INELASTIC RESPONSE OF ECCENTRIC BUILDINGS SUBJECTED TO BI-DIRECTIONAL GROUND MOTIONS

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SUMMARY

This paper presents an analysis of the inelastic response of torsionally unbalanced structures subjected to bi-directional ground motion excitations. The resisting elements are assumed to be of elastoplastic hysteretic type. The horizontal components of 1940 El Centro earthquake records are used as input. The response parameters of interest are the ductility demands on the resisting elements and the edge displacements of the building. By relating these responses to those obtained under uni-directional excitation, the adequacy of the simplification allowed in most building codes of considering ground motion effect one direction at a time is evaluated.

INTRODUCTION

Most building codes allow during the design process to consider the effect of horizontal ground motions to act along the axes of a building one direction at a time, even some members such as corner columns will be subjected to loadings from both components of horizontal ground motions acting simultaneously. As a result, most inelastic response studies have been on responses of symmetrical structures subjected to uni-directional excitation. In this paper, the effect of bi-directional excitation on the inelastic response of eccentric structures is examined. There are two main differences in studying the inelastic responses of eccentric structure under bi-directional excitation as compared to the conventional inelastic response of symmetric structures. First, there is the effect of coupling between the motions in the two lateral directions through torsional responses. There is only a limited number of inelastic torsional response studies on eccentric structures subjected to earthquake excitation (Refs. 1,2,3). All these studies have been based on unidirectional excitation on the system under study. Second, since the resisting elements will be subjected to strong excitations from two directions simultaneously, many of them will yield simultaneously in these directions. Therefore, the interaction effect of yielding in two orthogonal directions needs to be considered. Although the interaction effect of yielding for elasto-plastic material has been well-established (Ref. 4), its inclusion in dynamic response studies is limited. Nigam (Ref. 5) presented results of a single member symmetric structure subjected to bi-directional dynamic excitation. Kan and Chopra (Ref. 6) included such effect in their study on inelastic torsional responses. However, since only uni-directional excitation was used, the extent of yielding in the direction perpendicular to the excitation direction was likely to be small, and the full impact of the interaction effect of simultaneous yielding may not be apparent from their studies.

In this study, a single mass possessing two translational and one rotational degree of freedom system is used as the structural model. The resisting elements are assumed to have an elasto-plastic hysteretic force-displacement relationship. The scaled horizontal components of the 1940 El Centro earthquake records are used as input and the response is computed numerically. The response

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parameters of interest are the ductility demands on the resisting elements and the edge displacement of the structure when it is excited into the inelastic range. By varying the location of the resisting elements relative to the center of mass, three structures are studied corresponding to a symmetric configuration, a monosymmetric configuration with small eccentricity and a monosymmetric configuration with large eccentricity. Yielding interaction effect for each resisting element is taken into account in the computation. By relating the response quantities thus obtained to the corresponding quantities computed either neglecting the effect of eccentricity (symmetrical structure subjected to bi-directional excitation) or neglecting the effect of yield interaction (eccentric structures subjected to uni-directional excitation), the effect of eccentricity and interaction effect of yielding can be examined. The main purpose of this study is to determine the adequacy of the building code simplification allowing considerations of ground motion effect acting one direction at a time on the structure in the estimation of design loads on resisting elements.

FORMULATION OF PROBLEM

The system considered consists of a rigid rectangular deck of uniform thickness, size B by D, supported by four circular section columns on rigid footings located at the extremities of a rectangle having the same aspect ratio as the rigid deck, as shown in Fig. (1). Three structural configurations are studied, corresponding to a structure with two axes of symmetry ($e = 0$), monosymmetric buildings of small eccentricity ($e = 0.03D$) and large eccentricity ($e = 0.2D$). The mass of the deck and the column stiffnesses are adjusted such that the lateral periods T_x and T_y for the symmetrical structure are 0.5 sec. The distance between columns are adjusted so that the uncoupled torsional period is also 0.5 sec.

