COUPLED LATERAL-TORSIONAL RESPONSE OF BASE-ISOLATED STRUCTURES

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SUMMARY

A typical base-isolated structure on rubber bearings is liable to have a small eccentricity and closely spaced frequencies causing coupled lateral-torsional response. In seismic analysis, the structure is modeled as a rigid deck with lumped masses supported on axially inextensible bearings. The Green's functions for displacement response are derived for undamped and damped cases with small and large eccentricities. An interaction equation for normalized displacements is established for an idealized flat velocity spectrum. Numerical results for a specific building are carried out for comparison.

It is shown that the effect of torsional coupling on the response of base-isolated structures subjected to transient loadings is generally negligible due to the combined effects of the time lag between the maximum translational and torsional responses and the influence of damping in the bearings.

INTRODUCTION

Base isolation is an aseismic structural design strategy in which a building is uncoupled from the damaging horizontal components of an earthquake by a mechanism that attenuates the transmission of horizontal acceleration into the structure. The concept of base isolation has become a practical possibility with the recent development of multilayer elastomeric bearings. An extensive literature survey on the history of base isolation is given in Ref. 1, and the results of a series of experiments on this concept carried out at the Earthquake Engineering Research Center, University of California, Berkeley, have been published (Ref. 2-5). These results have established the effectiveness of this approach to aseismic design. While base isolation has generally been proposed for new construction (Ref. 6,7), the concept can be adapted to the rehabilitation of older buildings of architectural and historical merit. The economic feasibility of rehabilitation by base isolation has been studied for a building in downtown San Francisco (Ref. 8) from which physical parameters are taken for use in later analyses .

For an idealized base-isolated building, where bearings under columns are designed to carry exactly the vertical load and to have precisely the desired lateral stiffness, the center of mass of the superstructure will coincide with the center of rigidity of the bearings. In practice, however, this situation can rarely be achieved and there is generally an eccentricity. The dynamic response of such a structure is more complex when the natural frequencies are closely spaced.

The effects of torsion in buildings appears to have first been studied by Ayre (Ref. 9) for shear beam models. Recent studies of single and multiple story elastic systems through deterministic (Ref. 10) and probabilistic (Ref. 11) approaches have provided valuable insight into the general features of torsional coupling. It has been concluded by many studies that a strong coupling between lateral and torsional motions can occur if the corresponding frequencies are close together, even when the eccentricity between the centers of mass and rigidity is small (Ref. 9-12).

In this analysis, a base-isolated structure is idealized as a rigid deck with tributary masses lumped at column locations, Fig. 1. The rigid deck is supported on massless, axially inextensible bearings. The three degrees of freedom of the system are the horizontal displacements, u_x and u_y, at the center of mass of the system along the principle axes, x and y, of the structure and the rotation, θ, of the deck about the vertical axis. However, the rotational displacement $u_\theta = r\theta$, in which r is the radius of gyration of the deck, will be used in place of θ.

The dynamic response of this system to a horizontal ground motion along the x-axis is investigated. Closed form solutions for the coupled lateral-torsional response of the system are presented

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first. They will be followed by an interaction equation for normalized displacements of the system when an idealized flat velocity spectrum is used to characterize the ground motion.

It is shown in the analysis that coupling can be important, particularly in the estimation of the maximum displacement at the corners of the building. However, it is also shown that the coupling reduces the translational displacement at the center of mass. In addition, damping in the bearings has the effect of absorbing these coupled lateral and torsional motions. Damping in elastomeric isolation systems can be as high as 8% to 10% in the isolated modes and for these values torsional coupling is negligible for transient inputs, such as earthquake motions.

