LINEAR RESPONSE OF TORSIONALLY COUPLED BUILDINGS FOR MULTICOMPONENT EARTHQUAKE EXCITATIONS

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SUMMARY

The analytical model for the single-story system introduced in the companion paper (Ref. 1) is extended to describe the linear elastic behavior of multistory building systems subjected to three-translational-component ground motions. The structural dynamic characteristics are obtained by perturbing the eigenproblem of the corresponding two-fold symmetric building system (Refs. 2, 4). The eigenproperties of an important class of buildings--those exhibiting geometric similarity among stories--are easily synthesized from the eigensolution for the associated single-story system. The algorithm for the automatic earthquake analysis is currently being implemented on a microcomputer (IBM-XT).

INTRODUCTION

We preserve in this development the same kind of notation used in the presentation of the analytical model for the single-story system in the companion paper (Ref. 1). Fig. 1 shows the projections on floor (i) of the elastic systems of neighboring stories [(i-1), (i+1)], in addition to the resisting lateral-torsional and axial systems of the story (i) itself.

The displacement response of the lateral-torsional system of the total building is described by a 3N-component vector. The first N components represent the first L-T principal rigidity coordinates of individual stories grouped together in sequence. The following N components represent the second L-T principal rigidity coordinates of individual stories grouped together in sequence. The last N components represent the torsional principal rigidity coordinates of individual stories grouped together in sequence, as well. The displacement response of the axial system of the total building is described by another 3N-component vector--vertical translations and both rocking angles of individual stories--organized in a similar manner. The equations of motion of the building are referred to those coordinates. Therefore, the principal directions of inertia are determined for individual stories and the corresponding mass properties are transformed accordingly.

Due to the nature of the basic analytical model for supporting elements, the building L-T and axial systems behave independently for linear-elastic earthquake response. Future extensions of the model into the nonlinear range will introduce strong coupling between both systems during dynamic response, as suggested previously (Ref. 1).

SYSTEM STIFFNESS MATRIX

Separating terms into the L-T and axial systems, the elastic forces on floors referred to the principal rigidity coordinates are given by

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$$\underset{\sim}{F}_s^{CR} = [K] \cdot \underset{\sim}{D}^{CR} \tag{1}$$

or $$\underset{\sim}{F}_{s_{LT}}^{CR} = [K]^{LT} \cdot \underset{\sim}{D}_{LT}^{CR} \quad \text{and} \quad \underset{\sim}{F}_{s_{AX}}^{CR} = [K]^{AX} \cdot \underset{\sim}{D}_{AX}^{CR} \tag{1a}$$

where, $$\underset{\sim}{D}_{LT}^{CR} = \begin{Bmatrix} \underset{\sim}{D}_1 \\ \underset{\sim}{D}_2 \\ \underset{\sim}{D}_6 \end{Bmatrix} \quad \text{and} \quad \underset{\sim}{D}_{AX}^{CR} = \begin{Bmatrix} \underset{\sim}{D}_3 \\ \underset{\sim}{D}_4 \\ \underset{\sim}{D}_5 \end{Bmatrix} \tag{1b}$$

with $\underset{\sim}{D}_i = \{D_i^{(j)}\}$; j being the corresponding floor.

Thus, for the L-T system, $$[K]^{LT} = \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} & \underline{K}_{16} \\ & \underline{K}_{22} & \underline{K}_{26} \\ \text{SYMM.} & & \underline{K}_{66} \end{bmatrix} \tag{2a}$$

and for the axial system, $$[K]^{AX} = \begin{bmatrix} \underline{K}_{33} & \underline{K}_{34} & \underline{K}_{35} \\ & \underline{K}_{44} & \underline{K}_{45} \\ \text{SYMM.} & & \underline{K}_{55} \end{bmatrix} \tag{2b}$$

In general, the stiffness matrices $[K_{ij}]$ are tridiagonal. Fig. 2 shows schematically the details of the construction of some terms of the building stiffness matrix from first principles. Figs. 2a-2b refer to the L-T system and Figs. 2c-2d refer to the axial system.