The equations of motion of this dynamical system subjected to horizontal ground accelerations can be written as

$$[M]\,\underline{\ddot{u}} + \underline{F} = -[M]\,\underline{\ddot{u}}_g \tag{1}$$

where

$$\underline{u}^T = (u_x, ru_\theta, u_y)$$

$$\underline{\ddot{u}}_g{}^T = (\ddot{u}_{gx}, 0, \ddot{u}_{gy})$$

u_x, u_θ, u_y are two translational and one rotational degrees of freedom at the center of mass, and $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ represents the two horizontal components of ground motions experienced by the structure. In this study, $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ are identified with the scaled E-W and N-S components of the 1940 El Centro earthquake records respectively. Rotational ground motions are not considered. $[M]$ is the mass matrix and $\underline{F}$ is the restoring force vector. For computational convenience, the responses are calculated based on the incremental form of equation (1), namely

$$[M]\,\underline{\delta\ddot{u}} + [K_t]\,\underline{\delta u} = -[M]\,\underline{\delta\ddot{u}_g} \tag{2}$$

where $\delta()$ represents the incremental quantities of interest. $[K_t]$ is the instantaneous global stiffness matrix of the structure and can be obtained from the individual column stiffness matrix $[S]_i$ as follows:

$$[K_t] = \sum_{i=1}^{4} [D_i]^T [S_i] [D_i] \tag{3}$$

where

$$[D_i] = \begin{bmatrix} 1 & -y_i/r & 0 \\ 0 & x_i/r & 1 \end{bmatrix}$$

and (x_i, y_i) are the position coordinates of the i^{th} column with respect to the mass center. S_i is the column resistance function and it relates the incremental column lateral displacement (δv) and the lateral shear force (δV) by the following relation

$$\begin{Bmatrix} \delta V_x \\ \delta V_y \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} \delta v_x \\ \delta v_y \end{Bmatrix} \tag{4}$$

The derivation of [S] depends on the assumed reistance function. Two types of inelastic behavior are used in this study. If no interaction effect is considered, then each column is modelled as two independent elasto-plastic spring in the x and y directions. Hence, [S] is a diagonal matrix given by

$$[S] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \equiv [S]^e \tag{5}$$

where k_x and k_y are column stiffness and can take on either zero value (plastic state) or full elastic values (elastic state).

If the interaction effect on yielding in two directions are taken into account, the shear forces V_x and V_y on each column are related during yielding by a yield function ϕ given by

$$\phi = \left(\frac{V_x}{V_{px}} \right)^2 + \left(\frac{V_y}{V_{py}} \right)^2 \tag{6}$$

where V_{px} and V_{py} are shear forces that will cause yielding in the X and Y directions respectively.

The column is said to be elastic if $\phi < 1$; plastic if $\phi = 1$, associated with nonnegative values of incremental plastic work. Situations with $\phi > 1$ are considered inadmissible. Also, the range of behavior involving partial elastic state and partial plastic state is ignored in this modelling. Under these assumptions, it is shown by Nigam (Ref. 5) that [S] is given by

$$[S] = [S]^e - [S]^P$$

where

$$[S]^P = \frac{1}{k_x\left(\frac{\partial \phi}{\partial V_x} \right)^2 + k_y\left(\frac{\partial \phi}{\partial V_y} \right)^2} \begin{bmatrix} k_x^2 \left(\frac{\partial \phi}{\partial x} \right)^2 & \text{sym} \\ k_x k_y \left(\frac{\partial \phi}{\partial x} \right)\left(\frac{\partial \phi}{\partial y} \right) & k_y^2 \left(\frac{\partial \phi}{\partial y} \right)^2 \end{bmatrix} \tag{7}$$

The equations of motion are solved numerically using step-by-step integration technique, assuming linear variation of acceleration over a short time interval Δt. In the case where interaction effect on yielding is considered, a small value of Δt (= 0.001 sec) is necessary during the transition from elastic to plastic state, in order to keep the resultant force vector of V_x and V_y on the yield surface $\phi = 1$.

The two response parameters that are of special interests in design are the ductility demand in the resisting elements and the edge displacements of the structure. The former parameter is useful in the design and detailing of the resisting elements, and the latter is a good measure of the nonstructural damage potential. In this paper, the ductility demand is represented by the peak ductility ratio μ_x and μ_y at the column closest to the center of mass of the system. It is defined as the ratio of the absolute maximum displacement of column 1 in the x and y direction respectively to the yield displacement of the column. The edge displacements Δ_x and Δ_y are the x and y displacement of corner A, as shown in Fig. (1).

