SEISMICALLY INDUCED ROCKING
OF RIGID STRUCTURES

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SUMMARY

A study of the rocking response of a rigid block on a rigid plane subjected to a horizontal sinusoidal model of seismic acceleration has been made. It has been observed that, starting from rest, the block topples for certain combinations of excitation amplitude and frequency, while steady states are attained for other combinations. Thus, a safety criterion can be established. Furthermore, the results which pertain to the harmonic excitation case can be used as the basis of an approach to predicting toppling of a block subjected to seismic shaking.

INTRODUCTION

The destructive power of earthquakes with regard to structures or objects that can rock and topple is well known. Despite this danger, financial considerations often make it necessary to continue building in seismically active regions of the world. Thus, it is essential that a proper study be made of the behavior of those structures under strong ground shaking. This study aims to improve the current state of understanding of this rather complex dynamic problem. For this, a simple harmonic function will be used to model the ground shaking.

HARMONIC SEISMIC MODEL

Equation of Motion

A two-dimensional model is shown in Figure 1 of a rigid rectangular block which is free to rock, without slipping, on either of its base corners. The foundation is a rigid horizontal plane, which is excited in the horizontal direction relative to an inertial frame of reference. The vertical component of the foundation displacement is neglected for simplicity. Moreover, this simplification may be justified by the fact that, statistically, earthquake records show much smaller vertical than horizontal components. Taking moments about corner O, the governing equation of motion for the block O is written in the form

$$I_o\ddot{\theta} + mR\ddot{x}_g\cos(\theta_{cr} - \theta) + mgR\sin(\theta_{cr} - \theta) = 0\ ; \qquad \theta > 0 \tag{1}$$

In this equation, I_o is the moment of inertia of the block about O, θ is

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the clock-wise tilt of the block, θ_{cr} is the critical angle which separates toppling and recovering of the block in the static case, m is the mass of the block, R is the distance from 0 to the center of mass of the block, g is the gravitational acceleration, and $\ddot{x}_g$ is the horizontal ground acceleration. Similarly, the rocking about 0', for which $\theta < 0$, is governed by the equation

$$I_o\ddot{\theta} + mR\ddot{x}_g\cos(\theta_{cr} + \theta) - mgR\sin(\theta_{cr} + \theta) = 0 \ . \tag{2}$$

The transfer from rocking about one corner to that about the other is accompanied by an impact. The associated energy loss is accounted for by reducing the angular velocity of the block after impact. Specifically, it is assumed that

$$\dot{\theta}(t^+) = e\,\dot{\theta}(t^-) \ , \qquad 0 \le e \le 1 \ , \tag{3}$$

where e is the coefficient of restitution, t^+ is the time just after impact, and t^- is the time just before impact.

The nonlinear equations (1) and (2) can be linearized when θ and θ_{cr} are small. Specifically, they reduce to

$$I_o\ddot{\theta} + mR\ddot{x}_g + mgR(\theta_{cr} - \theta) = 0 \ ; \qquad \theta > 0 \ , \tag{4}$$

and

$$I_o\ddot{\theta} + mR\ddot{x}_g - mgR(\theta_{cr} + \theta) = 0 \ ; \qquad \theta < 0 \ . \tag{5}$$

In (Ref. 1), it is shown that overturning occurs when two empirical criteria are satisfied. First, the peak ground acceleration must be at least equal to the acceleration that initiates rocking. Second, the maximum ground velocity must be greater than 0.4 times the sudden velocity change that just causes toppling, if the excitation were a step velocity function. These criteria should be viewed as a first approximation to the exact solution since they do not account for changes of ground acceleration and velocity after the initial rocking. Using a half sine-wave to model ground acceleration, the authors of (Ref. 2) and (Ref. 3) produced a criterion for toppling during the first uplift. Clearly, the criteria of (Ref. 1) and (Ref. 3) are independent of the coefficient of restitution (e). However, when an impact occurs the magnitude of the coefficient of restitution becomes important in subsequent rocking or toppling. Thus, it is logical to seek conditions for toppling for a block which reflect the significance of the coefficient of restitution and involve more realistic models of ground shaking. Specifically, the ground acceleration is expressed as

$$\ddot{x}_g = Ag\cos(\omega t + \phi) \ , \tag{6}$$

where A is the <u>nondimensional</u> amplitude and ϕ is the excitation phase. Then, the nonlinear equations of motion become

$$I_o\ddot{\theta} + mgRA\cos(\omega t + \phi)\cos(\theta_{cr} - \theta) + mgR\sin(\theta_{cr} - \theta) = 0 \; ; \quad \theta > 0 \qquad (7)$$

and

$$I_o\ddot{\theta} + mgRA\cos(\omega t + \phi)\cos(\theta_{cr} + \theta) - mgR\sin(\theta_{cr} + \theta) = 0 \; ; \quad \theta < 0 \; . \qquad (8)$$

