A STATISTICAL BASIS FOR SPECTRUM SUPERPOSITION IN RESPONSE TO EARTHQUAKE EXCITATION

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SUMMARY

A theory is presented for evaluation of the expected and the most probable amplitudes of response peaks for different levels of multi-story structure in terms of: (1) root-mean-square amplitude of excitation; (2) a measure of the frequency "width" of the response spectrum; and (3) the total number of peaks of response. The statistical parameters are derived using a discrete transfer function model of a multi-degree-of-freedom system. Comparisons of the expected and most probable values of peaks with SRSS, SUM, and CQC methods have been presented.

INTRODUCTION

Dynamic response of the multi-level structures to strong earthquake shaking has been one of the most analyzed topics in earthquake engineering. Many methods have been introduced to evaluate this response. Some are based on a theoretical model of a structure and the exact dynamic response to strong earthquake ground motion; other methods employ response spectrum technique which permits separation of the characteristics of structures from those of the earthquake excitation. Several techniques for the response spectrum superposition have been developed in which an approximation to the maximum of the total response is obtained by combining the modal maxima. Some of these methods are the SUM, SRSS, CQC, NRL, GROUPING, TEN PERCENT, and DOUBLE SUM (Ref. 4, 5, 6, 7, 8, 9, 10). In this paper we present a new method based on a probabilistic technique for evaluation of the response of multi-degree-of-freedom structures to strong earthquake ground motion and we compare this method with other available methods.

EXPECTED AND MOST PROBABLE VALUE OF n-TH LOCAL MAXIMUM OF THE RESPONSE

It can be shown (Ref. 1) that the distribution of amplitudes of a random function, f(t), represented by the series

$$f(t) = \sum_n C_n \cos (\omega_n t + \phi_n) \qquad (1)$$

may be characterized by two parameters, $m_o^{1/2}$, and ε, where

$$\varepsilon^2 = 1 - \frac{m_2^2}{m_o m_4} \qquad (2)$$

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represents a measure of the width of the energy spectrum, $E(\omega)$, of $f(t)$. In equation (1), C_n are related to $E(\omega)$ through

$$\sum_{\omega_n=\omega}^{\omega+d\omega} \tfrac{1}{2}C_n^2 = E(\omega)\,d\omega \quad \ldots \quad (3)$$

where ω_n represents circular frequency; t = time; and ϕ_n are assumed to be randomly and uniformly distributed between 0 and 2π. In equation (2)

$$m_n = \int_0^\infty E(\omega)\,\omega^n\,d\omega \quad \ldots \quad (4)$$

represents the n-th moment of the energy spectrum $E(\omega)$.

Figure 1 shows an example of $f(t)$ for small ε. It represents relative displacement of a single-degree-of-freedom system for natural period T_n=2.5 sec., and fraction of critical damping ζ = 0.07, excited by the El Centro earthquake acceleration. The peak labeled $a^{(1)}$ represents the largest displacement during this excitation and corresponds to the relative displacement spectrum amplitude. The $a^{(2)}$, $a^{(3)}$, ..., $a^{(N)}$ represent the "second largest", the "third largest", ..., and the N-th or last peak of $f(t)$. Through generalization of Rice's work (Ref. 2), Longuet-Higgins (Ref. 3), and Cartwright and Longuet-Higgins (Ref. 1) it can be shown that the expected value of $a^{(n)}$ is given by (Ref. 4):

$$\frac{E(a^{(n)})}{\bar{a}} = n\,\{[\ln (1-\varepsilon^2)^{\frac{1}{2}} N]^{\frac{1}{2}} + \tfrac{1}{2}\gamma\,[\ln(1-\varepsilon^2)^{\frac{1}{2}}N]^{-\frac{1}{2}}\}$$

$$-\frac{n(n-1)}{2!}\{[\ln(1-\varepsilon^2)^{\frac{1}{2}}2N]^{\frac{1}{2}}+\tfrac{1}{2}\gamma[\ln(1-\varepsilon^2)^{\frac{1}{2}}2N]^{-\frac{1}{2}}\}$$

$$\vdots$$

$$(-1)^{n+1}\{[\ln(1-\varepsilon^2)^{\frac{1}{2}}nN]^{\frac{1}{2}}+\tfrac{1}{2}\gamma[\ln(1-\varepsilon^2)^{\frac{1}{2}}nN]^{-\frac{1}{2}}\} \quad \ldots \quad (5)$$

