NONLINEAR RESPONSE OF TORSIONALLY COUPLED BUILDINGS FOR MULTICOMPONENT EARTHQUAKE EXCITATIONS

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SUMMARY

A simple approach to the nonlinear analysis of rigid-floor, unsymmetrical single-story systems subjected to three translational components of ground motion is presented. The conventional concept of center of rigidity is extended by introducing the notion of axial center of rigidity. The nonlinear structure is idealized as a sequence of piecewise linear systems with successively changing stiffness properties during earthquake response. The equations of motion are formulated with reference to the dynamic principal rigidity coordinates; both geometric and material nonlinearities are considered. An example of application of the corresponding algorithm shows the economy and efficiency of its execution on a microcomputer.

INTRODUCTION

The single-story system is represented as a horizontal rigid-body floor standing on a number of supporting elements of arbitrary location and orientation in plan (Fig. 1). These column elements are assumed to offer resistance to lateral sway in any direction through shear forces associated with bending--like classical shear-beam elements--but in this model, they also offer resistance to axial deformation like conventional truss elements. End cross-sections of supporting elements may yield during earthquake response. The element local torsional stiffness is neglected. This analytical model is believed to represent the major components of the overall spring resistance of the structural system.

FORMULATION OF SYSTEM STIFFNESS

The structural system behaves in the elastic range as if composed by two parallel and independent parameter arrangements: the lateral-torsional (L-T) and the axial systems. However, as soon as yielding occurs at any element during the response, a strong interaction between both systems takes place, inducing a complex dynamic variation of the instantaneous system stiffness properties.

Ayre's description of the L-T system (Ref. 1) can be generalized after some algebraic manipulation to obtain the location of the L-T center of resistance $(\hat{a},\hat{b})$, the principal translatory and torsional rigidities (K_1,K_2,K_{tz}) and the principal directions of rigidity $\hat{\beta},\hat{\beta}+\pi/2$, for an arbitrary distribution and orientation in plan of the supporting elements.

To introduce the axial system, we apply the theory of unsymmetric bending (small displacement kinematics) to the analytic model of the single-story system as presented in Fig. 1.

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We define the axial center of rigidity as a point in the plan view of the floor such that when a force acting along a vertical axis through this point is applied to the rigid-body, it experiences pure vertical translation (no rocking). We also define the axial principal directions of rigidity as those mutually normal directions going through the axial center of rigidity such that when a bending moment vector is applied to the rigid-body floor along any one of these directions, it experiences pure rocking in the same direction (no vertical translation, no rocking in the orthogonal horizontal direction).

Then, if kz_i is the axial stiffness of element (i), the system vertical translatory stiffness is given by $K_3 = \sum_i kz_i$ (1)

and, the axial center of rigidity is located by

$$\tilde{a} = (\sum_i kz_i \cdot a_i)/K_3 \quad ; \quad \tilde{b} = (\sum_i kz_i \cdot b_i)/K_3 \tag{2}$$

The relationship between the effective bending moments applied at the axial center of rigidity in global directions and the corresponding rocking displacement coordinates is

$$\underset{\sim}{M} = [R]\cdot\underset{\sim}{\theta} \quad \text{or} \quad \begin{Bmatrix} M_x \\ M_y \end{Bmatrix} = \begin{bmatrix} R_{xx} & -R_{xy} \\ -R_{xy} & R_{yy} \end{bmatrix} \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} \tag{3}$$

[R] is the instantaneous rocking stiffness tensor with components given by

$$\begin{aligned} Rxx &= \sum_i kx_i \cdot y_i^2 = \sum_i (kz_i \cdot b_i^2) - K_3 \cdot (\tilde{b})^2 \\ Ryy &= \sum_i kz_i \cdot x_i^2 = \sum_i (kz_i \cdot a_i^2) - K_3 \cdot (\tilde{a})^2 \\ Rxy &= \sum_i kz_i \cdot x_i \cdot y_i = \sum_i (kz_i \cdot a_i \cdot b_i) - K_3 \cdot (\tilde{a} \cdot \tilde{b}) \end{aligned} \tag{4}$$