Furthermore, the piecewise linear equations become

$$I_o\ddot{\theta} - mgR\theta = - mgRA\cos(\omega t + \phi) - mgR\theta_{cr} \; ; \qquad \theta > 0 \qquad (9)$$

and

$$I_o\ddot{\theta} - mgR\theta = - mgRA\cos(\omega t + \phi) + mgR\theta_{cr} \; ; \qquad \theta < 0 \; . \qquad (10)$$

The general solutions for (9) and (10) are

$$\theta^+(t) = a^+\sinh \alpha t + b^+\cosh \alpha t + \theta_{cr} + \beta\cos(\omega t + \phi) \; , \qquad (11)$$

and

$$\theta^-(t) = a^-\sinh \alpha t + b^-\cosh \alpha t - \theta_{cr} + \beta\cos(\omega t + \phi) \; , \qquad (12)$$

where the superscript (+) indicates validity for $\theta > 0$, and the superscript (-) indicates validity for $\theta < 0$. Furthermore, a^+, b^+, a^-, b^- denote arbitrary constants, and α, β are defined by the equation

$$\alpha^2 = mgR/I_o \; , \qquad (13)$$

while

$$\beta = A\alpha^2/(\alpha^2 + \omega^2) \; . \qquad (14)$$

Response Under Quiescent Initial Conditions

In order to find conditions for toppling, transient responses of the system with quiescent initial conditions, $\theta(0) = 0$ and $\dot{\theta}(0) = 0$, are computed. By varying the excitation amplitude (A) and the frequency (ω), the results are conveniently plotted on the (A) versus (ω) plane. This procedure can be applied to a model with any combination of parameters. Herein, the model of (Ref. 4), with the specifications $R = 9.84$ ft (3 m), $H/B = 4$, $\theta_{cr} = 0.245$ rad, $m = 2.2$ lb (1 kg), $I_o = 284.8$ lb sq ft (12 kgm^2), $g = 32.2$ ft/s^2 (9.81 m/s^2), and $\alpha = 1.566$ is used. Note that taking m = 1 kg is not restrictive since the numerical results can actually be provided in terms of the ratio I_o/m. Throughout this paper all the transient responses are calculated with $\phi = 0$, but results corresponding to other logical values of ϕ are similar to those presented herein. The transient responses for this model are shown in Figure 2. Examining the nature of the response in the (A) versus (ω) plane, two points can be made. First, there is a "safe" region in which the responses are stable and approach steady states. Second, the stable steady states are of several types, to be defined shortly, which occupy definite regions of their own in the plane. Thus, from the first point, a safety criterion can be established. That is, for a rigid block rocking on a rigid foundation subjected to harmonic horizontal ground accelerations, (A) and (ω) must lie within the "safe" zone in Figure 2 to avoid toppling of the block. Note that the "safe" region expands in the (A) direction as (ω) increases, signifying that the block becomes increasingly robust against toppling as the excitation frequency increases.

In (Ref. 3) a continuous curve that separates the "safe" and the "unsafe" zones was produced, based on the response of an initially quiescent block to a half-sine wave pulse excitation. Such a curve is drawn on Figure 2 for comparison.

To examine the accuracy of the piecewise linear solutions, all the transient response calculations are repeated using numerical integration to solve the exact linear equations of motion. Figure 3 shows pertinent numerical results; they are almost identical to those obtained by using the piecewise linear equations of motion. Note that the small regions of the less prominent modes do not necessarily overlap. As far as computation time is concerned, the numerical integration of the nonlinear equations of motion requires approximately ten (10) times more than the solution which is based on the piecewise linear equations

It has already been mentioned that a variety of steady state models are observed; it is necessary to distinguish them. A steady state mode is called the (m,n) mode, where m,n are positive integers, if the mode has a minimum repetitive interval which equals n periods of the harmonic excitation and, during that interval, executes m cycles of oscillation. The (1,1) mode is also called the fundamental mode since it is predominant and has the same frequency as the excitation. Note that the system exhibits, as well, steady state modes which have non-zero means. Hence, the terms symmetric, zero mean, and unsymmetric, non-zero mean, are used as additional qualifiers. The (1,n) modes are called subharmonics since their frequencies are equal to 1/n times that of the excitation. Similarly, the (m,1) modes are called superharmonics because their frequences are m times that of the excitation frequency. Finally, for modes which have neither m nor n equal to 1, they are called, for convenience, combined modes.

The most prominent mode observed is the symmetric fundamental (1,1) mode, and the next most common mode is the subharmonic (1,3). Much scarcer is the unsymmetric (1,1) mode which has a pronounced non-zero mean. Other modes observed are the unsymmetric combined (2,2), the symmetric combined (3,3) and some other higher order (m,n) modes, all of which are quite rare. Note that, within the range of (A) and (ω) investigated, no superharmonics have been found.