EQUATIONS OF MOTION

Let k_{xi} and k_{yi} be the translational stiffnesses of the ith bearing in the x and y directions and $k_{xi} = k_{yi} = k_i$. The total translational stiffnesses of the bearings, $K_x = K_y = \sum k_i$, are simply the sum of individual bearing stiffness in the x or y direction. The total torsional stiffness of the isolation system defined at the center of mass is given by $K_\theta = \sum k_i(y_i^2+x_i^2)$ where x_i and y_i are the distances of the ith bearing measured from the center of mass.

Three frequency parameters ω_x, ω_y, and ω_θ are defined as follows:

$$\omega_x = \left(\frac{K_x}{m}\right)^{1/2}, \quad \omega_y = \left(\frac{K_y}{m}\right)^{1/2}, \quad \omega_\theta = \left(\frac{K_\theta}{mr^2}\right)^{1/2} \tag{1}$$

in which m is the total mass. These frequencies may be interpreted as the uncoupled frequencies of the system. In general, the individual bearing stiffness k_i will be selected to ensure that $k_i \approx m_i \omega_0^2$ where m_i is the tributary mass on the ith bearing and ω_0 is the design frequency for all the bearings. If this holds exactly, all the frequencies will be equal and result in $\omega_x = \omega_y = \omega_\theta = \omega_0$.

In this analysis, a Rayleigh damping equal to 2α times the stiffness matrix is assumed and the equations of motion of the system can be written as

$$\{\ddot{u}\} + 2\alpha[K]\{\dot{u}\} + [K]\{u\} = \{\ddot{u}_g\} \tag{2}$$

where $\{u\} = \{u_x, u_\theta, u_y\}^T$, $\{\ddot{u}_g\} = \{\ddot{u}_{gx}, 0, 0\}^T$ with $\ddot{u}_{gx}$ being the ground motion along the x-axis, and

$$[K] = \begin{bmatrix} \omega_x^2 & -\frac{e_y}{r}\omega_x^2 & 0 \\ -\frac{e_y}{r}\omega_x^2 & \omega_\theta^2 & \frac{e_x}{r}\omega_y^2 \\ 0 & \frac{e_x}{r}\omega_y^2 & \omega_y^2 \end{bmatrix} = \begin{bmatrix} \omega_0^2 & -\frac{e_y}{r}\omega_0^2 & 0 \\ -\frac{e_y}{r}\omega_0^2 & \omega_0^2 & \frac{e_x}{r}\omega_0^2 \\ 0 & \frac{e_x}{r}\omega_0^2 & \omega_0^2 \end{bmatrix} \tag{3}$$

in which e_x and e_y are the static eccentricities in the x and y directions, respectively. It should be noted that $[K]$ depends only upon the dimensionless parameters e_x/r and e_y/r.

ANALYSIS PROCEDURE

The frequency equation for the system is given by $[K]\{\phi_n\} = \omega_n^2[I]\{\phi_n\}$, in which ω_n is the eigenvalue of the nth mode; $\{\phi_n\}$ is the corresponding eigenvector; and $[I]$ is the identity matrix. The eigenvalues of the system take the form

$$\omega_1 = \omega_0(1-\frac{e}{r})^{1/2}, \quad \omega_2 = \omega_0, \quad \omega_3 = \omega_0(1+\frac{e}{r})^{1/2} \tag{4}$$

where $e^2 = e_x^2 + e_y^2$, with e being the eccentricity between the centers of mass and rigidity. The value of e/r will be small for an isolated building where the bearing stiffness is matched to the mass as mentioned earlier. The frequencies of the coupled response, therefore, can be approximated by

$$\omega_1 = \omega_0(1-\Delta), \quad \omega_2 = \omega_0, \quad \omega_3 = \omega_0(1+\Delta) \tag{5}$$

where $\Delta = e/2r$ is the shift of frequencies of the coupled system from the uncoupled system. In this analysis, Eq. (5) will be used instead of Eq. (4) for frequencies of the system. To the same order of approximation, $O(\Delta)$, the orthonormal eigenvectors of the system are given by

$$[\phi_1 , \phi_2 , \phi_3] = \begin{bmatrix} \frac{e_y}{\sqrt{2}e} & \frac{e_x}{e} & -\frac{e_y}{\sqrt{2}e} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{e_x}{\sqrt{2}e} & \frac{e_y}{e} & \frac{e_x}{\sqrt{2}e} \end{bmatrix} \tag{6}$$