Let $(\tilde{x},\tilde{y})$ be the axial principal directions of rigidity (Fig. 1), then, by tensor transformation, the directions of $\tilde{x}$ and $\tilde{y}$, are determined by

$$\tilde{\beta} = 1/2\cdot\tan^{-1}\left[-\frac{2R_{xy}}{R_{xx} - R_{yy}}\right] \quad (\tilde{\beta}) \quad \text{and} \quad (\tilde{\beta} + \frac{\pi}{2}) \tag{5}$$

and the principal components of the rocking stiffness tensor are obtained as

$$\begin{aligned} K_{tx} &= \left(\frac{R_{xx} + R_{yy}}{2}\right) + \left(\frac{R_{xx} - R_{yy}}{2}\right)\cdot\cos 2\tilde{\beta} - R_{xy}\cdot\sin 2\tilde{\beta} \\ K_{ty} &= \left(\frac{R_{xx} + R_{yy}}{2}\right) - \left(\frac{R_{xx} - R_{yy}}{2}\right)\cdot\cos 2\tilde{\beta} + R_{xy}\cdot\sin 2\tilde{\beta} \\ R_{\tilde{x}\tilde{y}} &= 0 \end{aligned} \tag{6}$$

Thus, Fig. 1 represents the resulting analytical model, with stiffness matrix

$$[K] = \left[\begin{array}{c|c} [K_{LT}] & \\ \hline & [K_{AX}] \end{array}\right] \text{ where, } [K_{LT}] = \begin{bmatrix} K_1 & & \\ & K_2 & \\ & & K_{tz} \end{bmatrix} ; \quad [K_{AX}] = \begin{bmatrix} K_3 & & \\ & K_{tx} & \\ & & K_{ty} \end{bmatrix} \tag{7}$$

The element axial forces can be recovered at the end of each time step during dynamic response by means of the expression

$$Pz_i = kz_i \cdot \left(\frac{M_{\tilde{x}} \cdot \tilde{y}_i}{K_{tx}} - \frac{M_{\tilde{y}} \cdot \tilde{x}_i}{K_{ty}} + \frac{F_Z}{K_3} \right) \tag{8}$$

$$\text{with,} \quad \begin{Bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{Bmatrix} = \begin{bmatrix} \cos\tilde{\beta} & \sin\tilde{\beta} \\ -\sin\tilde{\beta} & \cos\tilde{\beta} \end{bmatrix} \cdot \left(\begin{Bmatrix} a_i \\ b_i \end{Bmatrix} - \begin{Bmatrix} \tilde{a} \\ \tilde{b} \end{Bmatrix} \right) \tag{9}$$

$\langle FZ, M_{\tilde{x}}, M_{\tilde{y}} \rangle$ are the effective earthquake forces.

FORMULATION OF SYSTEM INERTIA

Control over element force response during earthquake action--an analysis/design consideration of utmost importance--is straightforward if the system equations of motion during any given finite time step are referred to the instantaneous principal rigidity coordinates so that, in general, the frame of reference varies during dynamic response. The resulting stiffness matrix is diagonal and the mass matrix referred to the principal inertial coordinates is transformed congruently:

$$\underline{M}^{CR} = M \cdot \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & \underset{\sim}{V}_1 \cdot \hat{n}_2 \\ & 1 & 0 & 0 & 0 & -\underset{\sim}{V}_1 \cdot \hat{n}_1 \\ & & 1 & -\underset{\sim}{V}_2 \cdot \hat{n}_5 & \underset{\sim}{V}_2 \cdot \hat{n}_4 & 0 \\ \hline & & & (\underset{\sim}{V}_2 \cdot \hat{n}_5)^2 + CS & -(\underset{\sim}{V}_2 \cdot \hat{n}_5)(\underset{\sim}{V}_2 \cdot \hat{n}_4) + S2 & 0 \\ & & & & (\underset{\sim}{V}_2 \cdot \hat{n}_4)^2 + SC & 0 \\ & SYMM & & & & |\underset{\sim}{V}_1|^2 + r_{zz}^2 \end{array}\right] \tag{10}$$