It is interesting to know how the "safe" region and the types of transient responses change with different values of the height to base ratio (H/B) and the coefficient of restitution (e). Thus, Figure 4 shows the kinds of transient solution for e = 0.6, H/B = 2, using the piecewise linear equations, while Figure 5 shows those corresponding to the nonlinear equations. Again, the agreement is quite good, proving that the piecewise linear equations describe the dynamics of the rocking block adequately.

Combining several numerical data, Figure 6 shows the "safe" region in the (A) versus (ω) plane for various values of (e) and (H/B). As might be expected, the "safe" zone increases as (e) and (H/B) decrease. This is because the block becomes more resistant to rocking as a result of the greater energy dissipation, and the greater restoring moment with decreasing (e) and (H/B) respectively. The corresponding curves of (Ref. 3) are also drawn on Figure 4 and Figure 6, for comparison. Clearly, these curves are not influenced by the coefficient of restitution (e) and therefore do not accurately predict the "safe" zone. It is also found that as (e) and (H/B) are reduced from 0.95 and 4, respectively, the symmetric (1,1) mode increases in dominance at the expense of the symmetric (1,3) mode, to the effect that when e = 0.6 and H/B = 2, the (1,3) mode is completely displaced in the (A) versus (ω) plane investigated.

Application for Real Seismic Motions

With regard to the applicability of the presented results to real seismic motions, (Ref. 1) can be used. In this reference, it has been demonstrated, through simulations, that the maximum velocity of an earthquake record can be used as a criterion for toppling. With that aim, Figure 7 shows the types of responses plotted on the velocity amplitude versus excitation frequency plane, (Ag/ω) versus (ω), for e = 0.95 and H/B = 4. Also shown is a curve which separates the "safe" and "unsafe" regions. Clearly, there exists a lower bound for the critical velocity amplitudes. Therefore, a reasonable criterion would be that toppling is not likely to occur if the maximum ground velocity of the earthquake is less than the lower bound of the critical velocity amplitude. Figure 8 shows the "safe" and "unsafe" regions, and the lower bounds of the critical velocity amplitudes for various (e) and (H/B) together with the bounds based on (Ref. 1). It is readily seen that the variation of the coefficient of restitution causes shifting of the bounds of the "safe" region; this is not reflected in the criterion of (Ref. 1).

CONCLUDING REMARKS

A study of the rocking response of a rigid block on a rigid plane subjected to a horizontal sinusoidal model of seismic acceleration has been made. First, by computing the transient responses of the system with quiescent initial conditions for various values of excitation amplitudes (A) and frequency (ω), it has been possible to determine safe, no-toppling, and unsafe regions in the (A) versus (ω) plane. Second, a safety criterion has been proposed for real earthquake excitations, specifically, the lowest velocity amplitude of sinusoidal excitation which causes toppling is set as the upper bound on the peak velocity of the real earthquake motion if toppling is to be avoided. The proposed criterion is deemed as an improvement on that presented in (Ref. 1). The improvement is associated with the fact that a harmonic function is a better, albeit still poor, approximation to an earthquake velocity record than a step function. It must be emphasized that this criterion is based on simulation data, and additional work needs to be done to test its reliability and level of conservatism. Note that piecewise linearization of the nonlinear equation of motion has been used in producing most of the results shown in Figure 2 through Figure 8. The reliability of this approximation has been tested by employing numerical integration of the nonlinear equation of motion; it has been found quite acceptable even for values of (H/B) as low as two.

ACKNOWLEDGEMENT

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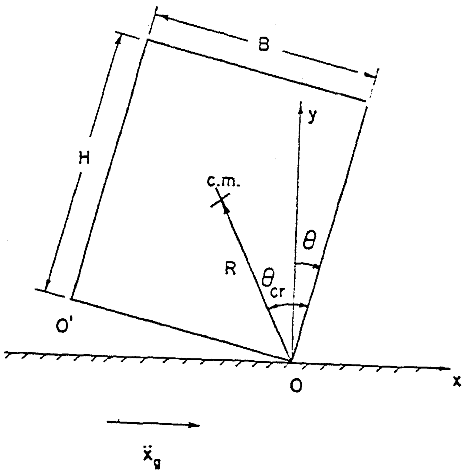

