MODAL COMBINATION IN RESPONSE SPECTRUM METHOD

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SUMMARY

A new method of combining modes in the response spectrum method of analysis is presented. The new method addresses one major concern viz, combination of modes at higher frequencies. Not only modes at frequencies beyond the rigid frequency are mutually correlated, but are also some of the modes below the rigid frequency. The correlation is measured in terms of the rigid response coefficient. The modal responses are represented in terms of the rigid response and the damped periodic response with the help of the rigid response coefficient. A new expression for the correlation between modes with closely spaced frequency is also presented. The response spectrum results obtained by using the new method show good agreement with the time history results.

INTRODUCTION

The response spectrum method of analysis gives maximum response in each mode of vibration. Since, maximum responses from different modes do not, in general, occur simultaneously, it is common to combine them by the SRSS (square root of the sum of the squares) rule. In many cases such a combination yields responses which are comparable to those from an equivalent time history analysis. The SRSS rule essentially assumes that the modal responses are statistically uncorrelated. It is know that this is not true when two modes have frequencies which are close, or when the modes have frequencies well beyond the frequencies contained in the input motion. The correlation between modal responses can be accounted for by a modification of the SRSS rule, by the so called double sum method.

$$R^2 = \sum_i R_i^2 + 2\sum_i \sum_{j>i} \varepsilon_{ij} R_i R_j \tag{1}$$

where R is the combined response; R_i and R_j, responses in the i^{th} and j^{th} modes, respectively; ε_{ij} represents the correlation between the two modes.

For modes with close frequencies, Rosenblueth and Elorduy (Ref. 1) have suggested an expression for ε_{ij} assuming that the earthquake is a finite segment of white noise. Several workers have suggested methods for accounting for correlation between modes at higher frequencies. A spectral frequency at which the spectral acceleration becomes almost equal to the maximum input acceleration plays an important role in these methods, and is often designated as the rigid frequency, f^r. Theoretically, the

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spectral acceleration should be exactly equal to the maximum input acceleration at the infinite oscillator frequency, i.e, at the zero oscillator period. The maximum input acceleration in the context of the response spectrum is, therefore, also called the zero period acceleration or the ZPA. It is generally accepted that the responses in modes having frequencies greater than the rigid frequency are almost perfectly correlated with the input acceleration history, and therefore are also mutually perfectly correlated. The rigid frequency in the USNRC Regulatory Guide 1.60 (Ref. 2) is 33 Hz. Therefore, Kennedy (Ref. 3) has suggested that all the modes having frequencies greater than 33 Hz be combined algebraically.

Various forms of relative response spectra based procedures have been proposed by Lindley and Yow (Ref. 4), Hadjian (Ref. 5), and by Singh and Mehta (Ref. 6). Lindley and Yow (Ref. 4) perform a static analysis using the ZPA, and the usual modal analysis using a relative acceleration response spectrum, ordinates of which are square root of the difference of the squares of the regular spectral ordinate and the ZPA. Hadjian (Ref. 5) essentially does something similar except that his relative response spectrum is obtained by directly subtracting ZPA from the regular spectral accelerations. Kennedy (Ref. 7) has pointed out that this procedure leads to an inconsistency in the method and suggests a modification which would make Hadjian's method very similar to Lindley's and Yow's. Singh and Mehta (Ref. 6) formulate the problem using the modal acceleration technique and suggest making use of relative velocity and relative acceleration spectra. In the present practice these spectra are not readily available.

Among the methods proposed by Kennedy (Ref. 3), Lindley and Yow (Ref. 4) and Hadjian (Ref. 5), Lindley and Yow's method appears to be most rational and is likely to give most accurate results for a structure having frequencies in the neighborhood of or greater than the rigid frequency. The method is likely to run into trouble for modes having frequencies significantly lower than the rigid frequency. Even for frequencies immediately below the rigid frequency, the method of calculating the relative spectral acceleration is somewhat arbitrary.

A new consistent method of combining modal responses has been developed by the writer and his coworkers at the North Carolina State University, and is presented in this paper.

THE NEW METHOD

The development and the application of the new method is detailed in Refs. 8, 9 and 10. The major issue addressed to is the correlation between modal responses at higher frequencies. It is widely accepted, and is true that the modes having frequency greater than the rigid frequency are almost perfectly correlated with the input acceleration history, and therefore, are also mutually perfectly correlated. But what happens at a frequency immediately below the rigid frequency? We call here the correlation between a modal response and the input acceleration as the rigid response coefficient, because at higher frequencies the response is considered "rigid" and the correlation is unity. Figure 1 shows the variation of rigid response coefficient for the San Fernando earthquake (Holywood Storage, 1971, EW). The rigid response coefficient becomes