(6x6)

where,

- $\underset{\sim}{V}_1$: Position vector of the lateral-torsional center of rigidity with respect to the principal inertial system.
- $\underset{\sim}{V}_2$: Position vector of the axial-rocking center of rigidity with respect to the principal inertial system.
- $\hat{n}_1$: $(\cos\alpha, \sin\alpha)$; $\hat{n}_2$: $(-\sin\alpha, \cos\alpha)$; $\hat{n}_2 \perp \hat{n}_1$
- $\hat{n}_4$: $(\cos\beta, \sin\beta)$; $\hat{n}_5$: $(-\sin\beta, \cos\beta)$; $\hat{n}_5 \perp \hat{n}_4$
- CS : $r_{xx}^2 \cos^2\beta + r_{yy}^2 \sin^2\beta$; SC : $r_{xx}^2 \sin^2\beta + r_{yy}^2 \cos^2\beta$
- S2 : $\frac{1}{2}\left(r_{yy}^2 - r_{xx}^2\right) \cdot \sin(2\beta)$

SYSTEM EIGENPROPERTIES

The formulation of the free vibration equations of motion of each independent system leads to the determination of its eigenproperties during a small time interval. For the lateral-torsional system, for example:

$$\left(\begin{bmatrix} \Omega_x^2 & & \\ & \Omega_y^2 & \\ & & \Omega_{\theta z}^2 \end{bmatrix} - \lambda^L \begin{bmatrix} 1 & 0 & (\underset{\sim}{V}_1 \cdot \hat{n}_2)/r_{zz} \\ & 1 & -(\underset{\sim}{V}_1 \cdot \hat{n}_1)/r_{zz} \\ \text{SYMM} & & 1 + \left(\left|\underset{\sim}{V}_1\right|/r_{zz}\right)^2 \end{bmatrix} \right) \cdot \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_6 \end{Bmatrix} = \underset{\sim}{0} \tag{11}$$

where, $\Omega_i^2 = k_i/m_i$, λ^L is an eigenvalue and $\langle\phi_1\phi_2\phi_6\rangle$ is a homogenized eigenvector.

Eigenproblem (11) has the characteristic equation $\sum_{i=0}^{3} c_i(\lambda^L)^{3-i} = 0$

$$\text{where,}\quad c_0 = 1 \;;\; c_1 = -\left\{\left[1 + \left(\frac{\underset{\sim}{V}_1 \cdot \hat{n}_2}{r_{zz}}\right)^2\right]\cdot\Omega_x^2 + \left[1 + \left(\frac{\underset{\sim}{V}_1 \cdot \hat{n}_1}{r_{zz}}\right)^2\right]\cdot\Omega_y^2 + \Omega_{\theta z}^2\right\} \;;$$

$$c_2 = \Omega_x^2\Omega_y^2 \left(1 + \left(\left|\underset{\sim}{V}_1\right|/r_{zz}\right)^2\right) + \Omega_{\theta z}^2 \left(\Omega_x^2 + \Omega_y^2\right) \;;\; c_3 \overset{\circ}{=} -\Omega_x^2\Omega_y^2\Omega_{\theta z}^2 \tag{12}$$

and has normalized eigenvectors

$$\underset{\sim}{\phi}_i^L = \left\langle -\frac{1}{\left(1 - \frac{\Omega_x^2}{\lambda_i^L}\right)} \cdot \left(\frac{\underset{\sim}{V}_1 \cdot \hat{n}_2}{r_{zz}}\right) \;,\; \frac{1}{\left(1 - \frac{\Omega_y^2}{\lambda_i^L}\right)} \cdot \left(\frac{\underset{\sim}{V}_1 \cdot \hat{n}_1}{r_{zz}}\right) \;,\; 1\right\rangle \tag{13}$$

