A NEW METHOD FOR SPECTRAL RESPONSE ANALYSIS

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SUMMARY

This paper presents a response spectrum analysis method in which the maximum modal responses are combined in a quadratic form using newly introduced modal coupling factor. It is called the ARC (Advanced Response Combination) technique, and is similar to the other quadratic modal response combination techniques except that the new method calls for an algebraic sum of the modal responses for sufficiently low or high modal frequencies. The examples presented here show that it is indeed an improvement over the existing quadratic modal response combination techniques such as the CQC method.

INTRODUCTION

The SRSS (Square Root Sum of Squares) technique has traditionally been used in the modal response combination for the response spectrum method of seismic analysis. Over the years, the SRSS technique has been found not desirable in several occasions, for example, when modes are closely spaced to each other. Analytically, it can easily be demonstrated that the maximum responses between two sufficiently closely spaced modes must be algebraically combined. Therefore, a more desirable technique for the modal response combination could take the following quadratic form:

$$R \doteq \sqrt{R_1^2 + R_2^2 + \ldots + R_j^2 + \ldots + 2 \sum_{j \neq k} c_{jk} R_j R_k} \qquad (1)$$

in which R_j, R_k are the maximum modal responses and c_{jk} the modal coupling factor. To satisfy the previously mentioned algebraic sum requirement for modes that are sufficiently closed to each other, the following conditions must be fulfilled:

(a) The modal coupling factor approaches the value of 1.0 when two modal frequencies converge to each other.

(b) The modal maximum response, R_j, does not assume an absolute value and it retains the algebraic sign associated with the product of all components (e.g., mode shape, modal participating factor, etc.) that constitute R_j, among which the modal spectrum input, S_j, is always assigned its absolute value.

The CQC (Complete Quadratic Combination) technique, which was proposed by Wilson and Kiureghian, fulfills the above minimum conditions (Ref. 1). The modal coupling factor in the CQC method is a function of both the modal damping and the ratios in frequency between modes. Therefore, there is no longer the need to specify an arbitrary threshold for defining when modes should be considered as closely spaced, such as the conventional 10% differential modal

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frequency criteria used in the nuclear industry.

The modal coupling factor specified in the CQC method was derived from the statistical theory assuming that the earthquake ground motions are ideal stationary random process, and it is independent of the values of the modal frequencies. This is an apparent drawback because earthquake ground motions are seldom stationary process and they have a limited frequency band. It can be proved analytically that the combination between the maximum responses of two modes converges to an algebraic sum when both modal frequencies are sufficiently low or high even though they may not be closely spaced with each other. Examining Equ.(1), this condition calls for c_{jk} to be also a function of the modal frequency such that it approaches the value of 1.0 at both sufficiently low and high frequencies. Thus, the desirable quadratic combination technique as defined in Equ. (1) not only must fulfill the aforementioned conditions, (a) and (b) for R_j and c_{jk} respectively, but also fulfill the condition of the modal frequency dependency described above for c_{jk}. To meet this end, a new modal coupling factor is introduced here. It was derived empirically from an extensive parametric study, and the new method is called the ARC (Advanced Response Combination) method. Numerical examples for a 3-story building are presented to compare the responses from the ARC method with the CQC, SRSS and time history methods.

THE MODAL COUPLING FACTOR

For the ARC method, the modal coupling factor was derived empirically based on an extensive parametric study of the maximum combined response between pairs of modes using the time history response analysis method. Several earthquake ground motions formed the basis of the study, and the following parameters were considered (Ref. 2).

- o Three modal damping values : β_j = 1%, 2%, 5%;
- o Six modal frequency differential : $\Delta_{jk} = |f_k - f_j|/\bar{f}_{jk}$ = .01, .02, .05, .1, .15, .2;
- o Twenty nine mean modal frequencies, $\bar{f}_{jk} = (f_j + f_k)/2$ = 0.1 to 40.

The resultant c_{jk} for a constant modal damping of 1% is shown in Fig. 1. The empirical results were then simplified to the ones illustrated in Figs. 2 and 3 for a constant modal damping of 1% and 5%, respectively. They can be expressed as follows.

$$c_{jk} = 1 - H(\bar{f}_{jk})\Delta_{jk}^2/[\Delta_{jk}^2 + 4(\beta_j + 0.01)^2] \tag{2}$$

in which $H(\bar{f})$ is a linear function between the following controling points:

$\bar{f}_{jk}$	=	0	1	5	15	25	33
H	=	0.	1.0	1.0	0.3	0.1	0.

