between horizontal and vertical excitations. Typical simulated results of overturning are shown in Figs. 5, 6.

PROPOSAL OF CRITERIA FOR OVERTURNING

Summarizing the results in the previous section, Fig. 7 is obtained to show the overturning conditions produced by the frequency sweep tests. In Fig. 7, there are three kinds of overturning (See Fig. 8). In the case of constant acceleration amplitude sweep tests the body usually initiates inverse phase rocking motion, and overturning from this inverse phase rocking occurs when the input excitations reach curve "a" in Fig. 7. But if the body initiates subharmonic rocking motion, the overturning from subharmonic rocking occurs at curve b in Fig. 7. It is difficult to tell the conditions that make the body undergo the inverse phase or subharmonic rocking motion. In the case of constant velocity or displacement amplitude sweep tests, the body repeats its "in phase" rocking motion only a few times and overturning from in phase rocking occurs at line c in Fig. 7.

A discrepancy between the experiments (See Fig. 4) and simulations (See Fig. 5) is that the overturning can occur at the lower acceleration than the value given by WEST's formula of Eq.(1) in the case of experiments (See the dotted curve in Fig. 7). In order to study this phenomenon, the frequency sweep simulation tests have been conducted with small initial angular displacements. The simulated results are shown in Fig. 6 which is much more similar to experimental results. Then, it is found that the body can be overturned by an acceleration which is less than one unit of the normalized acceleration, if some shocks initiate the rocking motion of the body.

From the results of experiments and simulations in the previous section, it is found that the lower limits of normalized acceleration and velocity necessary to overturn the body (point P in Fig. 7) are not affected much by b/h, size of the body, coefficient of friction, restitution coefficient or by the existence of vertical excitations. The dotted curve in Fig. 7 can be neglected, because the shocks which initiate the rocking motion of the body should be more than one unit of normalized acceleration.

Therefore, we can estimate criteria to overturn a body as follows: The acceleration amplitude a_o, which is the lower limit of the maximum horizontal acceleration to overturn the body, is approximately equal to the normalized acceleration.

$$a_o \simeq \frac{b}{h} g = \frac{B}{H} g \qquad (3)$$

where B and H are the breadth and height of a rectangular body, respectively. The velocity amplitude v_o, which is the lower limit of the maximum velocity to overturn the body, is approximately 0.4 times the normalized velocity.

$$v_o \simeq 0.4 \sqrt{\frac{2g}{r} (i^2 + r^2) \frac{1 - \cos \alpha}{\cos^2 \alpha}} \qquad (4)$$

In the case of a rectangular body,

$$v_o \simeq 0.4\sqrt{\frac{8gr}{3} \cdot \frac{1 - \cos\alpha}{\cos^2\alpha}} \tag{5}$$

If α is small,

$$v_o \simeq 0.4\sqrt{\frac{4gb}{3}}\,\alpha \simeq 10\frac{B}{\sqrt{H}} \qquad \text{(unit: cm, sec)} \tag{6}$$

Considering the displacement amplitude, it is greater than the value that makes the normalized acceleration equal to 1.0 and the normalized velocity equal to 0.4. Then using the relationship of $v_o^2 = a_o d_o$, the displacement amplitude d_o, which is the lower limit of the maximum displacement necessary to overturn the body, is given by

$$d_o = \frac{v_o^2}{a_o} \tag{7}$$

In the case of a rectangular body and if α is small,

$$d_o \simeq \frac{B}{10} \tag{8}$$

Using the criteria to overturn the body given by Eqs.(3), (5) and (7), the diagram to estimate the lower limits of the maximum acceleration, velocity and displacement from the size of an overturned homogeneous rectangular body can be made as shown in Fig. 9.

APPLICATION OF THE CRITERIA TO EARTHQUAKE EXCITATIONS

Motions of a rectangular body subjected to recorded earthquake horizontal excitations with or without the vertical components were simulated, considering various sizes of the body.