In equation (5), $a^{(n)}$ is the n-th largest peak of $f(t)$. Using

$$\alpha=\ln\left(N(1-\varepsilon^2)^{\frac{1}{2}}\left\{1-\frac{(n-1)\,[1-(1-\varepsilon^2)^{\frac{1}{2}}\,e^{-\alpha}]^N}{1-[1-(1-\varepsilon^2)^{\frac{1}{2}}\,e^{-\alpha}]^N}\right\}\right)$$

$$-\ln\left\{1-\frac{1}{2\alpha}\,[1-(1-\varepsilon^2)^{\frac{1}{2}}\,e^{-\alpha}]\right\} \quad \ldots \quad (6)$$

the most probable value of $a^{(n)}$, denoted by $\mu(a^{(n)})$ can be shown to be (Ref. 4):

$$\frac{\mu(a^{(n)})}{\bar{a}} = \sqrt{\alpha} \quad \ldots \quad (7)$$

In the above equations N is the total number of maxima given by (Ref. 5):

$$N = \frac{1}{2\pi} \left(\frac{m_4}{m_2}\right)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

The term γ is the Euler's constant which is approximately equal to 0.5772. For narrow-band function f(t) ($\varepsilon \cong 0$), $\overline{a}$ may be closely approximated by

$$a_{rms} = \left[\frac{1}{T}\int_0^T f^2(t)\,dt\right]^{\frac{1}{2}}, \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

the root mean square of f(t), as follows

$$\overline{a} = \sqrt{2}\; a_{rms} = \sqrt{2}\; m_o^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

Computation of $\overline{a}$, ε and N

To evaluate the expected and most probable values of the n-th local maximum of response of a multi-level structure to an earthquake excitation, the parameters $\overline{a}$, ε and N need to be evaluated. For this purpose a square of displacement response ($|R_j(\omega)|^2_D$), shear force ($|R_j(\omega)|^2_s$) and overturning moment ($|R_j(\omega)|^2_m$) of the j-th level of a multi-level structure are modeled as follows: (Ref. 5)

$$|R_j(\omega)|^2_D = \frac{\pi A_{j1}^2 \alpha_1^2 \overline{Z}^2(\omega_1)}{4\zeta_1 \omega_1^3}\,\delta(\omega-\omega_1) + \frac{\pi A_{j2}^2 \alpha_2^2 \overline{Z}^2(\omega_2)}{4\zeta_2 \omega_2^3}\,\delta(\omega-\omega_2)$$

$$+ \ldots + \frac{\pi A_{jn}^2 \alpha_n^2 \overline{Z}^2(\omega_n)}{4\zeta_n \omega_n^3}\,\delta(\omega-\omega_n), \quad \ldots\ldots\ldots \quad (11)$$

$$|R_j(\omega)|^2_s = \frac{\pi(M_1 A_{11} + M_2 A_{21} + \ldots + M_j A_{j1})^2 \alpha_1^2 \omega_1 \overline{Z}^2(\omega_1)}{4\zeta_1}\,\delta(\omega-\omega_1)$$

$$+ \frac{\pi(M_1 A_{12} + M_2 A_{22} + \ldots + M_j A_{j2})^2 \alpha_2^2 \omega_2 \overline{Z}^2(\omega_2)}{4\zeta_2}\,\delta(\omega-\omega_2)$$

$$+ \ldots$$

$$+ \frac{\pi(M_1 A_{1n} + M_2 A_{2n} + \ldots + M_j A_{jn})^2 \alpha_n^2 \omega_n \overline{Z}^2(\omega_n)}{4\zeta_n}\,\delta(\omega-\omega_n) \quad \ldots \quad (12)$$

and

$$|R_j(\omega)|_M^2 = h_1^2 |R_1(\omega)|_S^2 + h_2^2 \; |R_2(\omega)|_S^2 + \ldots + h_j^2 |R_j(\omega)|_S^2 \; . \; . \; . \; . \qquad (13)$$