Notice in Fig. 2a, for example, that by imposing $D_1^{(1)} = 1$,

- only the building L-T story systems are affected,
- only the elastic systems of 2 stories are excited: (i) and (i+1),
- rigidity system (i) is affected by displacements of $CR_{LT}^{(i)}$,
- rigidity system (i+1) is affected by displacements of $CR_{LT}^{(i+1)}$,
- the equilibrium equations are formulated at $CR_{LT}^{(i-1)}$, $CR_{LT}^{(i)}$, and $CR_{LT}^{(i+1)}$. Hence, external forces and reactions need to be transformed consistently to those reference systems.

Table 1 shows the components of the resulting system stiffness matrix.

SYSTEM MASS MATRIX

The companion paper (Ref. 1) presents the mass matrix of an individual story referred to the principal rigidity coordinates. This superelement mass matrix is used to assemble the system mass matrix. The assembly process of the inertia force vector leads to the construction of the following building mass matrix:

$$M_{CR}^{LT} = \begin{bmatrix} \underline{M}_{11} & \underline{0} & \underline{M}_{16} \\ & \underline{M}_{22} & \underline{M}_{26} \\ \text{SYMM.} & & \underline{M}_{66} \end{bmatrix} \quad , \text{ for the L-T system} \tag{3a}$$

and for the axial system, $$\underline{M}_{CR}^{AX} = \begin{bmatrix} \underline{M}_{33} & \underline{M}_{34} & \underline{M}_{35} \\ & \underline{M}_{44} & \underline{M}_{45} \\ & & \underline{M}_{55} \end{bmatrix} \tag{3b}$$

with, in general $[M_{ij}] = \underline{m} \cdot \underline{\tilde{m}}_{ij}$ (3c)

where, $\underline{m} = [m_i]$

and
$$\underline{\tilde{m}}_{11} = \underline{\tilde{m}}_{22} = \underline{\tilde{m}}_{33} = \underline{I}_{(NxN)} \quad ; \quad \tilde{m}_{66} = [(r_{zz}^2 + |\underset{\sim}{V}_1|^2)^{(i)}]$$

$$\underline{\tilde{m}}_{16} = [(\underset{\sim}{V}_1 \cdot \hat{n}_2)^{(i)}] \quad ; \quad \underline{\tilde{m}}_{26} = [(-\underset{\sim}{V}_1 \cdot \hat{n}_1)]^{(i)}$$

$$\underline{\tilde{m}}_{34} = [(-\underset{\sim}{V}_2 \cdot \hat{n}_5)^{(i)}] \quad ; \quad \underline{\tilde{m}}_{35} = [(\underset{\sim}{V}_2 \cdot \hat{n}_4)]^{(i)}$$

$$\underline{\tilde{m}}_{44} = [(\underset{\sim}{V}_2 \cdot \hat{n}_5)^2 + cs]^{(i)}] \quad ; \quad \underline{\tilde{m}}_{55} = [(\underset{\sim}{V}_2 \cdot \hat{n}_4)^2 + sc]^{(i)}]$$

$$\underline{\tilde{m}}_{45} = [-(\underset{\sim}{V}_2 \cdot \hat{n}_4)(\underset{\sim}{V}_2 \cdot \hat{n}_5) + s2]^{(i)}] \tag{3d}$$

THE EIGENPROBLEM

The system eigenproblem is broken down into its homogenized and normalized components:

For the lateral-torsional system, $\underline{\tilde{K}}_{CR}^{LT} \underset{\sim}{\phi}_L = \lambda_L \underline{\tilde{M}}_{CR}^{LT} \underset{\sim}{\phi}_L$ (4)

where, $\underline{\tilde{K}}_{CR}^{LT} = \underline{M}^{-1} \underline{P}_L \underline{K}_{CR}^{LT} \underline{P}_L \quad ; \quad \underline{\tilde{M}}_{CR}^{LT} = [\tilde{m}_{ij}]^{LT}$ (4a)

$$\underset{\sim}{\psi}_L = \underline{P}_L \cdot \underset{\sim}{\phi}_L \tag{4b}$$

and
$$\underline{M}^{-1} = \begin{bmatrix} \underline{m}^{-1} & & \\ & \underline{m}^{-1} & \\ & & \underline{m}^{-1} \end{bmatrix} \quad ; \quad \underline{P}_L = \begin{bmatrix} \underline{I}_{(NxN)} & & \\ & \underline{I}_{(NxN)} & \\ & & [r_{zz}]^{-1} \end{bmatrix} \tag{4c}$$