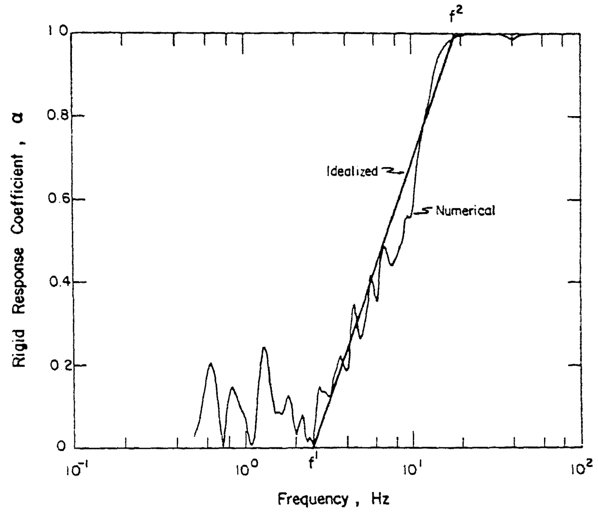

Fig. 1 - Rigid Response Coefficient for San Fernando Earthquake

almost unity at about 20 Hz, which is much less than rigid frequency which is about 30 Hz. Below 20 Hz the factor shows a gradual diminishing trend, and becomes zero at about 2.5 Hz. This means that even the modes having frequencies below the rigid frequency have a "rigid content". This rigid part is given by

$$R^r = \alpha_i \ R_i \tag{2}$$

where, α_i is the rigid response coefficient. The remaining part is designated as the damped periodic part, R_i^p, and is assumed to be statistically independent of the rigid part. Therefore,

$$R_i^p = \sqrt{(R_i)^2 - (R_i^r)^2} \tag{3}$$

The statement of Eqs. (2) and (3) above immediately leads to the new combination method. Since the rigid parts are all perfectly correlated, they are suumed algebraically

$$R^r = \sum_i R_i^r \tag{4}$$

The damped periodic parts are combined using the standard double sum equation

$$(R^p)^2 = \sum_i (R_i^p)^2 + 2 \sum_i \sum_{j>i} \varepsilon_{ij} R_i^p R_j^p \tag{5}$$

The total response becomes

$$R = \sqrt{(R^r)^2+(R^p)^2} \tag{6}$$

In Eq. (5) ε_{ij} represents the correlation between the modes due to closeness of frequencies. As indicated earlier, Rosenblueth and Elorduy (Ref. 1) have suggested an expression for ε_{ij} assuming that the earthquake is a finite segment of the white noise. Gupta and Cordero (Ref. 8) have modified the expression using actual earthquake records. The modified expression is

$$\varepsilon_{ij} = \left\{1 + \left[\frac{f_j - f_i}{\zeta(f_j+f_i)+c_{ij}}\right]^2\right\}^{-1} \tag{7}$$

in which

$c_{ij} = (1-3\zeta)(.036-|f_j^2 - f_i^2|)>0$

f_i, f_j = frequencies of modes i and j, respectively, Hz

ζ = critical damping ratio

The only difference between Eq. (7) and the similar equation in Ref. 1 is in the definition of the term c_{ij}. In Ref. 1, it is a function of the duration of the white noise, a quantity which is difficult to specify for the real earthquakes. The writer, therefore, prefers the form of Eq. (7).

Rigid Response Coefficient

The rigid response coefficient can be numerically evaluated if the earthquake time history is known. In many practical applications it is not so. As shown in Fig. 1, the numerically calculated rigid response coefficient can be idealized by a straight line on the semi-log graph. The idealized equation is given by

$$\alpha_i = \frac{\log f_i/f^1}{\log f^2/f^1}, \quad 0\leq\alpha_i\leq 1 \tag{8}$$

The key frequencies f^1 and f^2 can be expressed as

$$f^1 = \frac{S_{amax}}{2\Pi\, S_{vmax}}, \text{ Hz}; \quad f^2 = (f^1 + 2\, f^r)/3, \text{ Hz} \tag{9}$$

In Ref. 9, Eq. (8) has been applied to 11 earthquakes ground motions and 3 instructure motions, and good correlation between the idealized equation and the numerical values is shown. A different equation for f^2 is used in Ref. 9 than the one written above, Eq. (9). Equation (9) is simpler to use and gives almost same values of f^2 as Ref. 9.

An Alternate Method

It is shown in Refs. 9 and 10 that one only needs to calculate responses in modes having frequencies upto the rigid frequency. Responses in all the modes beyond that can be lumped into one residual rigid response, designated by R_o. The Eq. (9) is changed to

$$R^r = R_o + \sum_i R_i^r \qquad (4a)$$

All the other steps remain the same as before.

APPLICATION

The method was applied to analyze five 3 degrees of freedom system, each subjected to three actual earthquake ground records and three instructure calculated motions. The results are reported in Ref. 9. The five buildings have fundamental frequencies from 2 to 64 Hz. A comparison of the time history results with those from the response spectrum method (using the new modal combination rule) is shown in fig. 2 for the San Fernando earthquake. The dashed lines show the response spectrum results only when they are not superimposed by the time history results shown by the solid lines. The new combination rule does give responses which are quite close to the time history method. Story displacements were not compared because those were dominated by one mode. As such, it did not matter how the modes were combined.