Damped Case

The damped displacement responses of the first order approximation, $O(\Delta)$, for the system take the form

$$u_x = \frac{-1}{\omega_0}\int_0^t e^{-\alpha\omega_0^2(t-\tau)}\ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\left\{1-\left(\frac{e_y}{e}\right)^2\left[1-\cos\omega_0\Delta(t-\tau)\right]\right\}d\tau \tag{7a}$$

$$u_\theta = \frac{1}{\omega_0}\left(\frac{e_y}{e}\right)\int_0^t e^{-\alpha\omega_0^2(t-\tau)}\ddot{u}_{gx}(\tau)\cos\omega_0(t-\tau)\sin\omega_0\Delta(t-\tau)\,d\tau \tag{7b}$$

$$u_y = \frac{-1}{\omega_0}\left(\frac{e_x e_y}{e^2}\right)\int_0^t e^{-\alpha\omega_0^2(t-\tau)}\ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\left[1-\cos\omega_0\Delta(t-\tau)\right]d\tau \tag{7c}$$

Undamped Case

The undamped displacement responses of the same order of approximation take the form

$$u_x = \frac{-1}{\omega_0}\int_0^t \ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\left\{1-\left(\frac{e_y}{e}\right)^2\left[1-\cos\omega_0\Delta(t-\tau)\right]\right\}d\tau \tag{8a}$$

$$u_\theta = \frac{1}{\omega_0}\left(\frac{e_y}{e}\right)\int_0^t \ddot{u}_{gx}(\tau)\cos\omega_0(t-\tau)\sin\omega_0\Delta(t-\tau)\,d\tau \tag{8b}$$

$$u_y = \frac{-1}{\omega_0}\left(\frac{e_x e_y}{e^2}\right)\int_0^t \ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\left[1-\cos\omega_0\Delta(t-\tau)\right]d\tau \tag{8c}$$

It is clear from Eqs. (7a) and (8a) that the response in the x direction, u_x, of the coupled system can never be greater than the corresponding response in the uncoupled system. The torsional response, u_θ, increases as e_y increases. However, for two-way torsionally coupled systems ($e_x \neq 0$ and $e_y \neq 0$) the eccentricity in the direction of the ground motion *reduces* the maximum torsional response. This is consistent with a result obtained in a recent study (Ref. 11). The response in the y-direction, u_y, vanishes when either e_x or e_y vanishes.

Green's Functions of Undamped Case

For undamped two-way torsionally coupled systems with equal eccentricities, $e_x = e_y$, the responses can be demonstrated as follows:

$$u_x = \frac{-1}{2\omega_0}\int_0^t \ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\left[1-\cos\omega_0\Delta(t-\tau)\right]d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_1(t,\tau)\,d\tau \tag{9a}$$

$$u_\theta = \frac{1}{\sqrt{2}\omega_0}\int_0^t \ddot{u}_{gx}(\tau)\cos\omega_0(t-\tau)\sin\omega_0\Delta(t-\tau)\,d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_2(t,\tau)\,d\tau \tag{9b}$$

$$u_y = \frac{-1}{2\omega_0}\int_0^t \ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\Big[1+\cos\omega_0\Delta(t-\tau)\Big]d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_3(t,\tau)\,d\tau \tag{9c}$$

in which $G_1(t,\tau), G_2(t,\tau)$, and $G_3(t,\tau)$ are the Green's functions with the slowly oscillating envelopes $(1-\cos\omega_0\Delta t)$, $\sin\omega_0\Delta t$, and $(1+\cos\omega_0\Delta t)$ for u_x, u_θ, and u_y, respectively.