Standard Newton-Raphson procedure on the characteristic equation gives very efficiently one eigenvalue as precisely as desired. If the starting value is the Argand solution of the cubic: $\lambda_1 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3}\right)$

$$\text{with,}\quad \theta = \cos^{-1}\left(-R/\sqrt{-Q^3}\right) \;;\; R = \frac{c_1c_2}{6} - \frac{c_3}{2} - \left(\frac{c_1}{3}\right)^3 \;;\; Q = \frac{c_2}{3} - \left(\frac{c_1}{3}\right)^2 \tag{14}$$

After synthetic division, the remaining eigens are given by

$$\lambda = -\left(\frac{c_1 + \lambda_1}{2}\right) \pm \sqrt{\left(\frac{c_1 + \lambda_1}{2}\right)^2 - \left(\lambda_1^2 + c_1\lambda_1 + c_2\right)} \tag{15}$$

MATHEMATICAL FORMULATION OF NONLINEARITIES

Material nonlinearity is modelled by the incremental theory of plasticity as applied to the behavior of member end cross-sections, where yielding is assumed to be confined--spreading of plasticity is not allowed --so that the selected yield surface may strain-harden kinematically as conceived by Ziegler (Ref. 2). We plug a simple yield surface in this model:

$$\Phi_i = \left(\frac{F_x - \alpha_x}{F_{xo}}\right)_i^2 + \left(\frac{F_y - \alpha_y}{F_{yo}}\right)_i^2 + \left(\frac{F_z - \alpha_z}{F_{zo}}\right)_i = 1 \qquad (16)$$

with gradient $\underset{\sim}{N} = \langle 2Q_x, 2Q_y, Q_z \rangle$

where, $Q_x = (F_x - \alpha_x)/F_{xo}^2$; $Q_y = (F_y - \alpha_y)/F_{yo}^2$; $Q_z = 1/F_{zo}$ (17)

$\langle \alpha_x, \alpha_y, \alpha_z \rangle$ describes the current position of the yield surface center in the force-space of member i

and $\langle F_{xo}, F_{yo}, F_{zo} \rangle$ describes the acting-alone yield forces for the cross-section of member i

The structural system is assumed to behave linearly during each small time increment with the modified element stiffness properties obtained from the theory:

$$\underline{k}_i^* = \underline{k}_i - \{\underline{k}_i^T \underset{\sim}{N} \underset{\sim}{N}^T \underline{k}_i\}/\xi_i$$

where, $\xi_i = 4k_{xi}Q_x^2 (1 + \gamma_x) + 4k_{yi}Q_y^2 (1 + \gamma_y) + k_{zi}Q_z^2 (1 + \gamma_z)$

$$\gamma_j = (\text{strain-hardening coefficient } j)/k_{ji} \qquad (18)$$

and $\underline{k}_i$ is the element stiffness matrix from the previous time step.

P-Δ effects are represented in a simple form by means of a linearized geometric stiffness matrix (Ref. 3)

$$\underline{k}_{Gi} = -\left(\frac{W_i - P_{zi}}{h}\right) \cdot \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} \qquad (19)$$

where W_i is the dead load supported by the element

Then, $\underline{k}_i^{TOT} = \underline{k}_i^* + \underline{k}_{Gi}$, which is updated stepwise. (20)

The process is entirely consistent for elastic behavior. However, when yielding takes place, small off-diagonal coefficients appear in $\underline{k}_i^{TOT}$. This matrix is then diagonalized in a work equivalent sense to enforce consistency with the element analytical model.