One of the typical simulated results of overturning are shown in Fig. 10; where the abscissa and ordinate are the height breadth ratio H/B and the breadth H of the rectangular bodies, respectively. In the figure, the acceleration and velocity criteria for overturning -- i.e. Eqs.(3) and (5), respectively -- are also indicated; substituting the maximum acceleration and velocity of the earthquake excitations. From the simulated results by several earthquake excitations (i.e. El Centro, Taft, Miyagi-ken-oki) it can be seen that the criteria derived in the previous section are also applicable to earthquake excitations as summarized in Fig. 11. In this figure, A is the region of neither rocking nor overturning, B is the region of rocking but no overturning, and C is the region of rocking and probable overturning. The behavior of the bodies in region C is highly irregular, i.e. the taller bodies are sometimes more stable than shorter ones of the same breadth. The effect of vertical excitations is small and it does not always increase the probability of overturning. The effects of coefficient of friction and restitution coefficient are also small. As to the bodies in region C, the smaller ones have the higher probability of overturning.

Actually, all the bodies overturned are in region C -- or very close to -- of Fig. 11. Therefore, from the size of the body overturned, we can conclude that the maximum horizontal acceleration and velocity must have been greater than the values given by Eqs.(3) and (5), respectively. Considering the displacement criterion of Eq.(7), it seems that the criterion is too conservative, though it certainly gives the lower limit of the maximum horizontal displacement due to the earthquake excitations.

CONCLUSIONS

Criteria for overturning are studied and the following are concluded:

(1) Simulations and experiments of frequency sweep tests show that, in order to overturn a body, both the horizontal acceleration and the velocity of the floor must be greater than certain levels.

(2) These levels provide the minimum criteria for overturning the body, i.e. 1.0 times the acceleration given by West's formula and 0.4 times the velocity of the shock to overturn the body. Therefore it is possible to estimate the lower limits of the maximum horizontal acceleration a_o, velocity v_o and displacement d_o of the input excitations from the size of the body overturned.

In the case of a slender rectangular body whose breadth and height are B and H, respectively; they are

$$a_o \simeq \frac{B}{H} g \qquad \text{(g: gravitational acceleration)} \qquad (3)$$

$$v_o \simeq 10 \frac{B}{\sqrt{H}} \qquad \text{(unit: cm, sec)} \qquad (6)$$

$$d_o \simeq \frac{B}{10} \qquad (8)$$

(3) Simulations of actual earthquake excitations show that the criteria are also applicable to earthquake excitations. Therefore, we can estimate the lower limits of the maximum horizontal acceleration and velocity of the input excitations from the size of the body overturned.

ACKNOWLEDGEMENT

The author wishes to express deepest appreciation to Prof. Takuji SHIBATA of Hokkaido Univ., Prof. Toshihiro IRIE of Hokkaido Univ., Prof. Yutaka MATSUSHIMA of Tsukuba Univ., Prof. Michio OTSUKA of Kyushu Univ. and Dr. Makoto WATABE of I.I.S.E.E. B.R.I. for their valuable advice and encouragement; to Prof. Tetsuo KUBO of Nagoya Institute of Technology for providing corrected time histories of recorded earthquake excitations; to Miss Sumiko ENDO of I.I.S.E.E. B.R.I. for typing the manuscript; and to Mr. Nobuyoshi YAMAGUCHI of I.I.S.E.E. B.R.I. for computer and experimental working.

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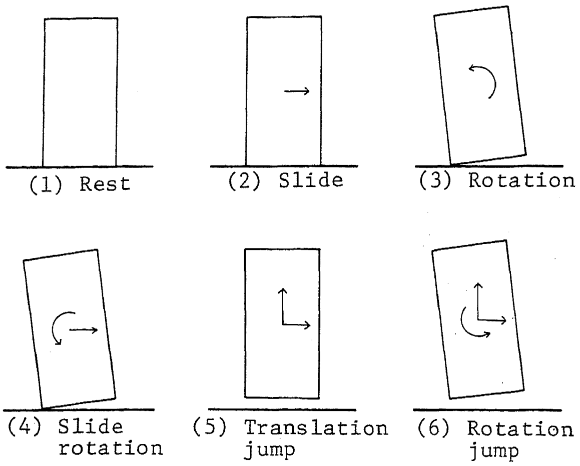