Equation (4) may be written as $\underline{\tilde{R}}_L \underset{\sim}{\phi}_L = \lambda_L \cdot \underset{\sim}{\phi}_L$, with $\underline{\tilde{R}}_L = (\underline{\tilde{M}}_{CR}^{LT})^{-1} \cdot \underline{\tilde{K}}_{CR}^{LT}$ (5)

Now, for the associated two-fold symmetric system (with superscript *):

$$\underset{\sim}{K}^*_{12} = \underset{\sim}{K}^*_{16} = \underset{\sim}{K}^*_{26} = [0]_{(N\times N)} \quad ; \quad \tilde{\underset{\sim}{m}}^*_{16} = \tilde{\underset{\sim}{m}}^*_{26} = [0]_{(N\times N)} \tag{6}$$

$$\therefore \quad \tilde{\underset{\sim}{R}}^*_L = \begin{bmatrix} \underset{\sim}{m}^{-1} \cdot \underset{\sim}{K}^*_{11} & & \\ & \underset{\sim}{m}^{-1} \cdot \underset{\sim}{K}^*_{22} & \\ & & \underset{\sim}{m}^{-1} \cdot \underset{\sim}{K}^*_{66} \cdot \underset{\sim}{r}^{-2}_{zz} \end{bmatrix} \tag{7}$$

and the eigenproblem becomes $\tilde{\underset{\sim}{R}}^*_L \, \underset{\sim}{\phi}^*_L = \lambda^*_L \, \underset{\sim}{\phi}^*_L$ with $\tilde{\underset{\sim}{M}}^{CR}_{LT} = \underset{\sim}{I}$ and $\tilde{\underset{\sim}{R}}^*_L = \tilde{\underset{\sim}{K}}^*_L$. (8)

The idea is to solve approximately the eigenproblem (5) by using the standard eigensolution yielded by (8)--classical shear-building eigenproblem.

The error matrix is easily computed (Ref. 4)

$$\underset{\sim}{E}_L = \tilde{\underset{\sim}{R}}_L - \tilde{\underset{\sim}{R}}^*_L = [E^{ij}_L \,_{(N\times N)}] \tag{9}$$

Observing the form of the eigenvectors of (5) and applying perturbation theory (Refs. 2 and 4), the corresponding eigens can be expressed as

$$\lambda^K_L = \lambda^{*K}_L + \alpha_{KK} + \sum_{i\neq K} \frac{\alpha_{iK}\alpha_{Ki}}{\left(\lambda^{*K}_L - \lambda^{*i}_L\right)} \qquad \text{(2nd order)} \tag{10}$$

$$\underset{\sim}{\phi}_{LK} = \underset{\sim}{\phi}^*_{LK} + \sum_{i\neq K} \frac{\alpha_{iK}}{\left(\lambda^{*K}_L - \lambda^{*i}_L\right)} \cdot \underset{\sim}{\phi}^*_{Li} \qquad \text{(1st order)} \tag{11}$$

where, $\alpha_{ik} = {}^p\underset{\sim}{\phi}^{*T}_{Lm} \cdot \underset{\sim}{E}_L^{pq} \cdot {}^q\underset{\sim}{\phi}^*_{Ln}$ (11a)

$p = (i + N - 1)$ int N ; $q = (K + N - 1)$ int N
$m = [(i-1) \bmod N] + 1$; $n = [(K-1) \bmod N] + 1$
int: integer division ; mod: modulo arithmetic (11b)