The Green's functions of Eq. (9) for a particular case where $\Delta=0.01$ and $\omega_0=\pi$, which may be considered representative of a base-isolated structure, are shown in Figs. 2 and 3. These plots demonstrate the Green's functions of the undamped response of the system for one half the period of the envelope functions. The time lag between the maximum translational and torsional responses is clearly shown for the soft system with small eccentricity. The torsional coupling effects for such systems are negligible when a short duration transient input is involved. Figure 4 shows the response of a corner point located at a distance a away from the center of mass, Fig. 1. The corner point experiences a response of $U=u_x+u_\theta a/r$, for which a/r is taken as 1.5 in this example.

Figure 4 reveals the importance of damping in the response of a base-isolated structure. The maximum displacement at corners could exceed the maximum displacement at the center of mass due to the buildup of the torsional response in an undamped system. This could result in impact against adjacent structures if sufficient clearance is not provided.

Green's Functions of Damped Case

For damped two-way torsionally coupled systems with equal eccentricities, $e_x=e_y$, and a constant damping ratio, $\xi\approx\alpha\omega_0=0.05$, the displacement responses take the form

$$u_x = \frac{-1}{2\omega_0}\int_0^t e^{-\xi\omega_0(t-\tau)}\ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\Big[1-\cos\omega_0\Delta(t-\tau)\Big]d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_4(t,\tau)\,d\tau \tag{10a}$$

$$u_\theta = \frac{1}{\sqrt{2}\omega_0}\int_0^t e^{-\xi\omega_0(t-\tau)}\ddot{u}_{gx}(\tau)\cos\omega_0(t-\tau)\sin\omega_0\Delta(t-\tau)\,d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_5(t,\tau)\,d\tau \tag{10b}$$

$$u_y = \frac{-1}{2\omega_0}\int_0^t e^{-\xi\omega_0(t-\tau)}\ddot{u}_{gx}(\tau)\sin\omega_0(t-\tau)\Big[1+\cos\omega_0\Delta(t-\tau)\Big]d\tau = \int_0^t \ddot{u}_{gx}(\tau)\,G_6(t,\tau)\,d\tau \tag{10c}$$

where $G_4(t,\tau)$, $G_5(t,\tau)$, and $G_6(t,\tau)$ are the Green's functions with the slowly oscillating envelopes. The Green's function of these responses for the particular case where $\Delta=0.01$ and $\omega_0=\pi$ are shown in Figs. 5 and 6 and the corner motion, $U=u_x+u_\theta a/r$ with $a/r=1.5$, is shown in Fig. 7.

The torsional response, u_θ, Fig. 6, can not build up to significant values because of the exponentially decaying effect of the damping. The influence of the damping during the time lag between the maximum lateral and torsional responses causes the torsional coupling to become negligible for the system with small eccentricity. This is clearly demonstrated in the response of the corner point, Fig. 7, which has virtually the same response as the center of mass, Fig. 5. The results indicate that for typical base-isolated systems with small eccentricity and 5% equivalent viscous damping, the torsional coupling effects are negligible.

Large Eccentricity

An artificial eccentricity equal to 5% of the maximum plane dimension, a typical value required by various seismic codes, is considered as the large eccentricity in this analysis. Increasing the eccentricity on this order will reduce the time necessary for the torsional response to build up and, as a result, the effects of torsional coupling will increase. However, the maximum displacements at the corner point and the center of mass are not noticeably different (Ref. 13).

INTERACTION EQUATION

For the purpose of design, it is more appropriate to characterize the expected ground motion by a response spectrum. An idealized flat velocity spectrum will be used in formulating an interaction equation for normalized displacements. For a typical isolated building, fundamental period around 2 sec, design spectra will generally approximate the idealized spectrum.