Fig. 1 Classifications of Motions of a Body

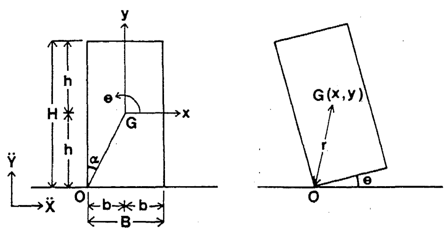


Fig. 2 A Body in Motion

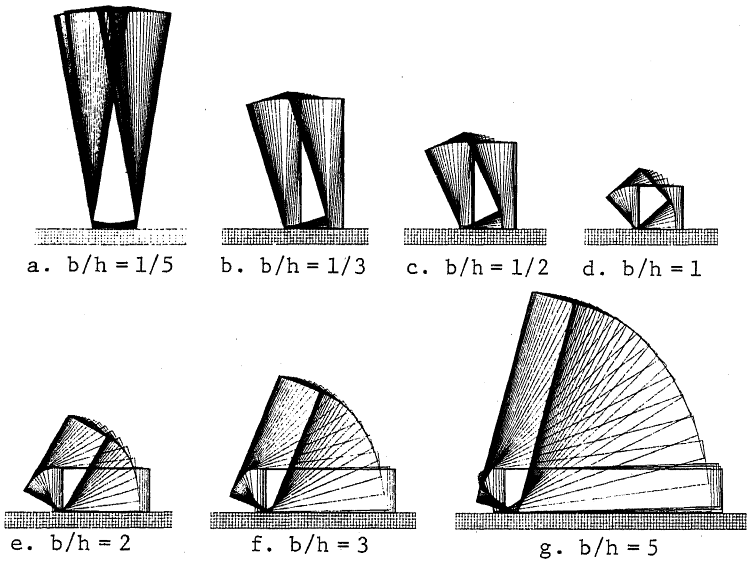


Fig. 3 Simulations of Motions

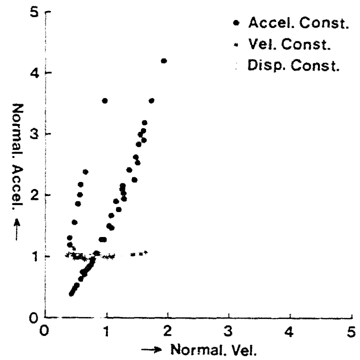


Fig. 4 Experimental Results of Overturning (Specimen: b × h = 5 × 25 cm hard rubber)

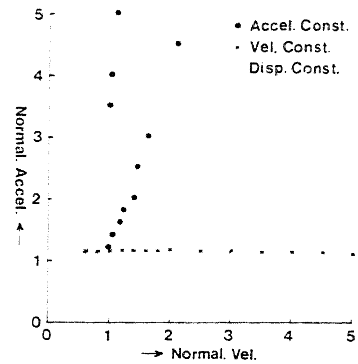


Fig. 5 Simulations of Overturning (b × h = 10 × 50 cm)

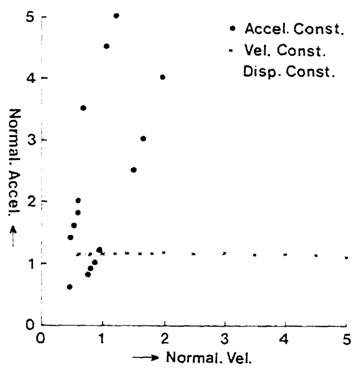


Fig. 6 Simulations of Overturning (b × h = 2.5 × 7.5 cm, Initial angular disp. = 0.1α)