In the above expressions A_{ij} is the j-th element of the i-th eigenvector, α_j is the j-th mass participation factor, $\overline{Z}\ (\omega_j)$ is the Fourier transform of ground acceleration at circular frequency ω_j. The term M_j is the lumped mass at the j-th level and h_j is the height of the j-th level of the structure.

To evaluate $\overline{a}$, ε and N of the response (displacement, shear force or overturning moment) at the j-th level of structure, equations (11), (12), or (13) along with equations (4), (10), (2) and (8) are employed. For example, for the case of displacement the k-th moment of $|R_j(\omega)|_D^2$ is given by (Ref. 5):

$$m_{jk} = \int_0^\infty \omega^k |R_j(\omega)|_D^2 \, d\omega$$

$$= \frac{\pi}{4} \Big[\frac{A_{j1}\ \alpha_1^2 \omega_1^k \overline{Z}^2(\omega_1)}{\zeta_1 \omega_1^3} + \frac{A_{j2}^2 \alpha_2^2 \omega_2^k \overline{Z}^2(\omega_2)}{\zeta_2 \omega_2^3} +$$

$$\ldots + \frac{A_{jn}^2 \alpha_n^2 \omega_n^k \overline{Z}^2(\omega_n)}{\zeta_n \omega_n^3} \Big] \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (14)$$

As an application example, a 7-story shear building is subjected to different ground accelerations. The expected and most probable, as well as the exact values of the maxima of displacement, shear force and overturning moment have been evaluated (Ref. 5). Figure 2 shows the exact $(a^{(n)})$, the expected value $(E(a^{(n)}))$ and the most probable value $(\mu(a^{(n)}))$ of the first five maxima of displacement at the 7th, 6th, and 5th levels of structure excited by San Fernando earthquake of 1971. The agreement of the estimated and the exact solutions is quite good. It is noted that the response spectrum displacement values, which are routinely available for any ground acceleration have not been used in this calculation.

Modified Root-Mean-Square Value

To improve upon the accuracy of the results, a method using the earthquake response spectra has also been developed. It is based on the simple idea that the "correction" for the nonstationarity of excitation and response must be contained in the relative response spectrum amplitudes computed directly from the recorded or assumed input acceleration.

It can be shown (Ref. 5) that a^2_{jrms} of the total response is the sum of a^2_{rms} values of n corresponding single-degree-of-freedom systems. Hence, the question that may be asked is: what should the a^2_{rms} for each single degree of freedom be, so that the expected and the most probable values of the largest peak (n=1), for the specific mode of vibration, are equal to its SD (spectral displacement) value. The value of $\bar{a}$ can then be shown to be (Ref. 5):

$$(\bar{a}_E)_j = \sqrt{2}\,(a_E)_{j\,rms} \quad (15)$$

$$(\bar{a}_\mu)_j = \sqrt{2}\,(a_\mu)_{j\,rms} \quad (16)$$

The terms $(a_E)_j$ and $(a_\mu)_j$ for the j-th level of structure are defined as (Ref. 5):

$$(a_E)^2_{j\,rms} = [(a_E)^2_{j1\,rms} + (a_E)^2_{j2\,rms} + \ldots + (a_E)^2_{jn\,rms}] \quad (17)$$