Equation (11) can be transformed into (Refs. 2 and 5)

$$\underset{\sim}{\phi}_{LK} \doteq \begin{Bmatrix} \eta_1 \cdot {}^1\underset{\sim}{\phi}^*_{LK} \\ \eta_2 \cdot {}^2\underset{\sim}{\phi}^*_{LK} \\ \eta_6 \cdot {}^6\underset{\sim}{\phi}^*_{LK} \end{Bmatrix} \; ; \quad K = 1\ldots N \text{ and } 3N \text{ sets of constants } \langle \eta_1, \eta_2, \eta_6 \rangle \tag{11c}$$

and, for the particular class of buildings exhibiting geometric similarity among stories

$$\underset{\sim}{\phi}_{LK} \doteq \begin{Bmatrix} n_1 \cdot {}^1\underset{\sim}{\phi}^*_{LK} \\ n_2 \cdot {}^1\underset{\sim}{\phi}^*_{LK} \\ n_6 \cdot {}^1\underset{\sim}{\phi}^*_{LK} \end{Bmatrix} \; ; \quad \begin{array}{l} K = 1\ldots N \text{ and } N \text{ sets of constants } \langle n_1, n_2, n_6 \rangle \\ \text{(similarly for } {}^2\underset{\sim}{\phi}^*_{LK} \text{ and } {}^6\underset{\sim}{\phi}^*_{LK}) \end{array} \tag{11d}$$

Enforcement of form (11d) into the original eigenproblem (5) leads to

$$\left(\begin{bmatrix} 1+\delta_{KK}^{11} & \delta_{KK}^{12} & \delta_{KK}^{16} \\ \delta_{KK}^{21} & 1+\delta_{KK}^{22} & \delta_{KK}^{26} \\ \delta_{KK}^{61} & \delta_{KK}^{62} & 1+\delta_{KK}^{66} \end{bmatrix}_{(1)} - \frac{\lambda_{LK}}{\frac{1}{\lambda^*_{LK}}} \underline{I} \right) \cdot \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \underset{\sim}{0} \tag{12}$$

$$\text{with } \delta_{KK}^{ij} = {}^1\underset{\sim}{\phi}^{*T}_{LK} \cdot \underline{E}_L^{ij} \cdot {}^1\underset{\sim}{\phi}^*_{LK} \tag{13}$$

This is the eigenproblem of the associated single-story building (cf. Refs. 1 and 2). Solution of equation (12) produces N eigenvalues and N eigenvectors {n}, complemented by equation (11d).

The same procedure applies for ${}^2\underset{\sim}{\phi}^*_{LK}$ and ${}^6\underset{\sim}{\phi}^*_{LK}$ to generate 3N eigens corresponding to the lateral-torsional system. Of course, in most cases only a few first modes contribute significantly to the response.

A similar process is then applied to the axial system to complete the building eigensolution (Ref. 5).

EARTHQUAKE ANALYSIS

The equations of motion of the building system during earthquake response are given by

$$\underline{M}_L \ddot{\underset{\sim}{D}}{}^{CR}_{LT} + \underline{C}_L^{CR} \dot{\underset{\sim}{D}}{}^{CR}_{LT} + \underline{K}_L^{CR} \underset{\sim}{D}{}^{CR}_{LT} = \underset{\sim}{P}{}^{CR}_{eff_L}(t) \tag{14a}$$

$$\underline{M}_A^{CR} \ddot{\underset{\sim}{D}}{}^{CR}_{AX} + \underline{C}_A^{CR} \dot{\underset{\sim}{D}}{}^{CR}_{AX} + \underline{K}_A^{CR} \underset{\sim}{D}{}^{CR}_{AX} = \underset{\sim}{P}{}^{CR}_{eff_A}(t) \tag{14b}$$