An estimation of the maximum of a response quantity R may be obtained by combining the modal maxima R_1, R_2, and R_3 according to the Complete Quadratic Combination (CQC) method (Ref. 14),

$$R^2 = \sum_{n,m} \rho_{0,nm} R_n R_m \quad n,m = 1,2,3 \tag{11}$$

in which the cross-correlation coefficients are given by

$$\rho_{0,nm} = \frac{2(\xi_n \xi_m)^{1/2}[(\omega_n+\omega_m)^2(\xi_n+\xi_m)+(\omega_n^2-\omega_m^2)(\xi_n-\xi_m)]}{4(\omega_n-\omega_m)^2+(\omega_n+\omega_m)^2(\xi_n+\xi_m)^2} \tag{12}$$

where ξ_n and ξ_m are modal dampings for modes n and m, respectively. The cross-correlation terms can be important under certain conditions, in particular when the natural frequencies of the structure are closely spaced. As this is often the case for buildings on a rubber isolation system, these terms are included in this analysis.

The modal response quantities u_{xn}, $u_{\theta n}$, and u_{yn} can be expressed in their normalized forms:

$$\bar{u}_{xn} = \frac{u_{xn}}{u_{x0}} \ , \ \bar{u}_{\theta n} = \frac{u_{\theta n}}{u_{x0}} \ , \ \bar{u}_{yn} = \frac{u_{yn}}{u_{x0}} \qquad n = 1,2,3 \tag{13}$$

where $u_{x0}=S_d(\omega_0,\xi)=S_v(\omega_0,\xi)/\omega_0$ in which S_d and S_v are the spectral displacement and velocity of the uncoupled system , respectively.

For a flat velocity spectrum in the x-direction, Eq. (13) can be written as

$$\bar{u}_{xn} = \frac{\omega_0}{\omega_n}\phi_{xn}^2 \ , \quad \bar{u}_{\theta n} = \frac{\omega_0}{\omega_n}\phi_{xn}\phi_{\theta n} \ , \quad \bar{u}_{yn} = \frac{\omega_0}{\omega_n}\phi_{xn}\phi_{yn} \tag{14}$$

By substituting Eq. (14) into Eq. (11) as well as making use of the orthonormal properties of the eigenvectors, the estimated maximum responses $\bar{u}_x$, $\bar{u}_\theta$, and $\bar{u}_y$, can be shown to satisfy the following interaction equation:

$$\bar{u}_x^2 + \bar{u}_\theta^2 + \bar{u}_y^2 = 1 + \delta \simeq 1 \tag{15}$$

in which $\delta=(e_y/r)^2/[1-(e/r)^2]$. The approximations made in Eq. (15) are consistent with those in the derivation of the equations of motion. The result is similar to that obtained in a previous study on elastic forces in torsionally coupled systems (Ref. 10).

For systems with small eccentricity, it is obvious from the interaction equation, Eq. (15), that u_x of the torsionally coupled system is not greater than u_{x0}, the displacement of the torsionally uncoupled system. This is consistent with the observation made for Eqs. (7a) and (8a) of u_x.

NUMERICAL EXAMPLE

In order to illustrate these analytical results, the previously mentioned building on an isolation system is analyzed. The system has a fundamental frequency of $\omega_0=\pi$ and the equivalent viscous damping is estimated to be 5% in the isolated modes. The El Centro earthquake record of May 18, 1940 and its corresponding response spectra are used as the input motion. Both the time history method and the response spectrum method using the modal superposition technique are implemented. For the small eccentricity case, the value of Δ is 0.01 and a/r is 1.5 for the corner point.

The results of the time history method are presented in Figs. 8 to 10. It can be seen that for Δ=0.01, the motion of the corner point, Fig. 10, is virtually the same as the motion at the center of mass, Fig. 8, and the torsional response is insignificant. It should, however, be noted that the 5% damping used is a conservative value, since elastomeric bearings can have an equivalent viscous damping ratio of up to 10%.