EARTHQUAKE ANALYSIS

Modal analysis for the finite time step (Ref. 3) leads to the uncoupled differential equations of motion

$$\ddot{q}_{iL} + (2\xi_i\omega_i)_L \dot{q}_{iL} + \lambda_{iL}q_{iL} = -\mathcal{L}_{ix}^L a_{gx} - \mathcal{L}_{iy}^L a_{gy}$$

and

$$\ddot{q}_{iA} + (2\xi_i\omega_i)_A \dot{q}_{iA} + \lambda_{iA}q_{iA} = -\mathcal{L}_{iz}^A a_{gz} \qquad (21)$$

where, $\mathcal{L}_{ij}^s$ are the earthquake participation factors, a_{gj} are the input ground accelerations, damping is assumed linear viscous and orthogonal, and the eigenvectors are normalized in such a way that $\underset{\sim}{\psi}_{iL}^T \underline{K}_L^{CR} \underset{\sim}{\psi}_{jL} = \underset{\sim}{\psi}_{iA}^T \cdot \underline{K}_A^{CR} \underset{\sim}{\psi}_{jA} = M\delta_{ij}$

(22)

The exact solution to eqs. (21) at the end of Δt is easily derived if the time increment is taken such that the three components of ground acceleration vary linearly during the whole interval. The initial conditions for the normal coordinates must be obtained from the response at the end of the previous time step (Ref. 4), and transformed to the updated system of reference. The element internal forces must be recovered in incremental form.

COMPUTER IMPLEMENTATION

We developed a microcomputer capability in structured COMPILER BASIC (IBM-XT with runtime module) for the automatic execution of the algorithm being discussed. This program consists of a preprocessor, a processor, and a post-processor. Each of these components is loaded sequentially into main memory during execution by a process of chaining.

The preprocessor allows fast and interactive input of building random access datafiles, so that after each analysis, the user may change supporting element properties and proceed to a new analysis. This feature helps the analyst to study particular sensitivities in the object structure and to construct parametric graphs. The processor does transient analysis for the assigned input accelerogram set and stores the results in output datafiles created dynamically during execution. The postprocessor drives an EPSON MX-80 printer that plots the time-histories of response requested by the analyst. Options presently implemented include the displacement response of the L-T and AXIAL centers of rigidity, the displacement response of the CG of floor, the variation of CR's locations and principal directions or rigidity, the effective earthquake forces applied at the centers of rigidity, and the time-history force response in elements selected by the analyst.

Fig. 2 presents the plan view and supporting element characteristics of an application example. This L-shaped system is subjected to the first 10 secs of the El Centro (1940) accelerogram set in directions A,B and vertical respectively. The processor execution time was 30 min. for the nonlinear problem. The linear-elastic version of the program executes in 4 min. Fig. 3 shows some representative examples of the final graphs rendered by this program.

CONCLUSIONS

An engineering approach for the 3-D earthquake analysis of nonlinear single-story systems has been presented. The method models the interaction between L-T and axial responses. Although the corresponding computer program is still in experimental phase, its execution in a microcomputer indicates that the method is efficient and very economical, and looks promising for the advancement of research and engineering practice.

REFERENCES

1. Ayre, R.S., "Interconnection of translational and torsional vibrations in buildings," Bulletin of the Seismological Society of America, Vol. 28, No. 2, April 1938, pp. 89-130.
2. Padilla-Mora, R. and Schnobrich, W.S., "Nonlinear response of framed structures to 2-D earthquake motion," Civil Engineering Studies, Structural Research Series #408, University of Illinois, Urbana-Champaign, July 1974.
3. Clough, R.W. and Penzien, J., Dynamics of Structures, McGraw-Hill, 1975.
4. Bathe, K-J. and Wilson, E.L., Numerical Methods in Finite Element Analysis, Prentice-Hall, Inc., 1976.