If the eigenvectors are normalized in such a way that

$$\underset{\sim}{\psi}_L^T \underline{K}_L^{CR} \underset{\sim}{\psi}_L = \underset{\sim}{\psi}_A^T \underline{K}_A^{CR} \underset{\sim}{\psi}_A = \begin{bmatrix} \underline{m} & & \\ & \underline{m} & \\ & & \underline{m} \end{bmatrix} \tag{15}$$

then, standard normal coordinate transformation (Ref. 3) leads to the uncoupled equations

$$\ddot{\underset{\sim}{q}}_L + [2\xi_L \omega_L] \dot{\underset{\sim}{q}}_L + \underline{\lambda}_L \underset{\sim}{q}_L = \underline{M}^{-1} \underline{\lambda}_L \underline{\psi}_L^T \underset{\sim}{P}{}^{CR}_{eff_L}(t) \tag{16a}$$

$$\ddot{\underset{\sim}{q}}_A + [2\xi_A \omega_A] \dot{\underset{\sim}{q}}_A + \underline{\lambda}_A \underset{\sim}{q}_A = \underline{M}^{-1} \underline{\lambda}_A \underline{\psi}_A^T \underset{\sim}{P}{}^{CR}_{eff_A}(t) \tag{16b}$$

To construct the vector of the effective earthquake forces, we use the results for the single-story building model (Ref. 1) in conjunction with the standard assembly process (Ref. 3):

$$\underset{\sim}{P}^{CR}_{eff_L} = -\left(\underline{M} \cdot \begin{Bmatrix} \underset{\sim}{c\alpha} \\ -\underset{\sim}{s\alpha} \\ \underset{\sim}{b}_1 \end{Bmatrix} a_{gx} + \underline{M} \begin{Bmatrix} \underset{\sim}{s\alpha} \\ \underset{\sim}{c\alpha} \\ -\underset{\sim}{a}_1 \end{Bmatrix} a_{gy} \right) ; \quad \underset{\sim}{P}^{CR}_{eff_A} = -\underline{M} \cdot \begin{Bmatrix} \underset{\sim}{1} \\ \underset{\sim}{\underline{m}}_{34} \cdot \underset{\sim}{1} \\ \underline{m}_{35} \cdot \underset{\sim}{1} \end{Bmatrix} a_{gz} \tag{17}$$

Thus, the typical uncoupled equations of motion result in

$$^{p}\ddot{q}_{Ln} + {}^{p}(2\xi\omega)_{Ln} \cdot {}^{p}\dot{q}_{Ln} + {}^{p}\lambda_{Ln} \cdot {}^{p}q_{Ln} = \frac{1}{m_n} \cdot {}^{p}\lambda_{Ln} \cdot [{}^{p}\underset{\sim}{\psi}^T_{Ln} \cdot \underset{\sim}{P}^{CR}_{eff_L}] \tag{18a}$$

$$^{p}\ddot{q}_{An} + {}^{p}(2\xi\omega)_{An} \cdot {}^{p}\dot{q}_{An} + {}^{p}\lambda_{An} \cdot {}^{p}q_{An} = \frac{1}{m_n} \cdot {}^{p}\lambda_{An} \cdot [{}^{p}\underset{\sim}{\psi}^T_{An} \cdot \underset{\sim}{P}^{CR}_{eff_A}] \tag{18b}$$

where, $K = 1 \ldots (2N)$; $(5N+1) \ldots (6N)$ for equations (18a)
and $K = (2N+1) \ldots (5N)$ for equations (18b)
$p = (K+N-1) \text{ int } N$; $n = [(K-1) \bmod N] + 1$.

which are solved by the same numerical procedure outlined in the companion paper (Ref. 1) for the single-story system. Once the normal coordinates are obtained, the determination of the time-histories of displacement response, of the effective earthquake forces, and, of element force response follows straightforward by a process of backsubstitution (Ref. 3).

CONCLUSIONS

The extension of the ideas presented in the companion paper (Ref. 1) for the earthquake analysis of single-story systems has been developed for multistory building systems behaving in the linear-elastic range. Judging from the efficiency of the resulting algorithm for single-story systems, great economy is expected in the resulting capability for multistory buildings--in particular, for structures exhibiting storywise geometric similarity.

The corresponding application program is presently being implemented on a microcomputer (Compiler Basic). In general, the use of a fixed-disk becomes necessary for the dynamic construction of the random-access time-history datafiles corresponding to the several structural parameters being investigated.