DISCUSSION OF RESULTS

Shown in Fig. (2) are the results for the symmetrical structure (e = 0). Plotted in Fig. (2a) are the ductility demand in the x and y direction as a function of excitation level parameter α. α is defined as the ratio $m(Sa)_{NS}/F_y$ where m is the mass of the slab, $(Sa)_{N.S.}$ is the undamped spectral acceleration of the N-S component El Centro ground record, at 0.5 sec and Fy is the yield strength of the structure. As a result, $\alpha = 1$ corresponds the structure just yield in the Y direction when it is excited by the El-Centro N.S. component. The results based on neglecting the yielding interaction effect (curve mark EP) are also shown. It can be seen that the yielding interaction effect is significant when α is large, or the system is excited well into the inelastic range. However, the spread between the ductility demands in the x and y directions decreases with the interaction effect taken into account due to coupling of the x and y direction displacements. Since there is no rotation of the structure, the corner displacements at A are identical to those at the columns. Therefore, the effect of yielding interaction on corner displacements can be inferred from the same graph.

Many seismic codes suggest that to allow for the bi-directional effect of earthquake ground motions, corner columns should be designed using a factor of 1.4 to magnify the earthquake effect from uni-directional consideration. Shown in Fig. (2b) is the ductility demand in the Y direction computed using N-S component of El Centro alone magnified 1.4 times. Plotted on the same curve is the "radial" ductility of the same column, with yielding interaction effect included in the computation. The radial ductility is computed as the root-sum-square of the ductilities in x and y directions. Comparison of the two curves would indicate the suggestion of the codes is conservative in this case.

For eccentric structural configurations, the corner displacement and column ductility demand in the Y direction will be discussed, as they are quantities most susceptible to rotational response of the structure. Shown in Fig. 3 are μ_y^*, the ductility demand of column 1 in the Y direction for structures with large and small eccentricity subjected to bidirectional excitation. This quantity is normalized to μ_{yo}, which represents the Y direction ductility demand of columns in the symmetric structure, subject to unidirectional N-S component El Centro excitation in the Y direction. μ_{yo} is also equal to the ductility demand for the symmetrical building under bi-directional excitation but neglecting the interaction effect of yielding as shown in Fig. (2). It is seen that in general, results based on neglecting the interaction effect overestimate the response. For building with small eccentricity, the ductility demand on the column is essentially the same as those in the symmetrical building, and the ductility demand can be estimated from the conventional uni-directional inelastic analysis, neglecting torsion. For buildings of large eccentricity, the ductility demand can be twice as large as that for unidirectional excited symmetrical system.

To investigate the cause of this large difference, the same ductility demand μ_y^* is re-normalized with $(\mu_y)_{NS}$, the ductility demand obtained using unidirectional excitation on the same eccentric system on one hand, and normalized with $(\mu_{yo})_{EPI}$ the ductility demand obtained using bi-directional excitation on a symmetrical system taking interaction effect into account on the other. These re-normalized curves are shown in Fig. (4a,b) respectively. Examination of Figure 4 shows that the ductility ratio μ_y^* is not much different from $(\mu_y)_{N.S.}$ while there is still considerable difference between μ_y^* and $(\mu_{yo})_{EPI}$. In other words, the difference of ductility demand μ_y^* from μ_{yo} as shown in

Fig. 3a is caused mainly by the effect of eccentricity. An analysis of the eccentric system using uni-directional excitation will give a good estimate of the ductility demand under bi- directional excitation with yielding interaction effect included.

The corner displacement in the Y direction of eccentric systems are shown in Fig. (5). Similar observation can be made on systems with small eccentricities that they behave essentially like symmetrical structures when they are excited into the inelastic range. There is substantial difference of corner displacements between buildings with large eccentricity as compared to symmetrical structures. Typically, the displacement of the eccentric system is twice that of the symmetric system, and can reach to four times for intense excitation levels. This large difference is again caused by the eccentricity effect. The corner displacement Δ_y* can again be estimated within design accuracy if an analysis is carried out on the eccentric system using uni-directional excitation. The accuracy of such estimation is shown in Fig. (6) where corner displacement Δ_y* is normalized with $(\Delta y)_{NS}$, the corresponding corner displacement value for uni-directional excitation of the same system.

CONCLUSION

Based on comparing the responses of eccentric structural systems subjected to bi-directional excitation to those of uni-directional excitation, the following conclusions can be drawn.

1. By neglecting the interaction effect of yielding in the columns, the resulting estimate of ductility demand and corner displacement generally is higher than those where interaction effect is included. With interaction, yielding occurs at a lower force level, causing inelastic action to occur earlier in the earthquake. This causes energy to be dissipated from the system and thus results in smaller responses.