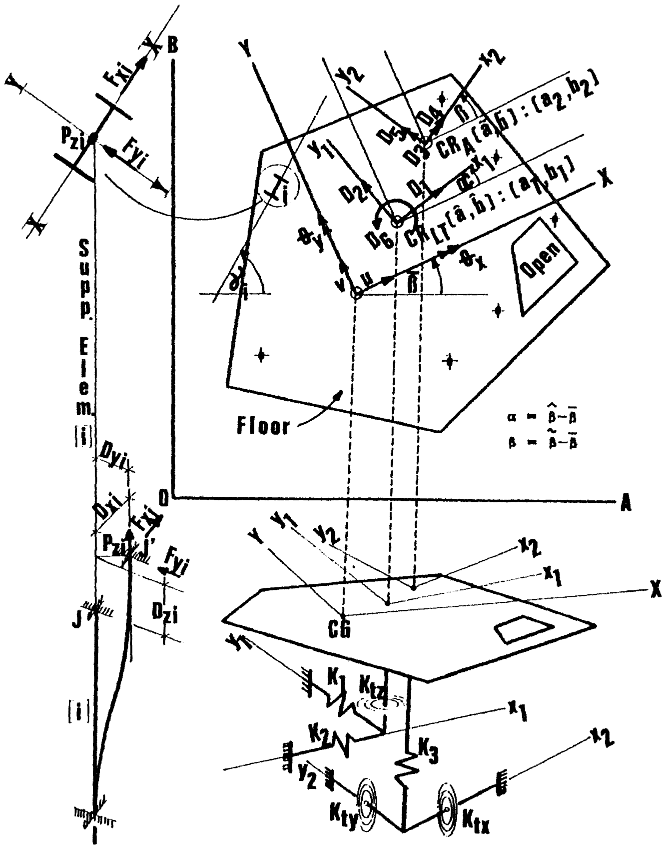


FIG. 1 SINGLE-STORY SYSTEM

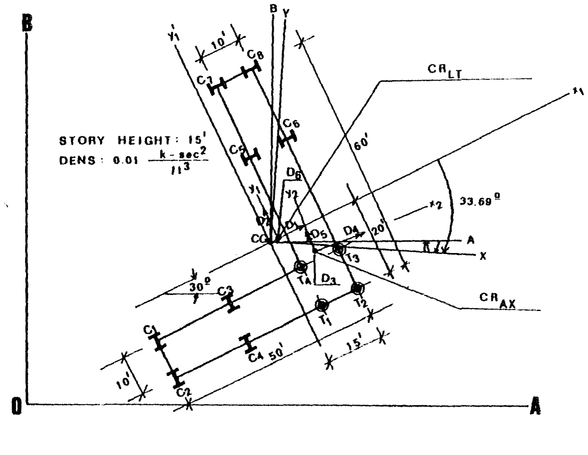
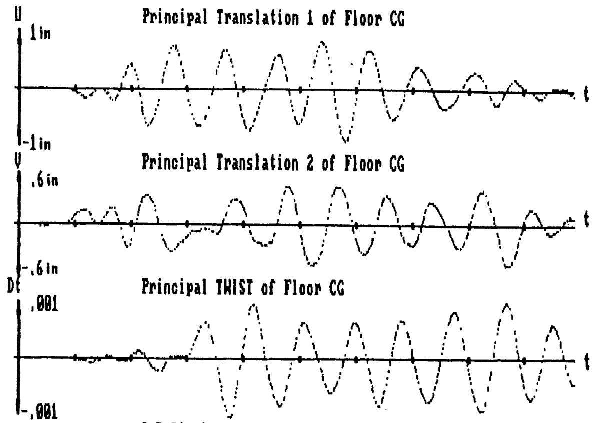