REFERENCES

1. Gergely, P., and de Béjar, L., "Nonlinear Response of Torsionally Coupled Buildings for Multicomponent Earthquake Excitations," VIII World Conference on Earthquake Engineering, San Francisco, Ca., 1984. (Companion paper #654 in this conference.)
2. Kan, C.L. and Chopra, A.K., "Elastic earthquake analysis of torsionally coupled multistoried buildings," Earthquake Engineering and Structural Dynamics, Vol. 5, 1977, pp. 395-412.
3. Clough, R.W. and Penzien, J., Dynamics of Structures, McGraw-Hill, 1975.
4. Wilkinson, J.H., The algebraic eigenproblem, Clarendon Press, Oxford, 1965, pp. 62-71.
5. de Béjar, L., "Dynamic Response of Torsionally Coupled Buildings for Multicomponent Earthquake Excitations," Ph.D. dissertation, Department of Structural Engineering, Cornell University, 1984 (in progress).

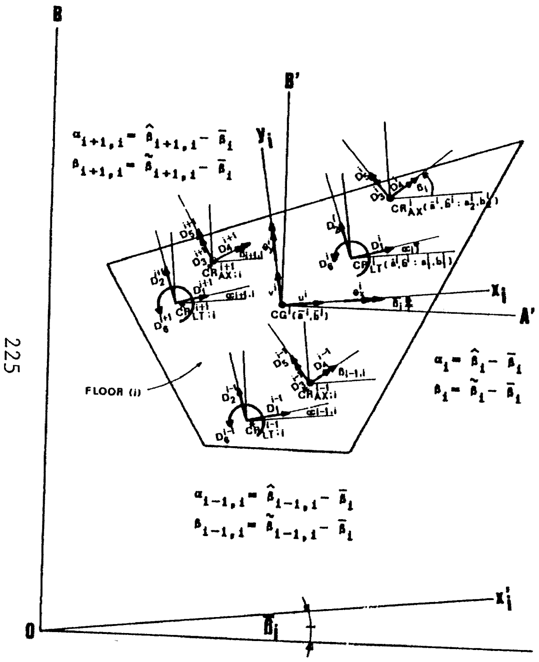

FIG. I. PROJECTIONS OF ADJACENT-FLOOR ELASTIC CENTERS ON FLOOR (i)

TABLE I

<u>Lateral-Torsional System</u>: Column (i) of $[K_{mn}]^{LT}$

Row	$[K_{11}]$ (Fig. 2a)	$[K_{zz}]$ (Fig. 2b)	$[K_{66}]$ (Fig. 2c)
i-2	$\vdots$ 0	$\vdots$ 0	$\vdots$ 0
i-1	$-K_x^{(i)} \cdot c\alpha_{i-1}^+$	$-K_y^{(i)} \cdot c\alpha_{i-1}^+$	$-K_{\theta z}^{(i)}$
i	$K_x^{(i)} + K_x^{(i+1)} c^2\alpha_i^+ + K_y^{(i+1)} s^2\alpha_i^+$	$K_y^{(i)} + K_y^{(i+1)} c^2\alpha_i^+ + K_x^{(i+1)} s^2\alpha_i^+$	$K_{\theta z}^{(i)} + K_{\theta z}^{(i+1)} + K_x^{(i+1)} \cdot DAMB^{2(i)+} + K_y^{(i+1)} \cdot DAPB^{2(i)+}$
i+1	$-K_x^{(i+1)} \cdot c\alpha_i^+$	$-K_y^{(i+1)} \cdot c\alpha_i^+$	$-K_{\theta z}^{(i+1)}$
i+2	0 $\vdots$	0 $\vdots$	0 $\vdots$