2. The ductility demand of eccentric system with small eccentricity is similar to that of symmetric system over the range of excitation levels considered.

3. The ductility demand of eccentric system with large eccentricity under bi-directional excitation can be substantially higher than that from a symmetrical system. The cause of this increase in demand is mainly due to the rotational response of the structure caused by eccentricity. The ductility demand can be estimated based on uni-directional excitation of the same system.

4. Requiring a factor of 1.4 to allow for the bi-directional excitation effect on corner columns is shown to be a reasonable, and conservative requirement.

All the above conclusions point favorably to justify using unidirectional excitation to represent the horizontal ground motion effect. However, one must recognize the limitation of the present study. The assumption involved in the bi-directional yield relationship of the columns is highly idealized and tends to downplay the effect of bi-directional loading. In real concrete columns, substantial cracking and spalling of the shell concrete may have taken place when the column yields in one direction. As a result, the resistance to excitation in a perpendicular direction will be substantially reduced. Similarly, in a tubular steel column, the large strain induced by yielding in one direction may cause local buckling, and this will reduce the strength of the column in a perpendicular direction. Unless these effects can be incorporated into the analysis, the justification of treating ground motion effect one direction at a time must still be considered an open question.

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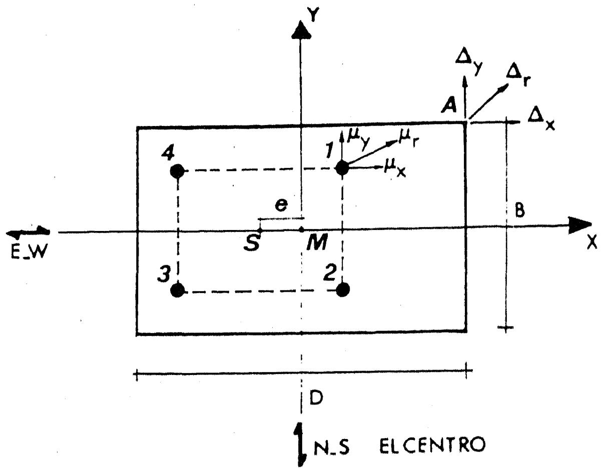


Fig. 1 STRUCTURAL PLAN

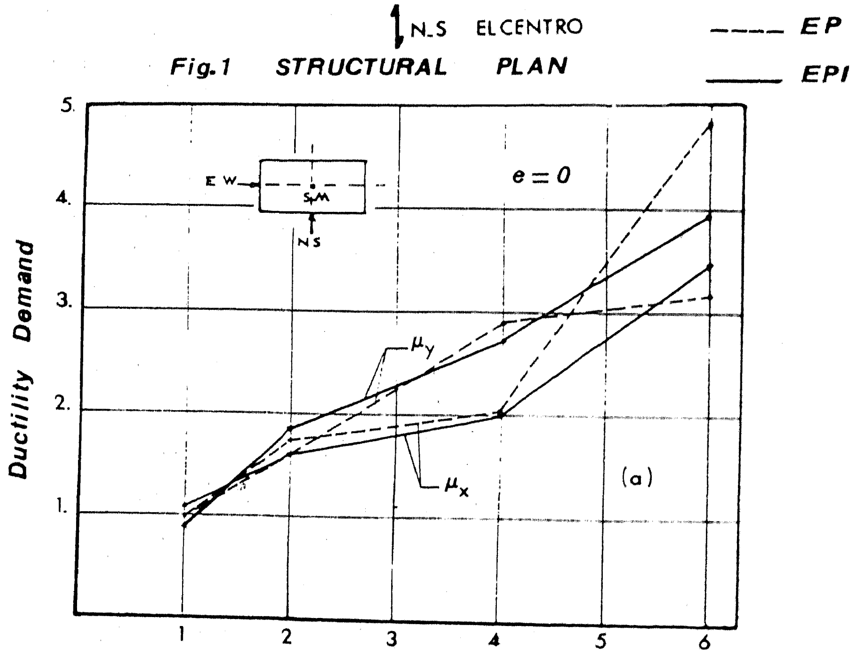
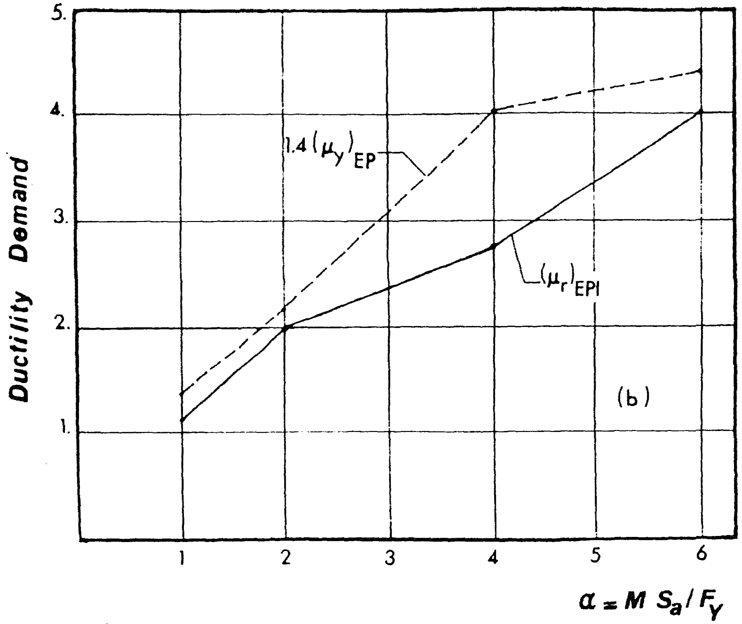