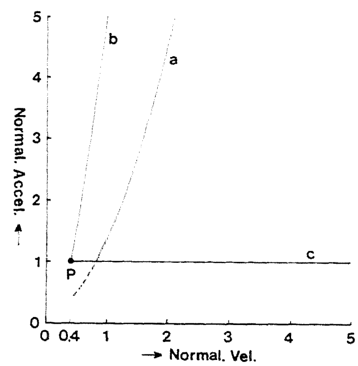
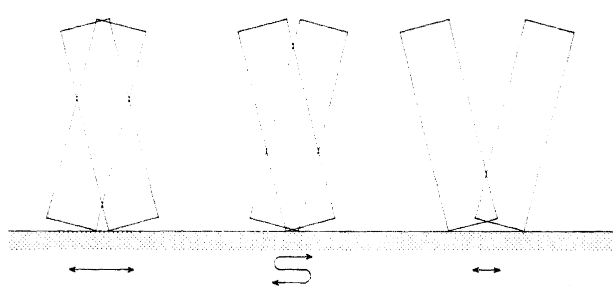


Fig. 7 Boundaries of Overturning



a. Inverse phase rocking b. Subharmonic rocking c. In phase rocking

Fig. 8 Three Kinds of Rocking Motions before Overturning

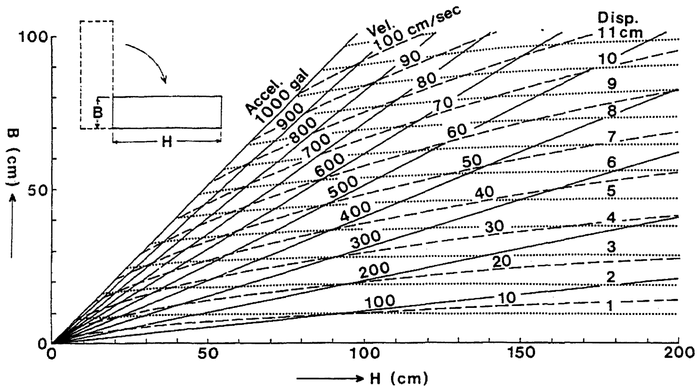


Fig. 9 Lower Limits of Acceleration, Velocity and Displacement to Overturn the Homegeneous Rectangular Bodies

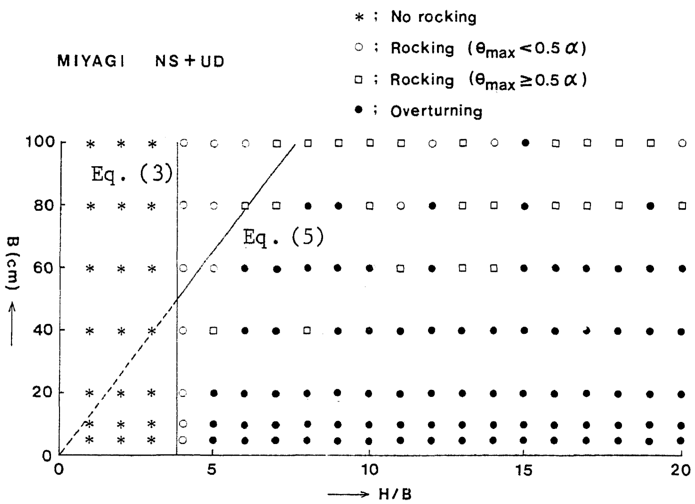


Fig. 10 Motions of Bodies by MIYAGI-KEN-OKI Earthquake (NS + UD component)

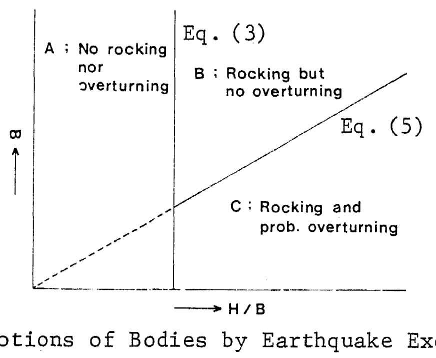


Fig. 11 Motions of Bodies by Earthquake Excitations