Row	$[K_{12}]$ (Fig 2b)	$[K_{16}]$ (Fig. 2c)	$[K_{26}]$ (Fig. 2c)
i-2	$\vdots$ 0	$\vdots$ 0	$\vdots$ 0
i-1	$K_y^{(i)} \cdot s\alpha_{i-1}^+$	0	0
i	$\frac{1}{2} \cdot [K_x^{(i+1)} - K_y^{(i+1)}] \cdot s(2\alpha_i^+)$	$K_x^{(i+1)} \cdot DAMB^{(i)+} \cdot c\alpha_i^+ - K_y^{(i+1)} \cdot DAPB^{(i)+} \cdot s\alpha_i^+$	$K_x^{(i+1)} \cdot DAMB^{(i)+} \cdot s\alpha_i^+ + K_y^{(i+1)} \cdot DAPB^{(i)+} \cdot c\alpha_i^+$
i+1	$-K_x^{(i+1)} s\alpha_i^+$	$-K_x^{(i+1)} \cdot DAMB^{(i)+}$	$-K_y^{(i+1)} \cdot DAPB^{(i)+}$
i+2	0 $\vdots$	0 $\vdots$	0 $\vdots$

where, $\alpha_i^+ = \alpha_{i+1,i} - \alpha_{i,i}$; $\alpha_{i-1}^+ = \alpha_{i,i} - \alpha_{i-1,i}$

$$DAMB^{(i)+} = [a_1^{(i+1)} - a_1^{(i)}] \cdot s\alpha_{i+1,i} - [b_1^{(i+1)} - b_1^{(i)}] \cdot c\alpha_{i+1,i}$$

$$DAPB^{(i)+} = [a_1^{(i+1)} - a_1^{(i)}] \cdot c\alpha_{i+1,i} + [b_1^{(i+1)} - b_1^{(i)}] \cdot s\alpha_{i+1,i}$$

<u>Axial System</u>: Column (i) of $[K_{mn}]^{AX}$

Row	$[K_{33}]$ (Fig 2d)	$[K_{44}]$ (Fig. 2e)	$[K_{55}]$ (Fig. 2f)
i-2	$\vdots$ 0	$\vdots$ 0	$\vdots$ 0
i-1	$-K_z^{(i)}$	$-K_{\theta x}^{(i)} \cdot c\beta_{i-1}^+$	$-K_{\theta y}^{(i)} \cdot c\beta_{i-1}^+$
i	$K_z^{(i)} + K_z^{(i+1)}$	$K_{\theta x}^{(i)} + K_{\theta x}^{(i+1)} \cdot c^2\beta_i^+ + K_{\theta y}^{(i+1)} \cdot s^2\beta_i^+ + K_z^{(i+1)} \cdot BAMB^{2(i)+}$	$K_{\theta y}^{(i)} + K_{\theta y}^{(i+1)} \cdot c^2\beta_i^+ + K_{\theta x}^{(i+1)} \cdot s^2\beta_i^+ + K_z^{(i+1)} \cdot BAPB^{2(i)+}$
i+1	$-K_z^{(i+1)}$	$-K_{\theta x}^{(i+1)} \cdot c\beta_i^+$	$-K_{\theta y}^{(i+1)} \cdot c\beta_i^+$
i+2	0 $\vdots$	0 $\vdots$	0 $\vdots$

Row	$[K_{34}]$ (Fig 2e)	$[K_{35}]$ (Fig. 2f)	$[K_{45}]$ (Fig. 2f)
i-2	$\vdots$ 0	$\vdots$ 0	$\vdots$ 0
i-1	0	0	$+K_{\theta y}^{(i)} \cdot s\beta_{i-1}^+$
i	$-K_z^{(i+1)} \cdot BAMB^{(i)+}$	$-K_z^{(i+1)} \cdot BAPB^{(i)+}$	$K_z^{(i+1)} \cdot BAMB^{(i)+} \cdot BAPB^{(i)+} + \frac{1}{2} \cdot [K_{\theta x}^{(i+1)} - K_{\theta y}^{(i+1)}] \cdot s(2\beta_i^+)$
i+1	$K_z^{(i+1)} \cdot BAMB^{(i)+}$	$K_z^{(i+1)} \cdot BAPB^{(i)+}$	$-K_{\theta x}^{(i+1)} \cdot s\beta_i^+$
i+2	0 $\vdots$	0 $\vdots$	0 $\vdots$

where, $\beta_i^+ = \beta_{i+1,i} - \beta_{i,i}$; $\beta_{i-1}^+ = \beta_{i,i} - \beta_{i-1,i}$

$$BAMB^{(i)+} = [a_2^{(i+1)} - a_2^{(i)}] \cdot s\beta_{i,i} - [b_2^{(i+1)} - b_2^{(i)}] \cdot c\beta_{i,i}$$

$$BAPB^{(i)+} = [a_2^{(i+1)} - a_2^{(i)}] \cdot c\beta_{i,i} + [b_2^{(i+1)} - b_2^{(i)}] \cdot s\beta_{i,i}$$