The results of the response spectrum method, along with the peak responses of the time history analysis, are presented in Table 1. The superiority of the CQC method over the conventional SRSS method is apparent. The SRSS method underestimates the corner motion and the response in the direction of the input motion, and overestimates the out-of-plane response. Hence, it is recommended that the CQC method be used in the response spectrum analysis for base-isolated buildings which are typically soft systems with small eccentricity and closely spaced frequencies.

CONCLUSIONS

As a result of this study the following conclusions can be drawn for a horizontal ground motion input in the x direction. They are confirmed by some general results obtained in previous studies using both deterministic and probabilistic approaches (Ref. 10,11).

1) Coupling induces the torsional response and reduces the translational response at the center of mass of the structure..

2) For two-way torsionally coupled systems where $e_x \neq 0$ and $e_y \neq 0$, the torsional response, u_θ, depends upon e_y/e, and the eccentricity in the direction of the ground motion, e_x, *reduces* the peak torsional response.

3) For one-way torsionally coupled systems where $e_x=0$ or $e_y=0$, the displacement response perpendicular to the direction of the ground motion, u_y, vanishes.

4) For a base-isolated structure with small eccentricity, the torsional coupling effect on the displacement response to transient loadings is negligible, due to both the time lag between the maximum lateral and torsional responses in the soft system and the influence of damping in the isolation system.

5) The displacements in a torsionally coupled system can be related to the displacements of the corresponding uncoupled system through an interaction equation when a flat velocity spectrum is used to characterize the ground motion. For the typical period range of isolated structures, *i.e.* 2.0 sec., many spectra, *e.g.* ATC-3-06, have this characteristic.

It is essential to obtain a reliable estimate of the maximum displacement response at the corner points of a base-isolated building. If sufficient clearance is not provided, impact against the adjacent structures may result. Because an isolated structure will inevitably have closely spaced frequencies, the CQC method should be used in the response spectrum analysis.

ACKNOWLEDGEMENTS

Partial support provided by the Malaysian Rubber Research and Development Board, Hertford, U.K. for this research is gratefully acknowledged. The authors would like to thank the Chief Civil/Structural engineer, F. E. Meyer, and Drs. F. J. W. Hsiu and K. M. S. Mark of NFO/Bechtel for their review of the paper. The first author is grateful to G. H. Borschel for her contribution. Excellent graphic work done by K. L. Dolan is acknowledged.

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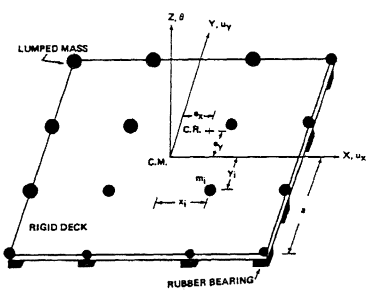

Fig. 1. Simplified 3-DOF System

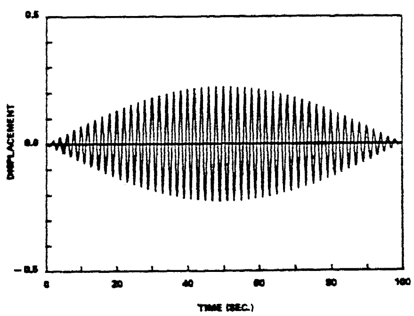

Fig. 3. Green's Function of u_θ (Undamped)

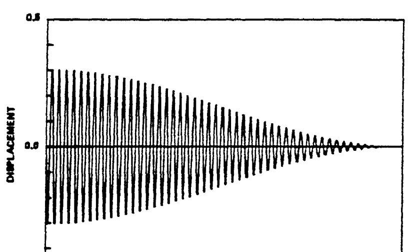

Fig. 2. Green's Function of u_x (Undamped)