Fig. 1 2-D Model

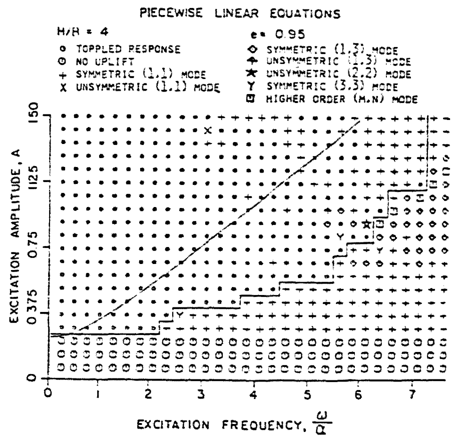

Fig. 2 Piecewise Linear Transient Responses

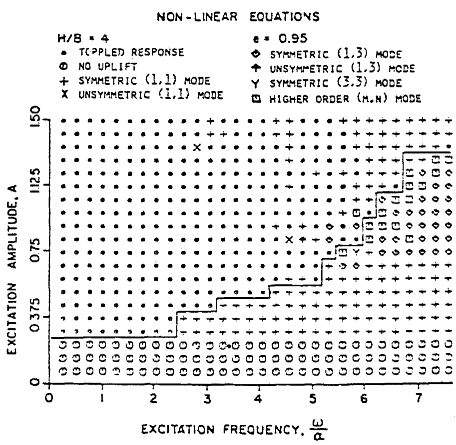

Fig. 3 Nonlinear Transient Responses

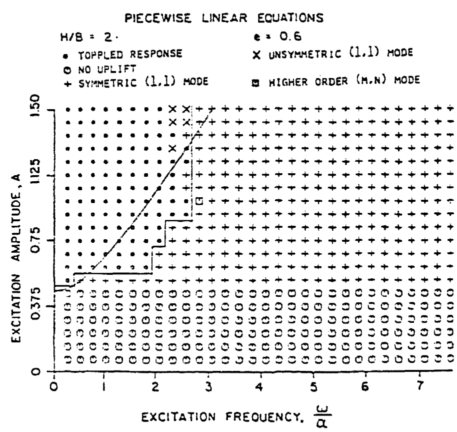

Fig. 4 Piecewise Linear Transient Responses

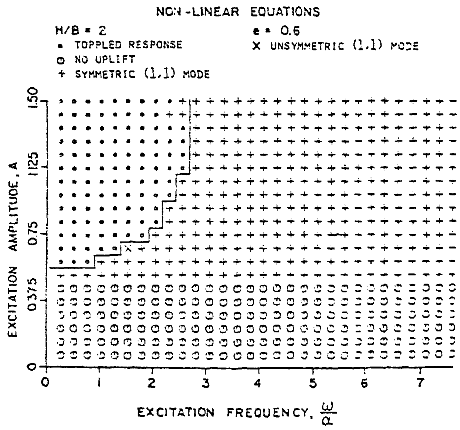

Fig. 5 Nonlinear Transient Responses

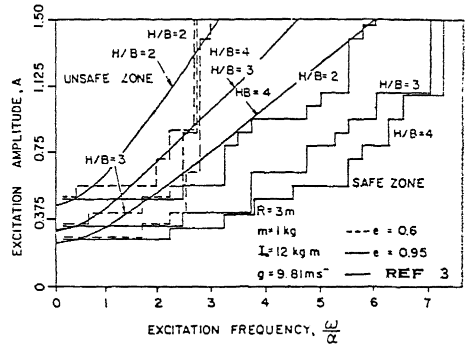

Fig. 6 Safe and Unsafe Regions in (A) versus (ω) plane

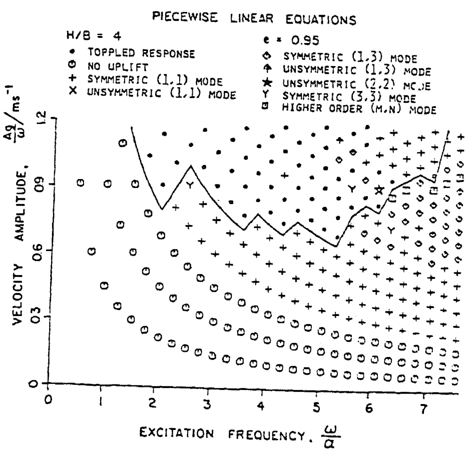

Fig. 7 Piecewise Linear Transient Responses

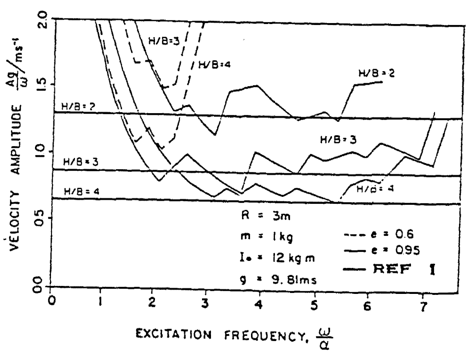

Fig. 8 Safe and Unsafe Regions in $(\frac{Ag}{\omega})$ versus (ω) plane