For purposes of comparison, the coupling factor from the CQC method for a constant modal damping may be expressed, equivalently, in terms of Δ_{jk} and β_j, as follows.

$$c_{jk} = \frac{\beta_j^2 \sqrt{[1 - (\Delta_{jk}/2)^2]^3}}{(\Delta_{jk}/2)^2 + \beta_j^2[1 - (\Delta_{jk}/2)^2]} \tag{3}$$

EXAMPLES

A three-story building from Ref. 3 was taken here for example. Fig. 4 shows the simplified 3-mass building model. Its interfloor shear stiffness and natural frequencies have the following properties: $k_1 : k_2 : k_3 = 1:2:3$, $f_1 : f_2 : f_3 = 1 : 2.137 : 3.178$. By keeping the masses unchanged and varying the stiffnesses uniformly, three building cases resulted that have the same mode shapes and participating factors but different modal frequencies, as shown below.

$$[\text{Mode Shape}] = \begin{bmatrix} .7449 & -.6381 & .2080 \\ .4797 & .3835 & -.5346 \\ .2235 & .4314 & .5138 \end{bmatrix} \quad [\text{Part. Factors}] = \begin{bmatrix} 1.9114 \\ 0.8000 \\ 0.4337 \end{bmatrix}$$

Building Case	f_1(Hz)	f_2(Hz)	f_3(Hz)
A	2.00	4.27	6.36
B	10.0	21.37	31.78
C	35.00	74.79	111.23

By inspection, the frequencies of building Case C are so high the building may be regarded as rigid and it would respond to earthquake ground motions like a rigid structure should.

Four earthquake ground motions were considered as input. Their durations and maximum ground accelerations are listed below.

Earthq. No.	Duration(sec)	Max. Ground Accel.(g)
1	16.0	0.225
2	14.2	0.476
3	10.4	0.457
4	27.1	0.482

We computed the building acceleration response, a_i, and inter-story shear V_i using both the time history method and the response spectrum method, the latter including the SRSS, CQC and ARC techniques. A constant modal damping of 2% of critical was assumed for the study.

The modal coupling factors for both the CQC and ARC methods were computed in accordance with Equ. (2) and (3), respectively. The results are shown in the following:

Method	Building Case	c_{12}	c_{13}	c_{23}
CQC	A, B, C	.0025	.0009	.0097
	A	.02	.02	.03
ARC	B	.71	.82	.90
	C	1.00	1.00	1.00

Note that the modal coupling factors from the CQC method for all three building cases and from the ARC method for Case A are so small that, for practical purposes, we may take the responses from such cases to be the same as those from the SRSS method, and no separate calculation is necessary.

The building acceleration responses and inter-story shears were computed. The maximum values from the time history analysis method are listed below.

Earthq. No.		Acceleration (g) A	B	C		Inter-Story Shear (kip) A	B	C
1	a_1	1.26g	0.89g	0.25g	V_1	492k	350k	98k
	a_2	0.94	0.66	0.23	V_2	911	726	225
	a_3	0.67	0.43	0.22	V_3	1130	1050	396
2	a_1	1.87	2.39	0.51	V_1	734	943	203
	a_2	1.27	1.63	0.48	V_2	1330	1860	477
	a_3	0.77	0.91	0.47	V_3	1640	2560	832
3	a_1	1.61	1.28	0.50	V_1	623	502	198
	a_2	1.08	0.89	0.47	V_2	928	1000	461
	a_3	1.02	0.59	0.45	V_3	1350	1430	801
4	a_1	1.56	0.70	0.49	V_1	610	273	193
	a_2	1.13	0.47	0.48	V_2	1010	542	466
	a_3	0.59	0.47	0.48	V_3	1380	808	839

The ratios in response between the response spectrum and time history methods of analysis, averaged over the four earthquakes, are summarized below.

Method		Acceleration Ratio Case A	Case B	Case C		Shear Ratio Case A	Case B	Case C
SRSS/T.H. & CQC/T.H.	a_1	0.97	1.10	1.44	V_1	0.97	1.09	1.43
	a_2	0.91	1.01	1.00	V_2	0.98	1.03	1.11
	a_3	0.92	0.75	0.61	V_3	1.00	0.94	0.84
ARC/T.H.	a_1	same as above	0.93	0.97	V_1	same as above	0.96	0.97
	a_2		1.01	1.00	V_2		0.98	0.99
	a_3		1.04	1.01	V_3		1.02	1.01

The above results indicate that, as the building becomes stiffer, the quality of the approximation in the responses from both the SRSS and CQC methods decreases. This is not the case with the ARC method simply because its modal coupling factor accounts for the dependency on frequency.

CONCLUSIONS

The examples presented previously confirmed the desirability of using a quadratic response combination technique that includes modal coupling factors being a function of modal damping, modal frequency and difference in modal frequency. The major difference between the presently proposed ARC method and the CQC method is in the dependency on the modal frequency. That is, the modal coupling factor in the CQC method is independent of the modal frequency whereas the coupling factor in the ARC method varies with the modal frequency and it converges to 1.0 at both zero and sufficiently high frequencies. For the rigid building in Case C in the examples, it was interesting to note that the ARC method was simply reduced to an algebraic combination of the modal responses because, in that case, all three modal coupling factors became 1.0. In addition, it is important to see that only algebraic sum of the modal responses would produce responses which one expects from a rigid structure.

The examples did not illustrate how the CQC and ARC methods compare with other in the case of closely spaced modes. However, both Equ. (2) and (3) show that the coupling factors in both methods converges to 1.0 as the modal frequency differential approaches zero. This implies that both methods are equally adequate as far as closely spaced modes are concerned because both call for an algebraic sum in the limiting case. The examples presented in Refs. 1 and 2 effectively illustrated the merit of algebraic sum over the SRSS combination in the case of closely spaced modes.