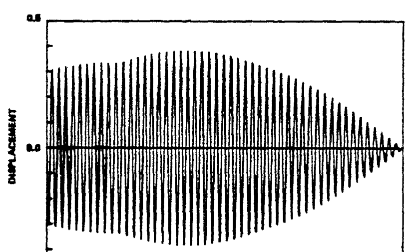

Fig. 4. Green's Function of U (Undamped)

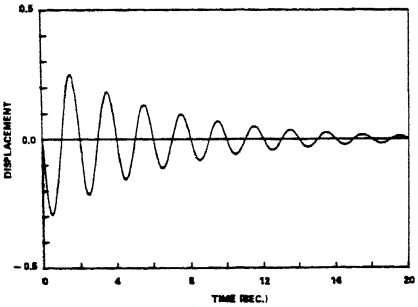

Fig. 5. Green's Function of u_x (Damped, ξ=0.05)

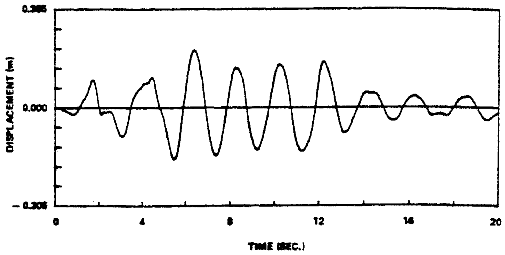

Fig. 8. Displacement Response, u_x, to the El Centro Earthquake, 1940

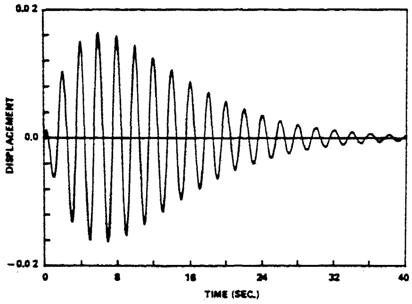

Fig. 6. Green's Function of u_θ (Damped, ξ=0.05)

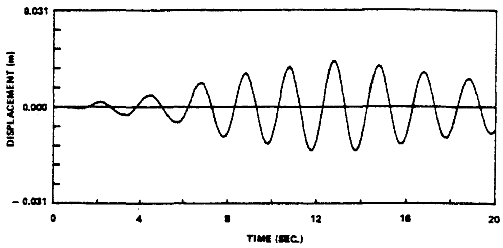

Fig. 9. Displacement Response, u_θ, to the El Centro Earthquake, 1940

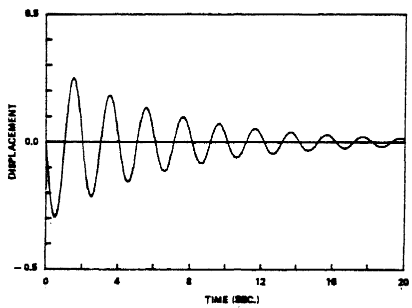

Fig. 7. Green's Function of U (Damped, ξ=0.05)

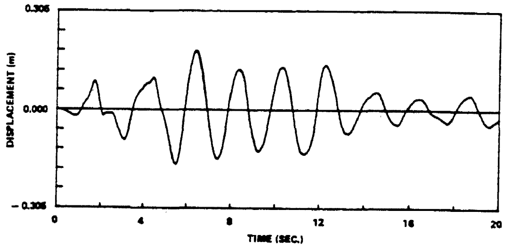

Fig. 10. Displacement Response, U, to the El Centro Earthquake, 1940

Table 1
Results of Time History and Response Spectrum Analysis; Unit: cm (in)

Small Eccentricity ($\Delta = 0.01$)			
Response	Time History	CQC	SRSS
u_x	17.78 (7.00)	17.58 (6.92)	10.82 (4.26)
u_θ	1.52 (0.60)	1.78 (0.70)	8.83 (3.48)
u_y	0.15 (0.06)	0.20 (0.08)	10.82 (4.26)
U	18.29 (7.20)	18.26 (7.19)	17.37 (6.84)