and

$$(a_\mu)^2_{j\,rms} = [(a_\mu)^2_{j1\,rms} + (a_\mu)^2_{j2\,rms} + \ldots + (a_\mu)^2_{jn\,rms}] \quad (18)$$

where

$$(\bar{a}_E)_{ji} = \frac{SD^*_{ji}}{\left(\frac{E(a^{(1)})}{\bar{a}}\right)_{ji}} = \sqrt{2}\,(a_E)_{ji\,rms} \quad (19)$$

and

$$(\bar{a}_\mu)_{ji} = \frac{SD^*_{ji}}{\left(\frac{\mu(a^{(1)})}{\bar{a}}\right)_{ji}} = \sqrt{2}\,(a_\mu)_{ji\,rms} \quad (20)$$

We are computing two different $\bar{a}$'s because $E(a^{(n)})/\bar{a}$ is slightly different from $\mu(a^{(n)})/\bar{a}$ (Ref. 4). The term SD^*_{ji} does not always represent the spectral displacement at frequency ω_i. The value of SD^*_{ji} is equal to SD_i (spectral displacement) for only those computations of single-degree-of-freedom system response involving estimation of displacement response. For a multi-degree-of-freedom system SD^*_{ji} for displacement, shear force and overturning moment are defined as follows

$$(SD^*_{ji})_D = A_{ji}\alpha_i SD_i \quad (21)$$

$$(SD^*_{ji})_S = (M_1A_{1i} + M_2A_{2i} + M_jA_{ji})\alpha_i\omega_i^2 SD_i \quad (22)$$

and

$$(SD^*_{ji})_M = [h_1M_1A_{1i} + h_2\,(M_1A_{1i} + M_2A_{2i}) + \ldots$$

$$+ h_j \, (M_1 A_{1i} + M_2 A_{2i} + \dots + M_j A_{ji})] \alpha_i \omega_i^2 \, SD_i . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The corresponding values of $E(a^{(n)})$ and $\mu(a^{(n)})$ for the modified root mean square are denoted by $\overline{E}(a^{(n)})$ and $\overline{\mu}(a^{(n)})$ respectively.

The computation of the response of the 7-story structure was repeated using this modified definition of $\overline{a}$, and more accurate results were obtained (Figure 3). Comparison of Figures 2 and 3 indicates the improvement in the accuracy of the response estimates ($\overline{E}(a^{(n)})$ and $\overline{\mu}(a^{(n)})$ with this modified definition of $\overline{a}$.

COMPARISON OF EXPECTED AND MOST PROBABLE VALUE METHOD WITH "SRSS", "SUM", AND "CQC"

To compare the results obtained here with more traditional modal superposition techniques, only $\overline{E}(a^{(1)})$ and $\overline{\mu}(a^{(1)})$ should be considered, since the methods such as SRSS, SUM, or CQC compute only the maximum response of each floor of a structure and do not deal with the evaluation of n-th local peak of responses. To show this comparison the response of the 7-story shear building to different earthquake excitations was computed using different techniques. The results of the displacement response to San Fernando earthquake of 1971 for all the floors are shown in Figure 4. The results based on different methods are in close agreement with the exact solution. The SUM method gives an overestimation of the results.

The above results represent a typical response. Several other accelerograms with long, medium and short duration as well as different responses (displacement, shear force and overturning moment) were considered (Ref. 5). The results show excellent accuracy for the method summarized here (Ref. 5).

CONCLUSION

The aim of this paper is not to advocate the use of the new proposed method when means are available to carry out exact calculations. However, if spectrum superposition is chosen, the method presented here will give more detailed and more complete information on the response of an n-degree-of-freedom system than any other technique available so far. The results on the distribution of $a^{(n)}$ amplitudes (Ref. 5) help a designer to consider the relationship between all response maxima (their number and amplitudes) and the physical characteristics of the structural system which is designed. The estimates of the amplitudes of the second $a^{(2)}$, third $a^{(3)}$, etc., largest peaks of the equivalent linear system should be helpful in understanding the number of times certain response levels may be exceeded as the structural system is progressing into nonlinear response. These results may further be useful for qualitative interpretation of the observed damage of structures in terms of the number of the equivalent linear excursions beyond the assumed design strength.