Fig. 2 DUCTILITY RATIO FOR SYMMETRIC SYSTEMS



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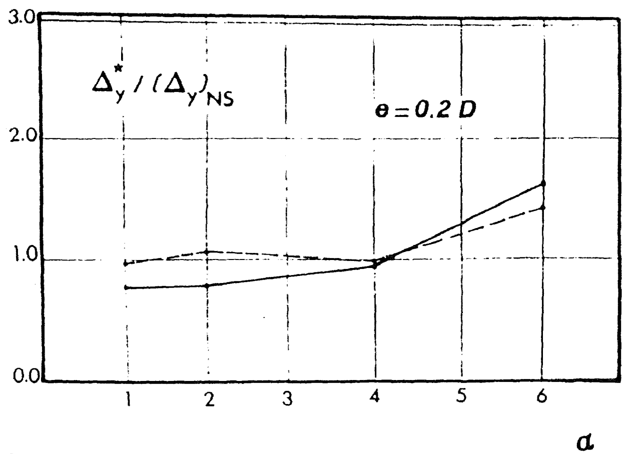

Fig. 6 CORNER DISPLACEMENT FOR ECCENTRIC SYSTEMS

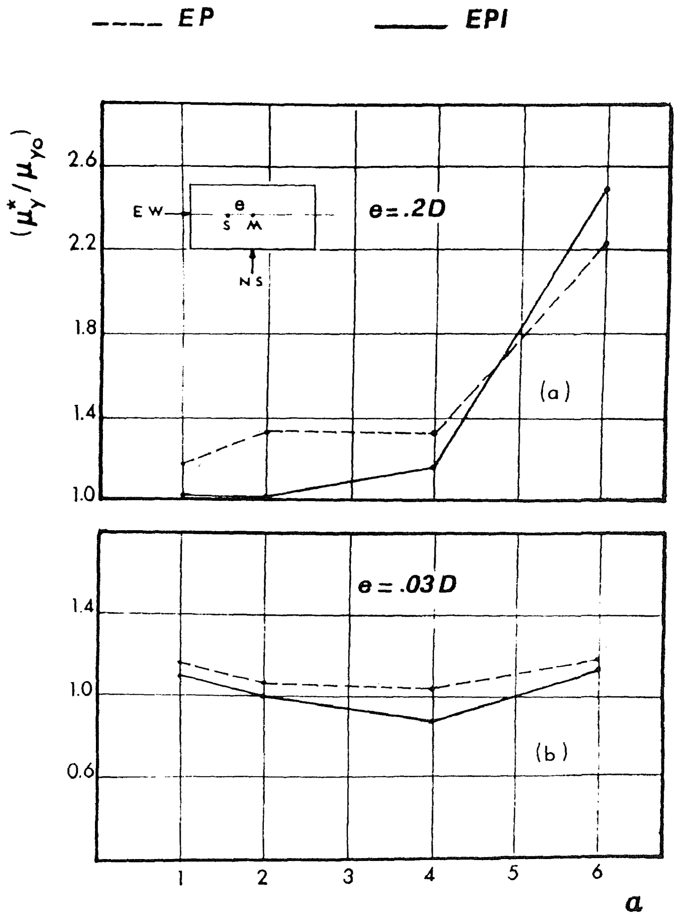