The method has also been applied to a rather large piping system and is reported in Ref. 10. Again, the response spectrum results combined by the new method give results which are quite close to the time history results.

CONCLUSIONS

The standard SRSS modal combination does not account for the possible correlation between modal responses. The modal responses may be correlated either when frequencies are closely spaced or when the frequencies are in the higher range. For modes with closely spaced frequencies, the writer recommends a modified version of the equation originally suggested by Rosenblueth and Elorduy. The new equation is based on actual earthquake records and does away with the earthquake duration term.

A number of techniques have been proposed in recent years to account for correlation between modes at higher frequencies. The writer found that even modes with a range of frequencies immediately below the rigid frequency continue to be perfectly correlated with the input acceleration. The correlation tends to diminish gradually, subsequent to that. The correlation coefficient between the modal response and the input acceleration is designated as the rigid response coefficient. A new method is proposed in which the modal response is repre-

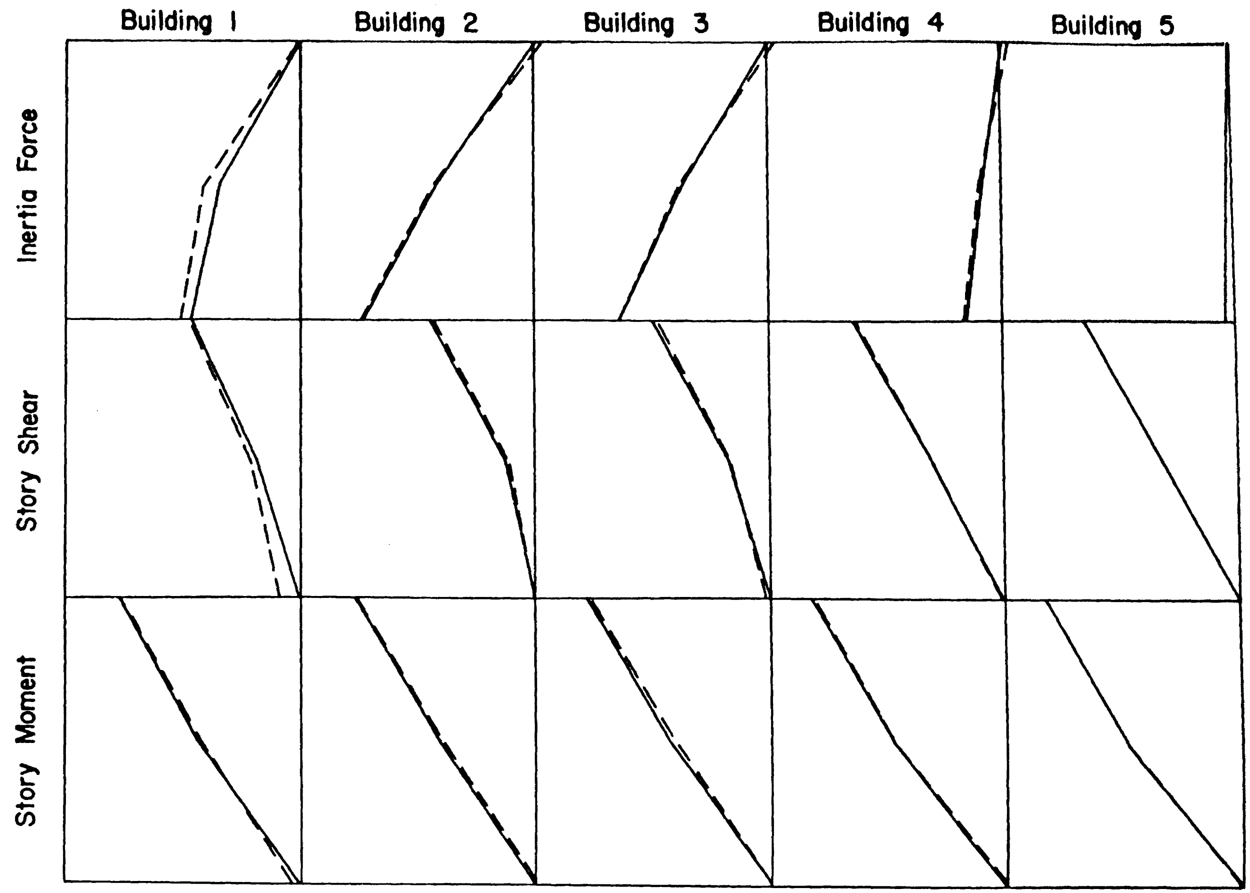

Fig. 2 - Comparison of Time History and Response Spectrum Results

sented by two mutually statistically independent parts, the rigid part and the damped periodic part. The rigid parts from various modes are summed algebraically. The damped periodic parts are combined by the SRSS equation.

The method has been applied to several problems and good agreement has been obtained with the time history results.

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