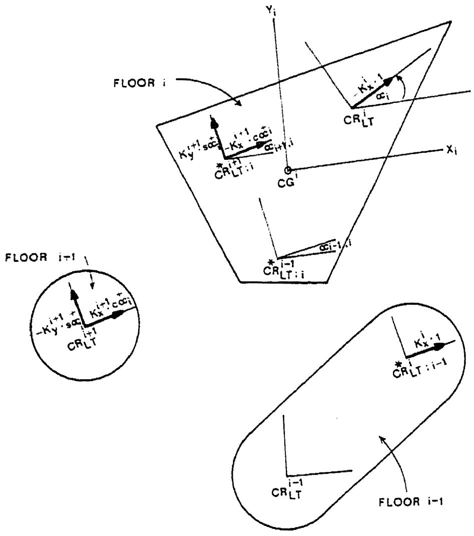

FIG. 2a. [Kj1]LT

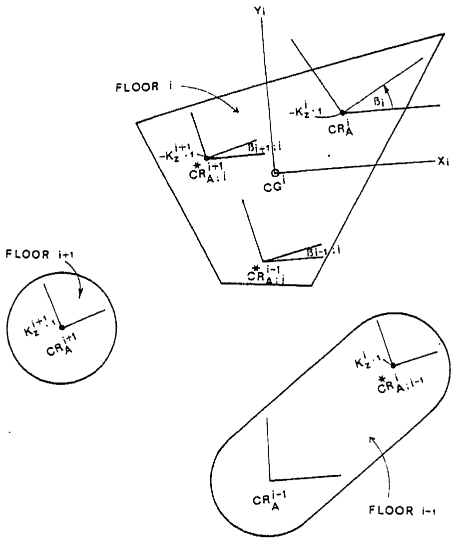

FIG. 2c. [Kj3]AX

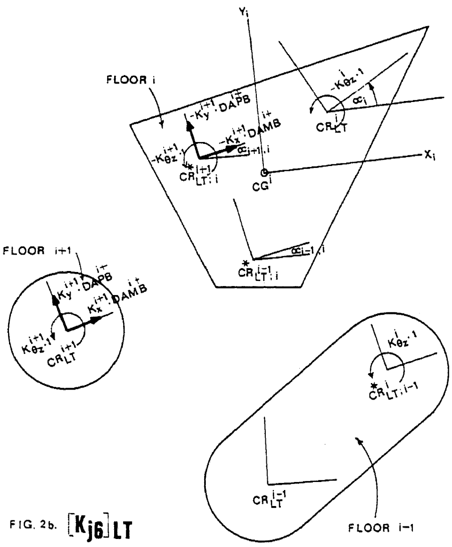

FIG. 2b. [Kj6]LT

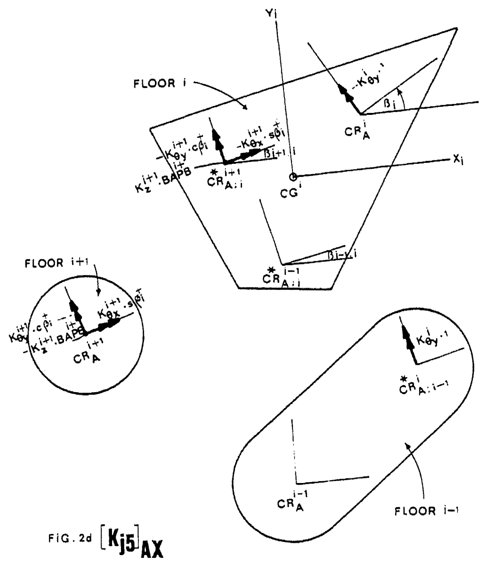

FIG. 2d [Kj5]AX

FIG. 2