ACKNOWLEDGMENT

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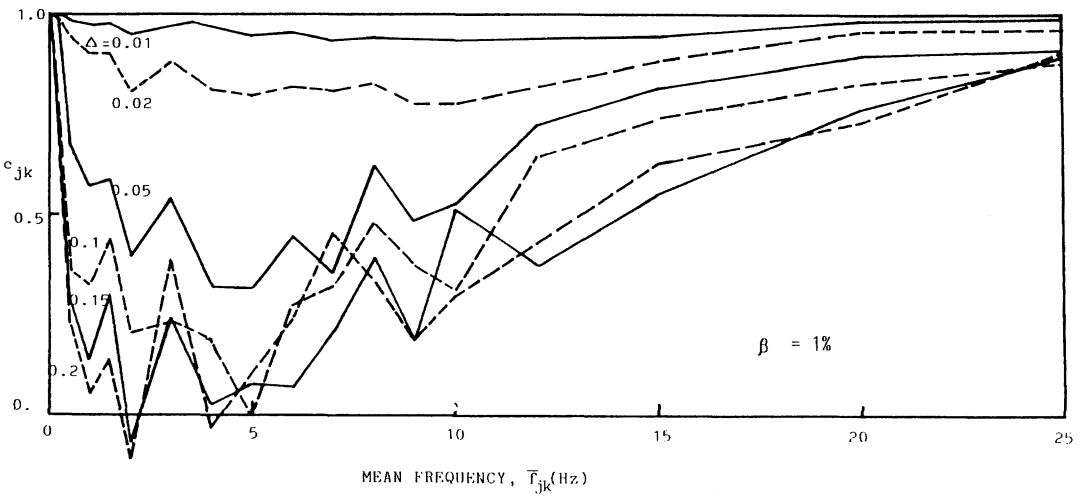

FIG. 1 AVERAGED MODAL COUPLING FACTOR FOR THE FOUR GROUND MOTIONS (DAMPING = 1%)

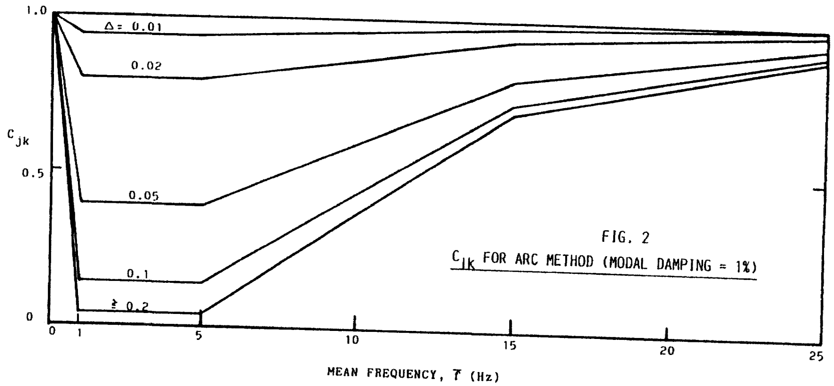

FIG. 2
C_{jk} FOR ARC METHOD (MODAL DAMPING = 1%)

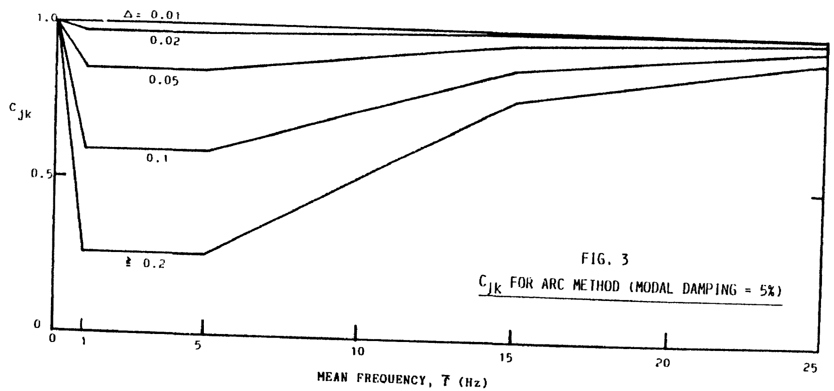

FIG. 3
C_{jk} FOR ARC METHOD (MODAL DAMPING = 5%)

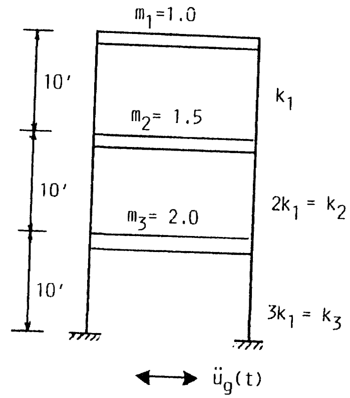

FIG. 4 THE 3-STORY BUILDING MODEL