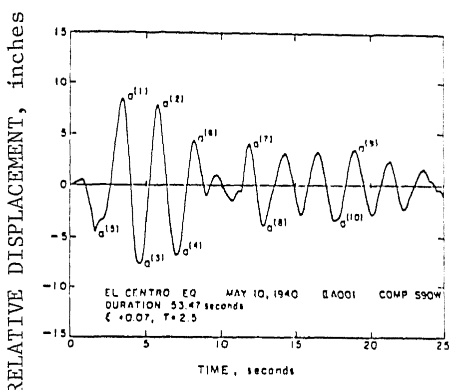

Figure 1. Relative Displacement of a Single-Degree-of-Freedom System.

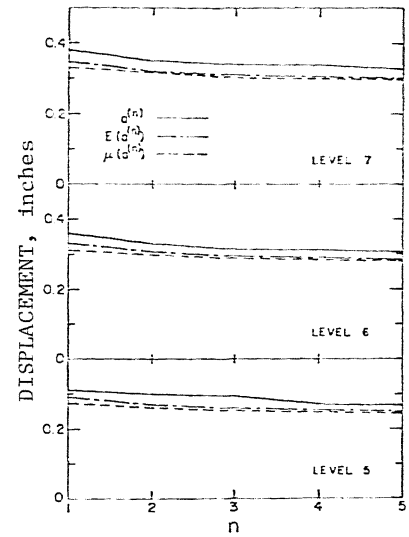

Figure 2. $a^{(n)}$, $E(a^{(n)})$, and $\mu(a^{(n)})$ versus n for Displacement, Levels 7, 6, 5 of a 7-story structure. (San Fernando Earthquake, Feb. 9, 1971, IIF103, Comp. N90W)

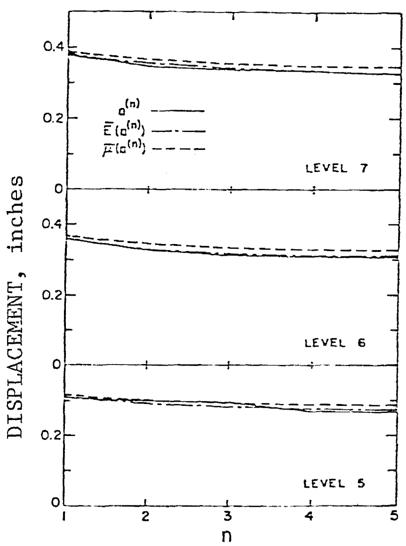

Figure 3. $a^{(n)}$, $\overline{E}(a^{(n)})$, and $\overline{\mu}(a^{(n)})$ versus n for Displacement, Levels 7, 6, 5 of a 7-story structure. (San Fernando Earthquake, Feb. 9, 1971, IIF103, Comp. N90W)

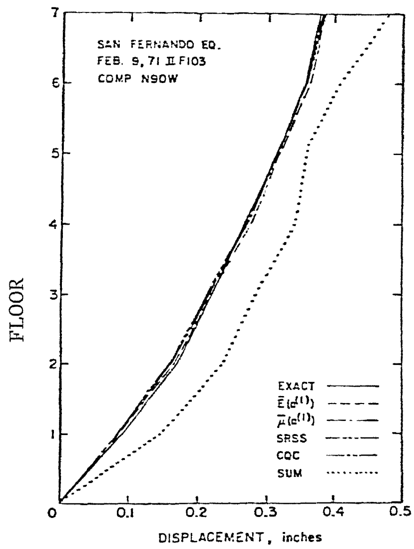

Figure 4. Maximum Displacement of Floors (San Fernando Earthquake, Feb. 9, 1971, IIF103, Comp. N90W)

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