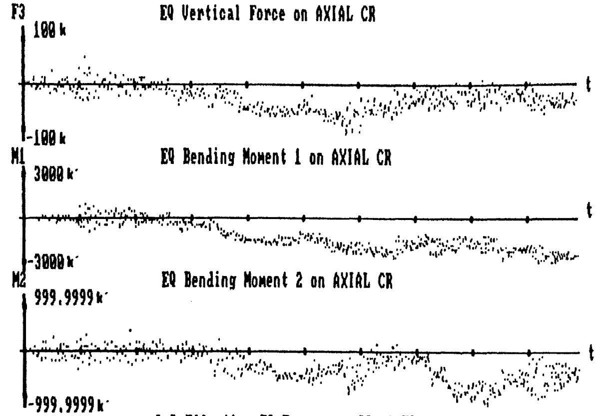
FIG. 2 APPLICATION EXAMPLE

Supporting Element	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	T_1	T_2	T_3	T_4
x (ft)	34.69	39.69	56.34	61.34	56.83	65.49	44.33	52.99	74.33	82.99	77.99	69.33
y (ft)	13.84	5.18	26.34	17.68	55.49	60.49	77.14	82.14	25.18	30.18	38.84	33.84
k_x $(\frac{k}{ft})$	4.	4.	5.	5.	80.	80.	40.	40.	100.	100.	100.	100.
k_y $(\frac{k}{ft})$	30.	30.	50.	50.	5.	5.	10.	10.	100.	100.	100.	100.
k_z $(\frac{k}{ft})$	5.	5.	10.	10.	20.	20.	10.	10.	50.	50.	50.	50.
orientation of stiffness (rad)	0.5236	0.5236	0.5236	0.5236	0.5236	0.5236	0.5236	0.5236	0.	0.	0.	0.
str-hard $coeff_x$ $(\frac{k}{ft})$	0.2	0.2	0.2	0.2	3.0	3.0	3.0	3.0	5.0	5.0	5.0	5.0
str-hard $coeff_y$ $(\frac{k}{ft})$	2.0	2.0	2.0	2.0	0.4	0.4	0.4	0.4	5.0	5.0	5.0	5.0
str-hard $coeff_z$ $(\frac{k}{ft})$	400.	400.	400.	400.	800.	800.	800.	800.	2500.	2500.	2500.	2500.
$F_{yield\ x}$ (k)	2.	2.	2.	2.	2.	2.	1.0	1.0	10.	10.	10.	10.
$F_{yield\ y}$ (k)	2.	2.	2.	2.	2.	2.	1.0	1.0	10.	10.	10.	10.
$F_{yield\ z}$ (k)	40.	40.	40.	40.	40.	40.	40.	40.	80.	80.	80.	80.

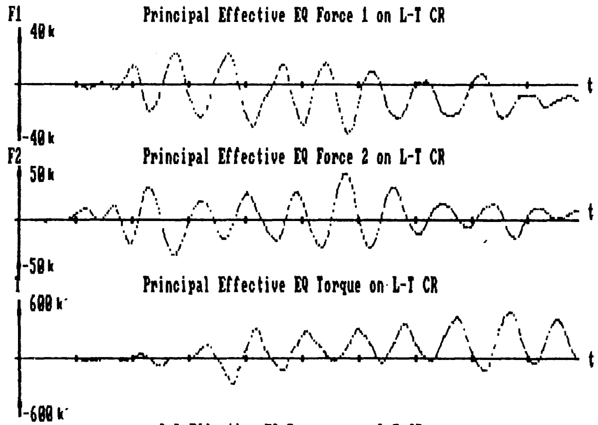
Resisting Elements in Application Example



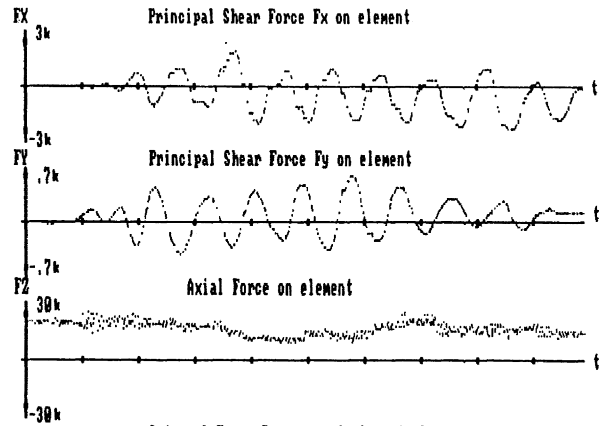
3-D Displacement response of Floor CG
Ground Motion : El Centro 1940



3-D Effective EQ Forces on CG of Floor
Ground Motion : El Centro 1940



3-D Effective EQ Response on L-T CR
Ground Motion : El Centro 1940



Internal Force Response of element 8
Ground Motion : El Centro 1940

FIG. 3