Fig. 3 Ductility Ratio For Eccentric Systems

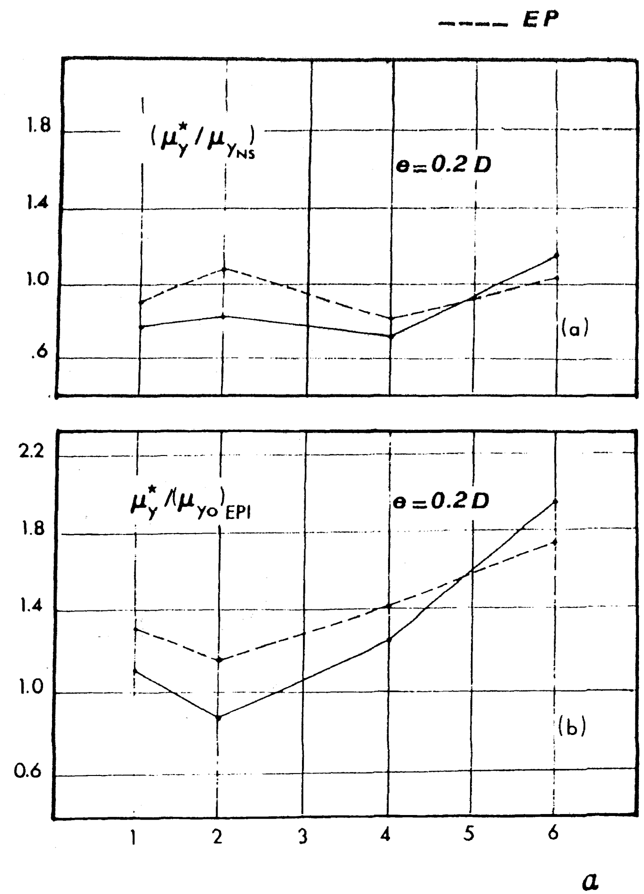

Fig. 4 DUCTILITY RATIO FOR ECCENTRIC SYSTEMS

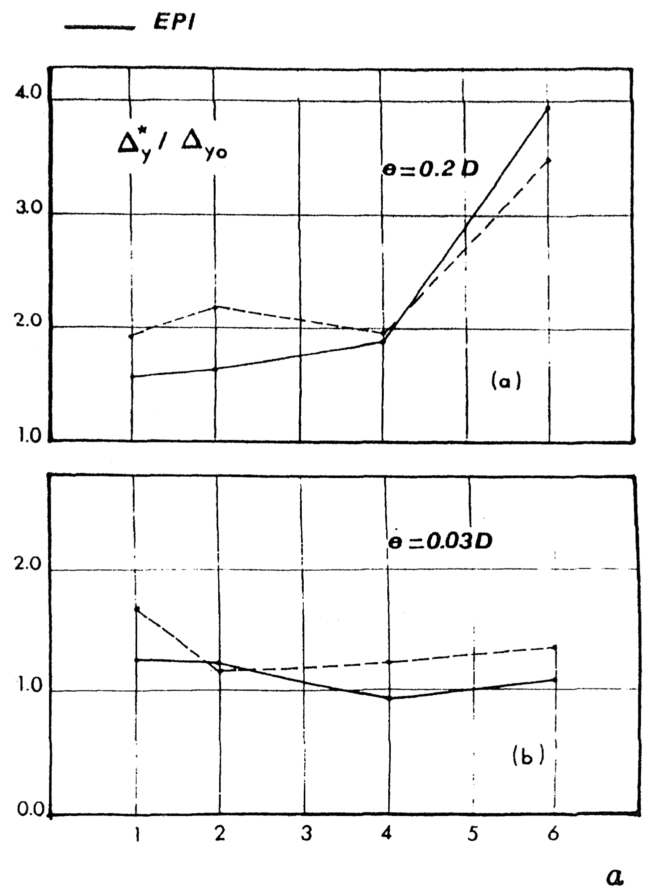

Fig. 5 CORNER DISPLACEMENT FOR ECCENTRIC SYSTEMS